Addiction to a Network*

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Abstract

We characterize dynamic rational addiction to a harmful product by informed individuals who are connected to a network of users of the addictive product. The network's accumulated stock of consumption harms each individual and imposes peer pressure on her to consume the addictive product. When harm is concave in aggregate stock, an increase in the network intensifies addiction, and when it is sufficiently convex, a larger network mitigates addiction. "Rehabilitation", achieved by total or partial disconnection from the network, can prevent addiction if implemented early enough . The results support regulation of social media platforms' practices encouraging expansion of the individual's network and use.

Keywords: network, addiction, peer pressure, social interactions, social network sites, social platforms, Facebook, Instagram, TikTok, rehabilitation, dynamic programming.

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1 Introduction

It is well documented in the psychological literature that people smoke, consume alcohol, and abuse elicit drugs more when around other people who do so and that direct or indirect peer pressure by the others is one of the main causes of initiation of consumption, increased consumption, and relapse after trying to quit (Larsen et al (2012); Bassiony (2013); Etcheverry and Agnew (2008); Bhad et al (2016); Mizanur et al (2016); Dimoff and Sayette (2016); Tikoo et al (2017); Lin et al (2017); Edwards et al (2017) Ramji et al (2019)).[1]

Similarly, psychological studies have found that individuals start using social media platforms, such as Facebook and Instagram, and find it hard to quit, due to indirect peer pressure and the fear of missing out (Subrahmanyama et al (2008); Pelling and White (2009); Kieslinger (2015); Abel et al (2016); Juergensen and Leckfor (2019); Blanca and Bendayan (2018); Tomczyka and Selmanagic-Lizdeb (2018); Pontes et al (2018); Liu and Ma (2018); McCrory et al (2022)). The fear of missing out is defined by these studies as the "feeling that others may be having rewarding experiences from which one is absent, characterized by the desire to stay connected with what others are doing" (Liu and Ma (2018); Przybylski et al. (2013)). This literature has also shown that excessive use of a social media platform is worse when the group-norm about the importance of the social network is stronger (Marino et al (2016)).

Indeed, in recent years, extensive psychological literature shows that many surveyed individuals develop symptoms that have the attributes of addiction to social media platforms, such as Facebook, Instagram and TikTok. The psychological literature shows that this behavior harms users and has the same attributes as ordinary addiction to chemical substances. In particular, such compulsive use of social media platforms has been shown to be associated with salience (permanent thinking about use), tolerance (increased consumption is required to reach previous utility levels) relapse (reverting to a pattern of earlier use after ineffective attempts to reduce consumption), withdrawal (becoming stressed when trying to avoid consumption), social overload due to excessive use, negative mood, depression, lower self-control, anxiety, insomnia, stress, envy, impair-

[1] With respect to smoking, for example, Malhorta et al (2009) find that "Nicotine users reported peer pressure as a single most important cause for initiation; however after a period of use nicotine withdrawal preempted them from stopping its use."

ment of offline activities, damage to relationships at work and at home (attributes also known as "conflict"), and, in extreme cases, suicide (Elphinston & Noller, (2011); Kuss & Griffiths, (2011); Andreassen et al, (2012); Maier et al (2012); Wilcox and Stephen (2013); Sagioglou & Greitemeyer, (2014); Andreassen (2015); Kircaburun and Griffiths (2018); Abbasia and Drouinb (2019); Marengo et al (2020); Brailovskaia et al (2020); Lemert (2022)). Economists have recently supported the psychological literature with randomized experiments (Allcott et al 2022).

It is well documented that, alongside various benefits, social media platforms cause harm. Allcott et al (2020)'s randomized experiment including 2,743 participants shows that social media users with intensive use would rather decrease their use of the social media site in order to improve their well-being. They find that deactivation of Facebook for four months is equivalent to added income of $30,000. Braghieri et al (2022)'s quasi experiment using Facebook's staggered rollout in US colleges finds that Facebook impaired college students' mental health and estimate that Facebook explains 24% of the increased prevalence of severe depression among college students over the last two decades. They show these mental health consequences to be higher the longer the college student was exposed to Facebook. Rosenquist et al (2022) cite internal Instagram and Facebook studies documenting such harm, particularly with respect to teenagers.[2] Harm to the individual by the social media platform has been shown to increase over time (Brailovskaia and Margraf (2017); Brailovskaia et al (2018))). Indeed, Mark Zuckerberg's published "Blueprint for Content Governance and Enforcement" reveals that user engagement peaks when the content is just barely allowed and when users' ex-post reported well being concerning it is negative.[3] It has been reported that Facebook's algorithm routing content to the user's feed places five times more weight on responses including angry emojies than on ordinary "likes" (Omerus et al (2021)). McCrory et al (2022)'s survey finds that features such as likes and infinite scrolling were associated with negative emotions. Similarly, Fan et al (2014) show that in Weibo, the largest Twitter-like service in China, anger by one user provokes anger by connected users more than joy provokes joy.

Shensa et al (2017) show, in a U.S. nationally representative sample of 1749 young

[2]Furman et al (2019) and Crémer et al (2019) report harm caused by social media platforms to the privacy of users.

[3]See Lemert (2022), and https://www.facebook.com/notes/751449002072082/.

adults aged 19 to 32, that 44% of U.S. young adults have symptoms of mild addiction to social media platforms and 14% have symptoms of more severe addiction. Reer et. al. (2020) show, in a nationally representative sample of 1929 German internet users aged between 14 to 39, that 2.6% of them suffer from disorders associated with addiction to social media platforms. De Cock et al. (2014) had similar findings, of 2.9% of internet users aged 18 and above in a nationally representative Belgian sample. Banyai et al (2017) find, in a nationally representative sample of adolescents in Hungary, that 4.5% of them were at risk of addiction to social media platforms. As Lemert (2022) reports, psychiatric experts on technology addiction estimate that 5% of all Facebook users (i.e., approximately 11.6 million Americans) experience addictive use. Also, studies have shown that adolescents spend 20% to 30% of their waking hours on digital platforms (Twenge et al 2019; Rosenquist et al 2022). As of November 2022, it has been reported that Facebook alone has 2.96 billion users and the 2022 figure for Instagram was 1.28 billion users.[4] Tiktok was reported being one of the world's fastest-growing social media platforms, with more than one billion active users worldwide.[5] By 2022, social media platforms are used by more than half of the world's population, and the average user spends around two and a half hours per day on social media platforms (McCrory et al (2022); Braghieri et al 2022).

In accordance with these empirical findings, in our framework consumption by the other members of the network harms the individual.[6] We assume the individual has no control over her connection to the network. For example, although an individual could physically disconnect from Facebook or Instagram, she cannot disconnect from her friends. Knowing that these friends will go on using Facebook or Instagram without her, the effects of the network on her (stress, envy, the fear of missing out, etc.) are assumed to persist.[7] Other examples of networks the individual must take as given

[4] See https://www.oberlo.com/statistics/how-many-users-does-facebook-have#:~:text=According%20to%20the%20latest%20data,users%20worldwide%20is%202.96%20billion; https://www.oberlo.com/statistics/how-many-people-use-instagram#:~:text=In%202022%2C%20the%20 number%20of,1.

[5] Anderson v. Tiktok, Inc. (United States District Court for the Eastern District of Pennsylvania), 2022 U.S. Dist. LEXIS 193841.

[6] Our framework could easily be extended to cases where harm from the network begins (or outweighs the benefit) when reaching some threshold of exposure to the network's use. This could capture cases in which moderate consumption (e.g., of alcohol or of using social media platforms) does not involve net harm to the individual.

[7] Our results would not qualitatively change if the individual is connected because she derives a large enough fixed benefit from being connected (or large enough fixed cost of disconnection). This benefit or cost notwithstanding, in our model the individual still prefers the network's consumption to be as

are adolescent school-mates, or members of the same household, who smoke, consume alcohol, consume drugs, or develop obesity, together.

We assume that the network exerts peer pressure on the individual to consume the addictive product. Peer pressure can involve direct pressure imposed on the individual to consume the product. Alternatively, peer pressure could be indirect: it may be the individual that feels pressured to consume the harmful product due to the fear of missing out. More precisely, in our model, each individual in the network decides on the quantity she consumes of a non-addictive product (e.g., offline activity) and an addictive product (e.g., use of the social media platform), both of which are assumed to give the individual current benefit. We model peer pressure by assuming that the higher the aggregate stock of consumption by the whole network, the lower the individual's marginal utility from consuming the non-addictive product rather than the addictive product. The effect of each network member i on the aggregate stock affecting another network member j could differ among individuals.

Hence the network in our framework can harm the individual either directly (as in the case of passive smoking or a social media platform exhausting the individual's time or invoking envy or frustration) or indirectly, via the peer pressure. For example, an adolescent who goes to a bar with her friends may individually prefer a soft drink to alcohol. Yet her friends' peer pressure reduces her marginal utility from the soft drink, so she consumes alcohol instead. Absent the network, she would have consumed the soft drink and derived a larger benefit.

In our model, it is the aggregate accumulated stock of consumption by the network that affects the individual and not only the peers' current or lagged consumption. Indeed, most of the psychological literature cited above shows that peer pressure is not caused only by the peers' current consumption of the addictive product, but also by the network's accumulated past consumption. In the case of social media platforms such as Facebook and Instagram too the psychological literature documenting the fear of missing out and peer pressure as a driver of use implies that this fear is increasing in the amount of accumulated use by others in the network, and does not hinge only on contemporary use (Turel and Osatuyi (2017)). For example, the larger the number

low as possible, so as to reduce the harm such consumption causes the individual. Take, for example, an individual who wants to be connected to a network of smoking bosses and colleagues at work, so as to keep her job or be promoted, yet she prefers that the quantity of smoking be reduced to zero.

of posts and stories the individual's friends have posted, the more the individual experiences the fear of missing out if she does not participate too. Our assumption that it is the past accumulated stock of the network's consumption that affects the individual also draws support from Arduini et al (2019)'s empirical study of high school students. Their data includes the amount of use of cigarettes or alcohol by an adolescent's close friends in previous years and they find that the parameters generated by this data affect the adolescent's current use.[8] Also, the harm that the network inflicts on the individual is more reasonably assumed to increase in the accumulated consumption of the network.

Our objective is to characterize equilibrium behavior of network members in a dynamic setting. Individuals in our model are strategic players, rational and informed. We wish to show under what circumstances a rational and informed individual can find herself addicted to a harmful product, when she is part of a network of other individuals who consume the product and exert peer pressure on her to consume. This can be a plausible cause for addiction in network settings (e.g., addiction to a social media platform) that may be extremely important. The underlying question is, will a rational individual reduce her own consumption of the addictive product, to counteract the future harm from the network's accumulated stock, or will the individual join in and consume more of the addictive product, due to the network's peer pressure? Which of these two opposing forces dominates?

We study addiction in a network setting under two scenarios. The first assumes that the individual's own consumption does not affect other members of the network. This case captures situations in which the individual does not play a pivotal role in the network, e.g., because she is unpopular or does not carry much weight within the network. This part of our analysis also captures situations where the network is large, as is the case of addiction to Facebook, Instagram, or TikTok, or smoking in a crowd. Accordingly, the individual assumes that the other members of the network are in a steady state, contributing a constant level of consumption per period.

A second scenario we explore is that of a network with strategic players. We characterize and study an open loop equilibrium (OLE), in which each individual in the

[8] Similarly, Larsen et al (2012) show that alcohol consumption of an individual is increasing in the size of her peers' consumption. See also Cutler and Storm (1975). Another example is obesity, where the literature shows that the chances of becoming obese rise when close peers are obese (e.g., Fowler and Christakis (2008)), and here too it is the accumulated consumption that matters.

network is assumed to know the initial state of aggregate stock but she does not observe the subsequent states. Hence, at the outset of the game, each individual chooses a sequence of consumption levels and commits to them. Each sequence of consumption levels is the best response to the sequences of the others.[9]

When discussing addiction by individuals who are not connected to a network, previous literature has dealt with either irrational or rational addiction. In the former strand, the individual does not weigh the product's virtues and harm in a rational way, and hence she becomes addicted (see, e.g., Winston (1980); Gruber and Koszegi (2001) and Gul and Pesendorfer (2007). The second strand of the literature assumes that the individual is rational, but nevertheless consumes an addictive product. Stigler and Becker (1977), and Becker and Murphey (1988)'s models of rational addiction imply that the individual is content with her addiction. In order to explain situations in which a rational individual regrets consuming the addictive product, Stigler and Becker (1977), and Becker and Murphey (1988)'s theory of rational addiction has been further developed by Orphanides and Zervos (1995), who assume that the individual is uninformed as to the harm she may suffer from the addictive product. Wang (2007) studies rational addiction by an individual who is misinformed regarding the level of consumption causing addiction and the level of consumption enabling the individual to quit successfully.

In contrast to this literature, in our paper the individual may consume an addictive product that harms her even though she is both rational and perfectly informed regarding future harm. This occurs when the individual is part of a network of other individuals who consume the addictive product, and is subject to peer pressure from the other members of the network.[10] Hence we offer an alternative explanation for initiation of consumption of a harmful and addictive product and of addiction to it that does not hinge on irrationality or misinformation about future harm or the ability to quit. Our framework also explains why rational and informed individuals initiate consumption

[9] In the online appendix, we show the existence of Markov Perfect equilibrium behavior, in which each individual's strategy relies on the current state of aggregate stock, in two extremes-of zero consumption or maximum consumption by the whole network. The online appendix is available at https://en-law.tau.ac.il/sites/law-english.tau.ac.il/files/media_server/Law/faculty%20members/David%20Gilo/addiction_network_online_appendix.pdf

[10] We do not claim that, in reality, most individuals involved in consumption of drugs, alcohol, or social media platforms are fully rational and informed. Yet we aim to show that even if we assume they are, they may find themselves in a harmful addiction. Naturally, assuming irrationality or under-estimation of harm would only exacerbate addiction in our framework.

of a harmful addictive product even absent a temporary stressful event, assumed to cause initiation in the Becker and Murphey (1988) framework. Allcott et al (2022)'s randomized experiment shows that individuals do not reduce their use of social media in a way that is consistent with their awareness of its addictive nature and they interpret this result as inattentiveness to self-control problems. Our theoretical result offers a rational foundation for Allcott et al (2022)'s finding, since in our model, although the individual is aware of the addictive nature of social media, peer pressure and the fear of missing out nevertheless induce her to use it extensively.

We show that being part of a network can encourage or discourage the individual to become an addict depending on whether the harm inflicted on the individual is concave or convex in the network's accumulated consumption. If the harm is concave, the concavity of harm and the peer pressure reinforce each other so that an individual who would have abstained from the addictive product absent the network may start consuming it and follow a consumption path that leads to her own addiction. Also, any increase in the network, its consumption, or the degree of influence among individuals causes addiction by the individual to be more severe, with larger consumption of the addictive product. This is consistent with Turel and Osatuyi (2017)'s empirical survey, showing that an observed increase in peers' use of a social media platform augments the individual's compulsive use. The reason for this result in our model is that the individual is between a rock and a hard-place: If she abstains, the network harms her anyway. She can mitigate this harm via current consumption, so she joins the rest of the network and consumes herself. If the harm inflicted by the network is concave in the accumulated stock of consumption, it is never the case that the individual restrains her own consumption in order to mitigate overall future harm.

Conversely, if the harm inflicted by the network is sufficiently convex, the network has a chilling effect on the individual's consumption. In such a case, being part of the network diminishes the individual's consumption of the addictive product. Here the future marginal harm inflicted by the network outweighs the peer pressure, so the individual counteracts the network's harmful consumption by mitigating her consumption. Furthermore, for a sufficiently convex harm function, adding a new member, increasing an influence parameter, or increasing a member's initial stock, reduce equilibrium consumption of all network members.

Although the network can cause and intensify addiction, there is always a welfare-maximizing consumption-less equilibrium in our framework, as long as members' initial stock of consumption is sufficiently small. Along-side this equilibrium, though, there are equilibria with consumption, including full consumption (of spending the individual's entire income on the addictive product). This implies that individuals can benefit from organizations such as Alcoholics Anonymous or weight loss groups, or appropriate guidance from teachers, which help them coordinate on the consumption-less equilibrium. Our focus on a network's accumulated past consumption as a factor that can induce the individual to become an addict also sheds a new light on the question of rehabilitation from the addictive product. One of the characteristics of harmful addictions that the rational addiction literature dealing with an individual not connected to a network has not yet been able to explain is the incidence of rehabilitation efforts (i.e., seeking external help to eliminate or reduce addiction). In particular, if an individual is content with her addiction, as in Stigler and Becker (1977) and Becker and Murphey (1988), then she would never want to go to rehab. In Orphanides and Zervos's (1995) framework of an uninformed individual who becomes addicted and regrets it, there is again no reason for rehab. In their model, if the individual becomes aware of her harmful addiction too late, she continues consuming the addictive product and does not want to go to rehab. If she grasps the situation early enough, on the other hand, she stops consuming the addictive product on her own and does not need rehab. Similarly in Wang (2007), the individual may be able to quit on time on her own, and if not, she will not want to go to rehab.

In our framework, in which there is a network whose accumulated stock of consumption pressures the individual to consume the addictive product, rational efforts to try to do something in order to free oneself from the addictive product become meaningful. In particular, the individual can rationally try to seek intervention that reduces the harm the network inflicts on her. Given that the individual has no control over the network's behavior, the individual may well require some external intervention or commitment mechanism that would help free her from the network's grasp. For example, a teenager that was induced to consume drugs or alcohol due to pressure from her network of friends may rationally prefer that some agency forbid her friends from consuming, or even from approaching her. Indeed, in the context of smoking, Dimoff and Sayette (2016) discuss disconnection from one's network of smoking friends as a means of helping a smoker quit smoking. Similar methods have been suggested in the context of addiction to alcohol by adolescents (Teunissen et al (2014); Kremer and Levi (2008)). Short of disconnection from the network, we show that a reduction in its size, or its consumption, can help the individual avoid addiction and mitigate the harm.

In the case of harmful addictions to social media platforms too, the individual may rationally prefer that some intervention be used to help her mitigate the harm inflicted by her peers' use of the network. Unlike disconnecting an adolescent from a network that uses drugs or alcohol, physical disconnection from a social media platform would not necessarily help, since the fear of missing out would persist. Still, the individual in our framework would benefit from external intervention (e.g., via regulation), that limits the nature and quantity of her friends' content she is exposed to.

In the former cases, where physical disconnection from the network is a viable option, we show that the question whether rehab, in the form of disconnection from the network, is effective depends on it's timing. If it is applied to the individual after aggregate stock passes a certain critical level, it is too late. Even if the individual is disconnected from the network, she goes on a binge and continues consuming the addictive product on her own. On the other hand, if such rehab is timed before aggregate stock passes this critical level, it succeeds in releasing the individual from the addictive product: the individual goes "cold turkey" and converges to a lower state of consumption of the harmful product. This is consistent with Malhorta et al (2009)'s finding that when the individual is disconnected from smoking peers too late, she typically continues smoking due to nicotine withdrawal.

Our formal results support legal policy papers (e.g., Rosenquist et al 2022; Griffin 2022; Langvardt (2019)) calling for regulation of social media platform's practices of pushing a user to expand her network or her usage of the network. Facebook, for example, has a generous ceiling, of 5000, on the number of friends an individual can have.[11] Instagram allows a user to follow up to 7500 other individuals,[12] and it was reported that TikTok's threshold is 10,000.[13] Within these generous ceilings, social media platforms consistently encourage individuals to expand their networks. All three networks have no limit on the number of followers a user, or a user's page, have.

[11]See https://www.facebook.com/help/211926158839933/?helpref=uf_share

[12]See https://help.instagram.com/408167069251249/?helpref=uf_share

[13]See https://www.itgeared.com/how-many-people-can-you-follow-on-tiktok/#:~:text=Unfortunately%2C%20TikTok%20 restricts%20the%20number,than%2010%2C000%20accounts%20in%2

Moreover all three networks do not limit the volume of content the individual's friends expose her to. Much to the contrary, social media platforms have been consistently using practices and algorithms that encourage excessive use. For example, they all feature "infinite scrolling", in which, without changing screens or having to click, the individual can endlessly scroll down content she is exposed to by others with no limit. Griffin (2022) surveys testimonies of industry experts according to which social media platforms use practices such as noisy and alerting notifications, "rewards" and invitations to react such as the "like" buttons, comment boxes, and "pull to refresh" buttons, designed to resemble slot machines, and proven by psychologists to encourage individuals to connect while exercising compulsive behavior.[14] As reported by Lemert (2022), a former Facebook executive revealed that "the thought process [behind Facebook's business model] was all about, "[h]ow do we consume as much of your time and conscious attention as possible?" and quotes Facebook's experts concluding that "we need to ... give you a little dopamine hit every once in a while, because someone liked or commented on a photo or a post ... and that's going to get you to contribute more content, and that's going to get you more likes and comments. It's a social validation feedback loop." As Neyman (2017) stresses, the "like" and analogous features build on individual's craving to receive social gratification.

While legal policy papers advocating for regulation of these practices take a behavioral approach and assume individuals become addicted because they are not informed or not rational, our results show that harmful addiction can occur even when the affected individual is completely rational and informed about the risk. This can further reinforce the case for regulation and copes with possible allegations from social media platforms that it is supposedly unreasonable to assume users are irrational and uninformed about the risk, given the salience of the possibility of addiction to social media platforms and the associated harm. Our results further imply that it may not be enough to change default features, as suggested by some of the current legal literature and proposed legislation, such as disabling infinite scrolling of content or frequent notifications about current and potential friends or limiting the duration of possible per-day use, while allowing the

[14] See, e.g., Hilary Andersson, Social Media Apps Are 'Deliberately' Addictive to Users, BBC NEWS (July 4, 2018), https://www.bbc.com/news/technology-44640959 [https://perma.cc/3WKV-9NCF]; Langvardt (2019).

individual to opt-in and restore such harmful features.[15] The reason is that due to the network's peer pressure, the individual may well opt-in and restore problematic features. A similar policy implication of our results is that merely making the individual aware of the risk of addiction, or warning her of her particularly extensive use (as offered for example by Nikbin et al (2020)) may not be helpful. In our framework, the individual is fully aware of the future negative repercussions stemming from excessive consumption of the addictive product. Yet she may consume excessively and become an addict due to the network's peer pressure. Similarly, the extensive regulations requiring suppliers of alcohol and cigarettes to disclose the dangers from consumption may not be effective if the individual is exposed to a network that consumes alcohol or smokes.

Another subtlety our analysis reveals is that in cases where the harm inflicted by the network is sufficiently convex in the network's consumption, increasing one's network, consumption stock, or the influence parameters actually reduces equilibrium consumption. Yet by revealed preference, if a social media platform such as Facebook, Instagram or TikTok had exerted efforts to expand individuals' networks and use, this implies that this social media platform understands the harm function not to be so convex so as to induce individuals not to use the network when it expands: this would harm the platform's profits. Accordingly, in this sense too the well-known efforts exerted by social media platforms to expand the network and its use can be used as evidence further supporting regulation. In response to public criticism, social media platforms have introduced features the user can choose to limit hours of use.[16] Yet given the network's peer pressure pushing the individual to expand her consumption, these modest efforts may not be enough to prevent addiction. These features are usually turned off by default and are difficult to find (Langvardt (2019)). In any case, all they do, if activated by the user, is make her aware of the time she had spent on the platform.[17] Our model demonstrates though that such awareness will not suffice.

Similar reasoning suggests that, at least in extreme cases of harmful addiction by an individual, ex-post tort or antitrust liability for the harm caused by the network's efforts to expand the individual's exposure should be considered. As legal scholars have

[15]For such legislative initiatives see Griffin (2022).

[16]See, e.g., Andersson 2018.

[17]See, e.g., https://m.facebook.com/help/www/1737706169659354; https://help.instagram.com/2049425491975359; https://support.tiktok.com/en/account-and-privacy/account-information/screen-time.

claimed, strong analogies can be drawn between tobacco companies inducing addiction to cigarettes and social media platforms inducing addiction to their networks (e.g., Rosenquist et al 2022; Griffin 2022; Lemert 2022). Just as ex-post tort liability of tobacco companies has been sought, similar tort liability could be considered in the case of social media platforms. Similarly, since the allegation is that these social media platforms have designed their product in a way that stimulates harmful addiction, liability related to product safety laws could be considered. In recent years, dozens of product liability suits across the U.S. were brought against Meta (the parent company of Facebook and Instagram), TikTok, Snapchat, and other social media platforms.[18] All of these suits were consolidated and transferred to the Federal district court in Northern California, holding that all of these suits "present common factual questions arising from allegations that defendants' social media platforms are defective because they are designed to maximize user screen time, which can encourage addictive behavior in adolescents. ... including whether Meta's platforms (Facebook and Instagram) encourage addictive behavior ... and inadequately safeguard against harmful content and/or intentionally amplify harmful and exploitative content."[19] Although the U.S. Congress has enacted, in 1996, section 230 of the Communications Decency Act,[20], which grants immunity from liability to a website platform for content created by third parties, the claim in these suits is that there is no such immunity for the actions of the platform itself that could encourage or intensify harmful addiction to the platform. Our results according to which an increase by the network and its usage harms the individual and can create or intensify addiction can support such legal implications.

Furthermore, social media platforms such as Facebook and Instagram are allegedly dominant in their relevant markets. In antitrust jurisdictions in which a dominant firm is liable not only for conduct excluding rivals, but also for conduct directly exploiting consumers, such as the EU, an antitrust claim could be brought, at least in extreme cases, according to which Facebook or Instagram had designed their services in a way that makes them unsafe and stimulates harmful addiction. Moreover, when a social media platform encourages addiction to its network, this also tends to exclude rivals,

[18]See In re Soc. Media Adolescent Addiction/Personal Injury Prods. Liab. Litig., 2022 U.S. Dist. LEXIS 227736.

[19]See id.

[20]47 U.S.C. §230.

in the sense that it exacerbates a network effect that causes the market to tip into monopoly in the first place. This exclusionary effect can, in principle, constitute an antitrust violation also in jurisdictions such as the U.S., in which there is no antitrust liability for direct exploitation of consumers.

Our paper contributes to the sparse literature that combines habit formation by forward-looking individuals with social interactions. Bisin et al (2006) study an equilibrium of habit formations and social interaction in which an individual's utility is affected by her neighbor's current actions and has disutility from changing habits. Ozgur et al (2017) extend Bisin et al (2006) and study a linear setting with random preference shocks. They assume that the individual has disutility from deviating from peers' current preference shocks, as well as from the individual's consumption in previous periods. Abel (1990) studies individuals affected by their peers' previous period average consumption as well as their own previous period consumption. He focuses on the equity premium puzzle. Reif (2018) studies addiction by an individual with quadratic utility who may be affected in a symmetric way by current actions of her peers . His model focuses on positive network effects, causing the individual to consume too little. Binder and Pesaran (2001) study individuals who are affected by their last-period decisions and have a taste for conformity to previous period's average behavior by others. Their focus is on consumption-savings.

Our paper differs from this literature in various ways. First, in our model accumulated consumption by the network directly affects each individual's state of aggregate stock. We believe this is an important avenue to study since, as noted, the psychological and empirical literature on peer pressure to consume addictive products usually implies that it is the peers' accumulated stock of consumption that matters. This includes the phenomenon of excessive use of social media platforms, where the psychological literature implies that the fear of missing out driving such excessive use is increasing in the accumulated stock of consumption by the rest of the network. Moreover, unlike previous literature, we study the case of a harmful product and of a network's aggregate stock that harms the individual. Also, our model utilizes the possible multiplicity of steady states and equilibria to study the effect of rehabilitation (i.e., disconnection from the network) on binge and cold turkey behavior. Another difference between our paper and previous literature is that we examine how a change in the network, such as the addition of new members, or a change in members' initial stock or the influence matrix, affect the equilibria of the strategic game. This has policy implications for instances in which an entity influencing network size and use, such as a social media platform, utilizes this influence to encourage addiction. To the best of our knowledge, ours is the first theoretical paper to model how actions of social media platforms affect addiction.The rest of the paper is organized as follows. Section 2 describes the model. Section 3 studies the case of a constant network and Section 4 studies the strategic game. Section 5 discusses the case where sufficiently convex harm can eliminate addiction. Section 6 summarizes our policy implications and Section 7 concludes.

2 Model

Consider n individuals who form a network, individuals $i = 1, 2, ..., n$. In each period, individual i can consume two products: the non-addictive product, c, and the addictive product, a. The n individuals are connected to the network at time $t = 0$. Time is discrete and there is an infinite number of periods. In each period t individual i consumes a quantity c_t^i of product c and a quantity a_t^i of product a. The aggregate stock of product a, the addictive product, affecting individual i until the beginning of period t, is s_t^i. Accordingly:

$$s_{t+1}^i = \delta s_t^i + \sum_{j=1}^{n} \gamma_{ij} a_t^j \tag{1}$$

Where $\gamma_{ij} \geq 0$ is a parameter depicting how consumption by individual j (including individual i herself) affects individual i's utility, through its effect on i's aggregate stock. The weights γ_{ij} form the influence matrix $\varGamma \equiv (\gamma_{ij})$. For example, a particularly popular Instagram celebrity j is expected to have a large γ_{ij} for all $i \neq j$.[21]. We assume that the network is well connected in the following sense: for every two individuals i, j, if $\gamma_{ij} = 0$ then there exist k individuals $l_1, ..., l_k$ such that $\gamma_{il_1}, \gamma_{l_1 l_2}, ..., \gamma_{l_k j} > 0$, i.e. j influences l_k, which in turn influences l_{k-1} and so on, until l_1 who influences i. Hence, indirectly, every two individuals influence each other.

The contribution to the aggregate stock of individual i at time t by all others is de-

[21]Furthermore, $\gamma_{ii} = 1$, so the individual's own consumption enters her aggregate stock as is

noted by $\xi_t^i = \sum_{j \neq i} \gamma_{ij} a_t^j$. In Facebook, for example, the accumulated stock each individual is exposed to is placed in her "feed", using the infinite scrolling feature mentioned in the introduction, which includes all content and updates from her friends and from pages she follows. The content posted by the individual is placed in her profile, which includes a time-line in which the individual can share, on an ongoing basis, posts, text, photos and videos, with her friends and followers.[22] Instagram has similar features, such as the "feed" (with the infinite scrolling) and the profile, and it also features the "story", in which content expires after 24 hours, though the individual can store this content permanently by including the content in the "highlights" feature.[23]

According to equation 1, the state of the game is an n dimensional vector comprised of the aggregate stocks of all n individuals and is denoted by $\underline{s}_t = [s_t^1, s_t^2, ..., s_t^n]^T$. Thus, the evolution of this n dimensional state in matrix notation is:

$$\underline{s}_{t+1} = \delta \underline{s}_t + \Gamma \underline{a}_t \tag{2}$$

Where $\delta \in (0, 1)$ is the past consumption's dissipation factor and $\underline{a}_t \equiv [a_t^1, a_t^2, ..., a_t^n]$.

Each individual derives current positive benefit from the addictive product, but at the same time she suffers harm from the aggregated stock accumulated from past consumption by the individual and the other members of the network, s^i. All individuals have the same utility function and the utility of individual i in period t is denoted $u(c_t^i, a_t^i, s_t^i)$. Furthermore, the network inflicts peer pressure on the individual, which reduces the individual's marginal utility from product c, the non-addictive product ($u_{13} < 0$).

Each individual has a fixed income per-period, y. We shall normalize the price of product c to 1 per unit and assume, for simplicity, that the "price" per unit of product a, the addictive product, is also 1.[24] All individuals discount future utility by $\beta \in (0, 1)$. Accordingly, the problem faced by individual i when the initial stock vector is $\underline{s}_0$ is:

[22] See https://www.facebook.com/help/396528481579093/?helpref=hc_fnav.

[23] See https://help.instagram.com/381013822382269/?helpref=uf_permalink&parent_cms_id=1986234648360433 (for the "feed"); https://help.instagram.com/110121795815331/?helpref=uf_share (for the profile); and https://about.instagram.com/blog/announcements/introducing-instagram-stories; https://about.instagram.com/blog/announcements/introducing-stories-highlights-and-stories-archive (for story and highlights).

[24] The price the individual pays for the addictive product need not be monetary. For example, in the case of addiction to social media platforms such as Facebook or Instagram, although the individual does not pay in monetary terms, she can be assumed to pay by devoting time or privacy. We assume, for concreteness, that this sacrifice is deducted from the individual's per-period income, just like a monetary price.

$$max \left\{ (1-\beta) \sum_{t=0}^{\infty} \beta^t u(c_t^i, a_t^i, s_t^i) \right\} \quad (3)$$

$s.t.\ c_t^i + a_t^i \leq y,$

$c_t^i, a_t^i \geq 0,$

$s_{t+1}^i = \delta s_t^i + \sum_{j=1}^n \gamma_{ij} a_t^j$

Where we multiply the stream of utility by $1-\beta$ to obtain the average utility per-period that individual i obtains. We make the following assumptions:

Assumption 1: The function $u(c_t^i, a_t^i, s_t^i)$ is twice continuously differentiable for $c_t^i, a_t^i, s_t^i \geq 0$, including one-sided differentiation in corners.

Assumption 2: The function u is increasing and strictly concave in c^i and a^i ($u_1^i > 0$, $u_2^i > 0$, $u_{11}^i < 0$, $u_{22}^i < 0$).

Assumption 3: The function u is decreasing with aggregate stock: $u_3 < 0$.[25]

Assumption 4: Aggregate stock reduces the individual's marginal utility from the non-addictive product ($u_{13} < 0$). [26]

Assumptions 1 and 3 enable us to focus on addiction: although aggregate stock (that includes the individual's own past consumption) harms the individual, her consumption of the addictive product gives her immediate benefit: this formally captures the characteristics of withdrawal and tolerance that are typical to addiction. Under assumptions 3 and 4, however, it is not only the individual's own accumulated consumption that affects her, but also the whole network's aggregate consumption, weighed by the influence parameters γ_{ji}. This capture's our focus on the effect of the network on the prospects of harmful addiction, through the harm and peer pressure that the network causes the individual.

Assumption 5: $u_{33} > 0$: aggregate stock harms the individual in a decreasing way (that is, harm itself is concave in aggregate stock). This reinforces the peer pressure inflicted on the individual in a way that encourages addiction. As we shall see, if this

[25]Assumptions 1 and 2 are consistent with McCrorry et al (2022)'s qualitative survey of social media users finding that individuals convey a short-lived positive experience together with a long-run negative experience.

[26]To focus on peer pressure, we further assume for concreteness $u_{23} = 0$ (aggregate stock has no effect on the individual's marginal utility from the addictive product). Similarly, $u_{12} = 0$: current consumption of the addictive product does not affect the marginal utility from the non-addictive product and vice versa. Our results would not be affected by allowing $u_{12} \neq 0$, and $u_{23} \neq 0$, as long as $u_{23} - u_{13} > 0$ and $u_{11} + u_{22} < 2u_{12}$.

assumption is relaxed and u^i_{33} is sufficiently negative (i.e., harm is sufficiently convex in aggregate stock), belonging to a network can actually cause the individual to mitigate her consumption of the addictive product and comparative statics in equilibrium are reversed.

Individual i's problem can be simplified by noting that her utility is strictly increasing in c^i so that her budget constraint is always binding. Thus $c^i = y - a^i$ and the individual's per-period utility and harm can be represented by $w(a^i, s^i) \equiv u(y - a^i, a^i, s^i)$. It follows directly from assumptions 1 to 5 above that $w_1 > 0, w_2 < 0$, $w_{11} < 0, w_{12} > 0, w_{22} > 0$.

The next section discusses the network's effect on an individual whose behavior does not affect other network members, who are assumed to be in a steady state, so that they contribute a constant level of aggregate stock each period. We shall call this the constant network case. Then, in Section 4, we extend the analysis to a strategic network where individuals optimally respond to each other's consumption.

3 Constant network

Consider now a special case of the general model depicted in section 2, in which for a certain individual i, $\gamma_{ji} = 0 \, \forall j \neq i$, i.e., the individual's consumption of the addictive product does not affect the other network members. Nevertheless, consumption by the other network members affects individual i. This corresponds to cases in which individual i is not an important member of the network (e.g., an unpopular adolescent who is part of a network engaged in consumption of cigarettes, drugs or alcohol). Another case this variant of the model depicts is that of a network that is large, so that the individual does not affect it, as is usually the case with social media platforms such as Facebook and Instagram. Since individual i does not affect the rest of the network, assume the rest of the network is at some steady state, so that its effect per-period on individual i is constant: $\sum_{j \neq i} a^j_t \gamma_{ij} \equiv \xi \quad \forall t$. As before, the individual has two available products: an ordinary product c and an addictive product a. Denote the individual's consumption of product a at period t as a_t and the aggregated stock affecting the individual (contributed by the individual and the network) until the beginning of period t as s_t. The evolution of aggregate stock affecting the individual is given by:

$$s_{t+1} = \delta s_t + a_t + \xi, \tag{4}$$

Hence s_t is the (one-dimensional) state at time t.

The individual's utility in period t is $w(a_t, s_t)$. Clearly, the optimal strategy is stationary and depends solely on s_t.

Thus, the individual's value function can be represented recursively as::

$$V(s_0) = \max_{\forall t: a_t \in [0,y]} (1-\beta) \sum_{t=0}^{\infty} \beta^t w(a_t, s_t).$$

$$s.t.\ s_{t+1} = \delta s_t + a_t + \xi$$

The corresponding Bellman equation is:

$$V(s) = \max_{a \in [0,y]} (1-\beta) w(a, s) + \beta V(\delta s + a + \xi), \tag{5}$$

Proposition 1 establishes the existence and uniqueness of this value function and the existence of the individual's policy correspondence. All of the proofs are in the appendix:

Proposition 1. *In the constant network case, there is a unique continuous value function, $V(s)$, which satisfies equation (5) and there exists a non-empty, upper hemicontinuous, policy correspondence:*

$$\Phi(s) = \{s' :\ V(s) = (1-\beta) w(s' - \delta s - \xi, s) + \beta V(s')\} \tag{6}$$

We now derive an equation that the individual's optimal consumption path must fulfill. Denote by $\mu_{1t} \leq 0$ the Lagrange multiplier on the constraint $a_t \geq 0$ and $\mu_{2t} \geq 0$ the Lagrange multiplier on the constraint $a_t \leq y$. The Lagrangian associated with Equation (5) is

$$L_t = (1-\beta) w(a_t, s_t) + \beta V(\delta s_t + a_t + \xi) + \mu_{2t}(a_t - y) - \mu_{1t} a_t. \tag{7}$$

Denote $\mu_t \equiv \mu_{2t} - \mu_{1t}$. Lemma 1 derives the individual's first order condition:

Lemma 1. *(a necessary condition for optimal behavior along the optimal path of an individual exposed to a constant network) The individual's first order condition is:*

$$w_1(a_t, s_t) + \mu_t + \beta[w_2(a_{t+1}, s_{t+1}) - \delta w_1(a_{t+1}, s_{t+1}) - \delta\mu_{t+1}] = 0. \tag{8}$$

Equation (8) describes individual i's optimal reaction to the network's consumption. The network's consumption affects the aggregate stock of consumption, s, which, in turn, affects individual i's utility from consumption. Also, since we have not imposed a condition of joint concavity, there may be more than one steady state (i.e., a state satisfying $\overline{s} \in \Phi(\overline{s})$) to which optimal paths converge, depending on the initial level of aggregate stock. When studying the case of an individual not connected to a network, Orphanides and Zervos (1994) and Dechert and Nishimura (1983) have shown that in this case, even with multiple steady states, optimal paths of consumption monotonically converge to a steady state. Our next result is that this monotonicity property carries over to the case of an individual connected to a constant network.

By the principle of optimality, one of the optimal paths starting from the steady state of aggregate stock $\overline{s}$ involves retaining the same aggregate stock ad-infinitum. Proposition (2) below shows that retaining the same aggregate stock is actually the only optimal path. The proposition also shows that between any two consecutive steady states there is a unique critical level such that an optimal path starting below the critical level converges to the lower steady state and an optimal path starting above the critical level converges to the upper steady state.

Using the individual's first order condition in (8) we can prove that the aggregate consumption paths are separated (i.e., if a path of consumption starts at a higher level, it always continues at a higher level each period) and monotone:

Proposition 2. *Fix $\xi \geq 0$. The optimal path of aggregate consumption of the addictive product has the following properties:*

(i) At least one steady state exists.

(ii) Any optimal path of aggregate stock monotonically converges to a steady state.

(iii) There is one critical level between any two consecutive stable steady states.

The possibility that optimal paths and steady states are not unique and the feature that an optimal consumption path beginning above a critical level monotonically converges to an upper steady state while an optimal path beginning below the critical level monotonically converges to a lower steady state is meaningful for modeling addiction. It can capture behavior such as going on a binge (converging to an upper steady state of consuming the addictive product) or go cold turkey (converging to a lower steady state). Also, note that we do not require that optimal paths be interior solutions.[27] This allows for optimal abstinence from the addictive product or acute addiction of using the individual's entire income to consume the addictive product.

Next we wish to emphasize that in our model, the network and aggregate stock harm the individual, as described in the next lemma:

Lemma 2. *(the individual suffers from the network) Fix s_0 to be some initial state and define by $V(s_0;\xi)$ the value of the decision problem in (5) for a specific ξ. Then $V(s_0;\xi)$ is decreasing in both arguments.*

This follows directly from our assumption that $w_2(a_t, s_t) < 0$. For example, if the individual is present in a network of smokers, the larger this network, the worse off is the individual (regardless of her own smoking), due to passive smoking. That is, the increase in the individual's own smoking brought about by an increase in the network's smoking, despite its immediate benefit to the individual, never outweighs the harm inflicted on the individual by the aggregate stock of consumption. The same could apply to social media platforms, such as Facebook and Instagram: Obviously, these networks bring utility to the individual, and this utility in and of itself may well be higher for a larger network. We focus, however, on the harm: Suppose that a multitude of friends on Facebook causes the individual harm that outweighs the above-mentioned benefit. This harm could stem, for example, from distress, envy, less productivity, problems with relations, and so forth, as documented by the vast psychological and economic literature and internal Facebook and Instagram studies cited in the introduction. The harm inflicted on the individual need not be direct. A network of teenagers consuming alcohol together inflicts harm on an individual teenager in the sense that the network's peer pressure reduces the individual's marginal utility from consuming the non-addictive product, e.g., a soft drink. Consequently, the network causes the individual to consume a smaller quantity of soft drinks and a larger quantity of alcohol and hence the peer pressure harms the

[27] In particular, $w_1(0, s) \neq \infty$.

individual. Also, the lemma shows that the individual in our framework always prefers to begin with lower stock (s_0), regardless of the network's size.

Suppose that absent the network, the individual would have abstained from the harmful product. Can a large enough network induce a rational and informed individual to become an addict? We show that the answer is yes. To prove this result, we first show that if a path of aggregate stock is increasing for a particular network size, it must be increasing for a larger network. This further implies that a critical level (above which if the individual begins consumption then her future consumption converges to a higher steady state) is non-increasing with the network. These findings are established in the following lemma:

Lemma 3. *(i) Let $\widehat{\xi} < \xi$ be two network levels and let s_0 be an initial state. If the path of aggregate stock starting from s_0 increases over time when the network is $\widehat{\xi}$, it also increases over time when the network is ξ.*

(ii) Critical levels are non-increasing in the size of the network ξ.

According to the first part of Lemma 3, the individual does not react to an increase in the network by reducing her own consumption in a way that makes aggregate stock decrease over time (so as to reduce future harm from aggregate stock). The driving force of this result is the peer pressure inflicted by the network ($w_{12} > 0$) and the fact that the harm is concave in aggregate stock ($w_{22} > 0$). Intuitively, what could have caused the individual to try to counteract the harm caused to her by the network by significantly decreasing her own consumption is the detrimental long-term effects of aggregate stock on her utility. Yet the individual knows that the network's peer pressure, reducing her marginal utility from the non-addictive product, will grow into the future, while the marginal harm diminishes.

The second part of Lemma 3 follows directly from the first: If an optimal path starting from any s_0 slightly above the critical level of a small network is rising, we know from the first part of the proposition that the optimal path must also rise from s_0 for a larger network. Hence it cannot be that the critical level is higher in the larger network. The following figures illustrate an example in which a larger network strictly lowers the critical level.[28] Figure 1 depicts the $\xi = 0$ case while in figure 2 there is a small network

[28]In this example $w(a, s) = \frac{ln(1.1-a)}{s+0.1} + (s + 0.1)ln(0.1 + a)$, $y = 1$, $\beta = \delta = 0.9$

of $\xi = 0.01$, causing the critical level to decrease from 0.73 in the no-network case to 0.59 with a network.

The results in the first part of Lemma 3 can be used to show that a large enough network induces an individual who would have abstained from the addictive product to become an addict. This is shown in the following proposition:

Proposition 3. *(a large enough network induces an abstainer to become an addict) Suppose that for* $\xi = 0$ *the lowest steady state is* 0 *(accompanied by a constant consumption of* 0*) and the second lowest steady state is* $\overline{s}$*, and let* s_c *be the critical level between these two steady states. Then for any* $\xi > (1-\delta)s_c$*, the individual never consumes* 0 *in a steady state.*

Proposition 3 shows that an individual who, but for the network, would have abstained from the addictive product, becomes addicted for a large enough network. The intuition for this result is similar to that of Lemma 3: the network changes the individual's priorities, through its effect on the individual's marginal benefit from the non-addictive product. Thise peer pressure exerted by the network induces the individual to consume the addictive product, despite the self-inflicted long-term harm involved in such consumption. The individual fully understands that her current consumption will cause her future harm. Nevertheless, she consumes the addictive product, because her marginal benefit from the non-addictive product is decreased, and will continue to decrease further in the future, due to the network's consumption. The network will harm the individual more and more into the future, and the individual cannot do anything about it. Thus, the individual joins in on consumption, in order to minimize the harm via current benefit from consumption.

Proposition 3 also establishes an upper threshold for what constitutes a sufficiently large network so as to induce addiction. It suffices that the network be larger than $(1-\delta)s_c$, where s_c is the critical level (absent the network) between the zero consumption steady state and the higher steady state. Such a network accumulates aggregate stock that converges (even absent any consumption by the individual) to a steady state above s_c (the steady state for $\xi = (1-\delta)s_c$ absent consumption by the individual is precisely $\frac{(1-\delta)s_c}{1-\delta} = s_c$).

The fact that critical levels can strictly decrease with the network (see part (ii)

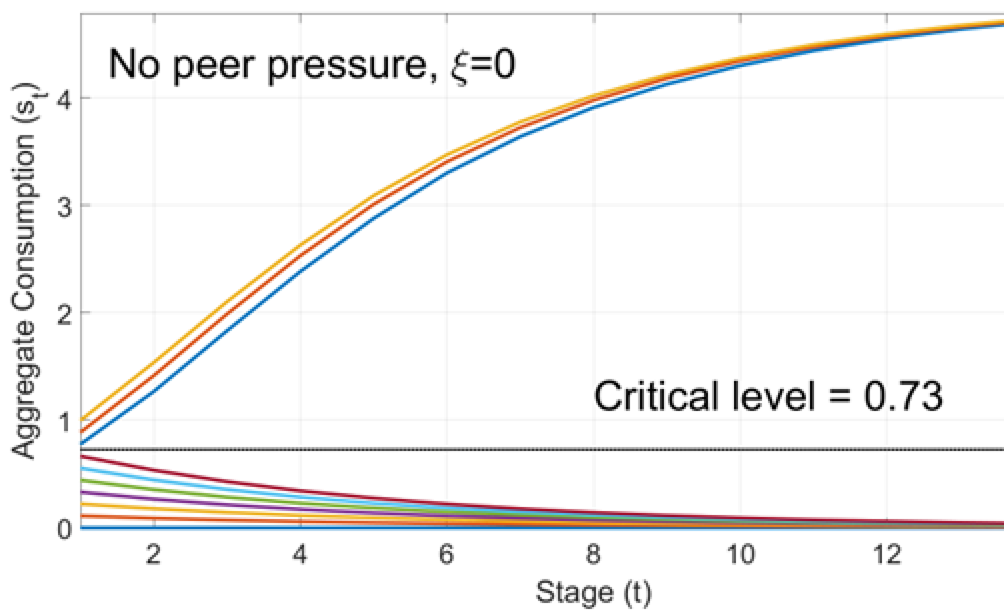

Figure 1: No network

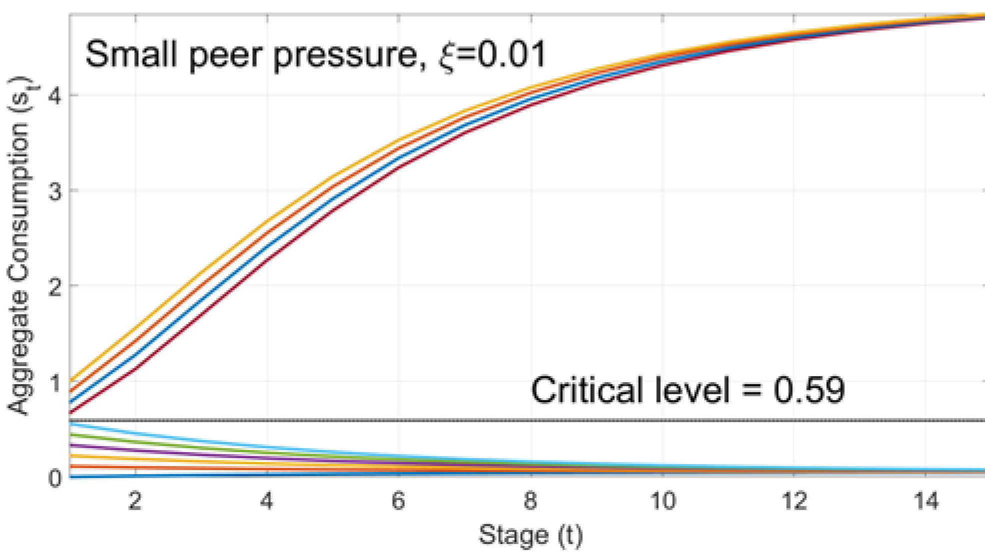

Figure 2: Small network

of Lemma 3 and the example in Figure 3), introduces a second mechanism, besides that of Proposition 3, by which an abstainer can start consuming the addictive product when connected to a network. Without a network, the critical level may be above the individual's initial stock. Hence her aggregate stock converged to a lower steady state, corresponding to zero consumption. When connected to the network, the critical level may be reduced below the individual's initial stock. In such a case, the individual starts consuming and aggregate stock converges to a positive level. This mechanism is not relevant to an abstainer with zero initial stock. Since the critical level cannot be reduced below zero, only the first mechanism, of Proposition 3, can cause her to consume.

The second part of Lemma 3 could have implications for an extended model in which the individual is not aware of the harmful effects of the addictive product, and experiments with it as in Orphanides and Zervos (1995)'s framework. Only after such experimentation the individual can verify whether the network is harmful or not. Once the individual verifies this, however, it may already be too late: if such awareness occurs after the individual has passed the critical level, she becomes addicted and continues consuming up to a higher steady state. When the individual is connected to a network, Lemma 3 shows that the larger the network, the lower this critical level. It follows that the larger the network, the more likely is experimentation by the individual to end up in addiction.

Note that, both under the mechanism of Proposition 3 and the mechanism described above, of reduction of the critical level, the individual need not be exposed to the network for a long period to start consuming the addictive product. Positive consumption by the (former) abstainer, due to the network, begins immediately when the individual connects to the network. She understands that, due to connection to the network, aggregate stock will converge to a positive steady state anyways, so she joins in with consumption at the outset.

The conclusion that for a network larger than $\xi = (1-\delta)s_c$, stock crosses the critical level that prevailed even absent the network has another important implication: once aggregate stock passed this critical level, "rehab" in the form of disconnecting the individual from the network, is no longer effective. This is summarized in the next corollary:

Corollary 1. *(condition for timeliness of rehab) For any network of size $\xi > (1-\delta)s_c$, if rehabilitation of the individual, via an intervention that disconnects her from the network,* *is implemented after period* $t_c = \frac{\ln\left(1-\frac{s_c}{\xi}(1-\delta)\right)}{\ln\delta} - 1$, *it is no longer effective.*

This follows from proposition 3 and from a calculation of the number of periods that it takes the network (even absent consumption by the individual) to reach the critical level s_c. If the network is larger than $(1-\delta)s_c$, it eliminates zero consumption as a steady state and will cause aggregate stock at some point to cross s_c, which is the critical level above which the individual becomes hooked on the addictive product even absent a network. Hence if intervention that disconnects the individual from the network (immediately reducing ξ to zero) occurs after this time window, and therefore after aggregate stock had already crossed s_c, it will not help and the individual will continue consuming on her own up to her upper steady state. Note also that t_c is decreasing with ξ,[29] i.e., the larger the network, the higher the chances of untimely rehab. We also know that the individual is worse off than in the zero-consumption steady state since by 2, the individual's value function $V(s_0, \xi)$ is decreasing in both arguments. Hence the value is lower due to the inflated aggregate stock, even after disconnection from the network that reduced ξ to zero. Conversely, if disconnection from the network occurs early enough, and in particular before aggregate stock reaches s_c, such rehab is timely and effective. Once the individual is disconnected from the network's peer pressure, her aggregate stock of consumption converges downwards, back to the steady state of zero consumption. Note, however, that the time window provided in Corollary 3 is just an upper threshold. In particular, depending on the extent of the individual's own consumption until aggregate stock reaches s_c, earlier interventions than this time window may be ineffective.[30]

Next we examine the network's effect on an individual who is prone to be a heavy user of the addictive product, but may also be in a lower steady state of consumption. The next proposition shows that a network of modest per-period consumption causes the individual's lower steady state to disappear so that her only steady state is that of maximum consumption of the addictive product:

Proposition 4. *(a network consuming more than y retains the individual's maximum consumption as a unique steady state) Suppose that for* $\xi = 0$, *one of the individual's*

[29] Recall that $ln(x) < 0$ for $x < 1$, so the argument in the numerator of the formula for t_c is negative.

[30] As noted, physical disconnection from a social media platform does not necessarily help, due to the individual's fear of missing out even when disconnected. In such cases the type of external intervention that would be helpful is some regulation reducing the scope and type of content the individual's peers expose her too.

steady states involves maximum consumption of the addictive product (a steady state of $\frac{y}{1-\delta}$, with a constant consumption of y by the individual). Then, for $\xi > y$, the unique steady state is the one where the individual consumes y every period.

The intuition for Proposition 4 is that without a network, if initial stock corresponds to consuming y per period, this is better for the individual than any lower consumption, since maximum consumption is one of the individual's steady states. A network contributing more than y per period to the stock affecting the individual eventually brings her to such a situation, so that maximum consumption becomes her only steady state. This result applies also to an individual who, alongside her maximum consumption steady state, also has a steady state of total abstention. Absent the network her steady state is zero-consumption, while when connected to the network her steady state is maximum consumption.[31]

While Proposition 4 warns us that even a network of modest size could cause the individual's steady state to be one of maximum consumption, we next show that when the network is large enough, the individual consumes the maximum quantity of the addictive product from the outset, once she joins this large network:

Proposition 5. *Suppose that for $\xi = 0$, one of the individual's steady states involves maximum consumption of the addictive product. For a large enough network, the individual's optimal consumption plan is to consume y in each period for every initial state.*

We now turn to the effect of the network on internal steady states (that involve neither zero consumption nor maximum consumption). Suppose the individual is already addicted, in the sense that she consumes a non-zero quantity of the addictive product in a steady state.

Let us now derive a feature of the individual's second order condition that will prove useful in what follows. Suppose that the initial state is $\overline{s}$ and consider a consumption policy $a_t = \overline{a} + \epsilon b_t$ where ϵ is small enough and b_t is some bounded series. The term ϵb_t represents possible small deviations from the individual's optimal consumption strategy in an internal steady state. The overall payoff (divided by $(1-\beta)$) when using this policy is $f(\epsilon) = \sum_{t=0}^{\infty} \beta^t w(\overline{a} + \epsilon b_t, \overline{s} + \epsilon \sum_{k=0}^{t-1} \delta^{t-k-1} b_k)$. The first argument is the individual's

[31] Note that the condition in Proposition 4 is sufficient and not necessary. A smaller network could also eliminate the low-consumption steady state.

consumption at time t and the second argument is aggregate stock, after calculating the cumulative effect of the deviation terms. Because consumption of $\overline{a}$ supports the steady state, the function $f(\epsilon)$ attains its maximum at 0, which means that $f'(0) = 0$ and $f''(0) < 0$.

The small deviations from the optimal consumption path, ϵb_t, could take various forms. For our purposes, it suffices to consider constant deviations each period from the optimal strategy ($\underline{b} = b(1, 1, \ldots)$). Then the second order condition $f''(0) < 0$ yields the following lemma:

Lemma 4. *According to the individual's second order condition, the requirement that constant deviations from the optimal strategy are not profitable demands that:*

$$(1 - \beta\delta) w_{11}(\overline{a}, \overline{s}) + 2\beta w_{12}(\overline{a}, \overline{s}) + \tfrac{\beta(1+\beta\delta)}{1-\beta\delta^2} w_{22}(\overline{a}, \overline{s}) < 0. \tag{9}$$

The intuition for Lemma 4 is that in a steady state, the individual balances between the marginal benefit she derives from consuming more of the addictive product and the marginal loss caused by the future increase in aggregate stock this consumption causes. The individual rationally anticipates that, although an increase in her current consumption of the addictive product benefits her in the present, it entails negative repercussions in the future. These negative repercussions are caused by the fact that current consumption increases future aggregate stock. Future aggregate stock further induces the individual to consume more in the coming periods (causing further future harm, and so forth). This is especially so if w_{12} (corresponding to the peer pressure inflicted by the network) is large. Indeed, a large w_{12} increases the left hand side of (9), making it more easily violated. Furthermore, the individual knows that she will not be able to restrain herself from consuming in the future if w_{22} (the concavity of the harm from increased stock) is too large and w_{11} (the concavity of marginal utility due to consumption) is too small in absolute value. Indeed, (9) is violated when w_{12} and w_{22} are too large relative to w_{11}'s absolute value.

We can use the result in Lemma 4 to prove that any increase in the network increases both the aggregate stock and the individual's own consumption in any internal steady state:

Proposition 6. *(steady state consumption and stock increase with the size of the network) Fix $\xi \geq 0$. Let $\overline{s}$ be an internal steady state (i.e. a steady state that is supported by a non-corner consumption, $\overline{s} \neq \frac{\xi}{1-\delta}, \frac{y+\xi}{1-\delta}$) and let $\overline{a} = (1-\delta)\overline{s} - \xi$ be the supporting consumption. Both the steady state and the individual's consumption increase with ξ.*

Proposition 6 shows that the network has a positive effect not only on the occurrence of addiction, as shown in propositions 3 and 4, but also on the severity of addiction: In our framework, an addict connected to a network always consumes larger quantities of the addictive product when the network is larger. At first blush, there could have been an opposite intuition, where a rational individual faced by a larger network reduces her own consumption so as to counteract the future harm involved in the higher aggregate stock of consumption. When the network grows, the individual faces a trade off: on the one hand, the network's peer pressure presses her to consume more. On the other hand, the individual knows that current consumption has negative effects on her well being in the future. Nevertheless, proposition 6 shows that when the network grows, the first effect always dominates, as long as harm is concave in aggregate stock: the addicted individual responds by increasing her consumption of the addictive product. In particular, the proposition shows that had the individual attempted to mitigate the effect of a larger network on her by consuming less, she would have deviated from what is optimal for her and violated the second order condition in Lemma 4. This second order condition already takes account of the above-mentioned trade-off between the current benefit from consuming the addictive product and the negative future repercussions. This second order condition is more binding on the individual than the trade off that an increase in the network creates.

4 Strategic network members

In this section we extend our analysis to a strategic game in which individuals in the network react optimally to each other's consumption. This is unlike the previous section, in which other network members were assumed to be in a steady state and the individual was assumed not to affect their behavior. We focus on an Open Loop Equilibrium ("OLE") solution concept. Based on the initial state vector of stock, $\underline{s}_0$, each individual in the network chooses a time-dependent consumption plan and commits to it. These strategies form an equilibrium in time-dependent strategies: given the consumption paths of all players $j \neq i$, the best response of player i is her equilibrium strategy. The strategies are not state-dependent, so the consumption of the players remains the same even if others deviate or make errors.[32] This solution concept can be either interpreted as individuals' ability to commit to a consumption path or as a lack of the individual's ability to observe whether her peers had deviated from their equilibrium consumption paths. As noted by Fudenberg and Tirole (1995, p. 131-132), the OLE solution concept, in addition to its tractability, can be a good approximation for a Markov Perfect Equilibrium if there are many individuals. In such a case, an unexpected deviation by one individual can have little influence on player i's optimal strategy, so we assume she does not observe this deviation. We start from the existence of a zero-consumption OLE (Section (4.1)) and of a maximum-consumption OLE (one where all individuals in the network spend their entire income y on the addictive product each period) (Section (4.2)). Then, in Section (4.3), we prove the existence of an OLE for any vector of initial states and any influence matrix and study comparative statics and the possibility that an addition of an individual to the network will cause a cascade of consumption of the addictive product by other network members.

4.1 Zero-consumption equilibrium

Suppose that individuals' utility function w is such that supports zero consumption in the no-network case such that a single individual with this utility function, with an initial state $s_0 = 0$ and without a network optimally will abstain from the addictive product. The following proposition shows that, even when such an individual is connected to a network of similar individuals, with a low enough initial state, the entire network will avoid consumption along the equilibrium path.

Proposition 7. *(Consumption-less OLE) Suppose that the optimal strategy of an individual without a network with a utility function w and initial state $s_0 = 0$ is to avoid consumption for all t. Then there exists $\overline{\xi}$ such that for all initial states in the set $[0, \overline{\xi}]^n$,*

[32]The online appendix discusses a closed loop equilibrium solution concept, and in particular a Markov Perfect Equilibrium ("MPE"), in which each individual's strategy relies on the current state of aggregate stock. We show that our results, to be elaborated on below, of co-existence of a consumption-less equilibrium path and an equilibrium in which all network members spend their entire income on the addictive product each period, carry over to this case as well.

avoiding consumption for all t is an OLE.

Proposition 7 shows that a network need not induce the consumption of the addictive product if all individuals manage to coordinate on zero consumption. This may be true even if individuals reach the network after consuming minor amounts of the addictive product on their own, as long as the initial stock of consumption is small enough. The size of this interval depends on the individuals' utility function and on the influence matrix Γ. Similarly, depending on the parameters of the case, small enough consumption by network members will not break the consumption-less equilibrium.

Since, as we show below, there are also other equilibria, with consumption, we note here that the consumption-less equilibrium characterized in Proposition 7 is the best one in terms of welfare. Indeed, suppose there exists an equilibrium where individual i consumes (a_t^i) and the rest consume according to their best-response (a_t^{-i}). Let $w_i(a_t^i, a_t^{-i})$ be the utility of individual i. Since the payoff is decreasing with the network, she prefers everyone else to consume zero, thus $w_i(a_t^i, a_t^{-i}) < w_i(a_t^i, 0_t^{-i})$. However, if everyone else consumes 0, we get the consumption-less equilibrium (provided initial consumption was small enough) since by Proposition 7 i's best reply is also 0: $u_i(a_t^i, 0_t^{-i}) < u_i(0_t^i, 0_t^{-i})$. Thus: $u_i(a_t^i, a_t^{-i}) < u_i(0_t^i, 0_t^{-i})$ and the consumption-less equilibrium yields the best payoff to all individuals. Accordingly, members of the network face a coordination game. They collectively prefer the consumption-less equilibrium, but may find themselves in an inferior equilibrium with positive consumption.[33] For example, a group of adolescents with a small enough initial stock of smoking, or the use of drugs or alcohol, would benefit from guidance, from teachers or instructors, that helps them coordinate on collectively remaining with no consumption. Joint guided discussions, such as those of Alcoholics Anonymous, weight loss groups and similar organizations, can help solve this coordination problem as well.

The consumption-less equilibrium, however, is fragile, in the sense that if an individual with a high enough initial state of consumption of the addictive product joins the network, this may induce consumption of the addictive product by others in the network- a cascade. Indeed, as we shall see in Proposition (10) below, when the consumption-less equilibrium is broken, an OLE with consumption always exists. Hence the addition

[33]As shown in the online appendix, the result of an optimal consumption-less strategy profile when initial stock is small enough carries over to a Markov-Perfect Equilibrium framework.

to the network of a problematic individual, with high enough consumption, causes the network to switch to a new equilibrium with consumption by other individuals, to the detriment of the whole network. We explore this possibility in Lemma 7 and the discussion following it.

Individuals' coordination problem may lead them to the worse kind of equilibrium, where all network members spend their entire income on the addictive product. This is studied in the next section.

4.2 Maximum consumption equilibrium

This section shows that if network members are prone to severe addiction, in the sense that for a large enough stock of consumption, they consume their entire income y per period on the addictive product, then for a large enough network, it is an OLE for all of them to consume y each period:

Proposition 8. *Suppose that for all individuals there exists an initial state and a constant network $\xi_0 \geq 0$, such that the optimal action is to consume y each period. Then there exists N' such that if $n > N'$ there exists an OLE in which all the individuals consume y in each period, regardless of the initial state.*[34]

Recall though that by Proposition 7, if network members are prone to severe addiction on one hand but also have a steady state of total abstention, on the other, a consumption-less OLE always exists as well, regardless of the number of individuals. Our next result is that when the network is large enough to support a maximum-consumption equilibrium, these two extremes are the only OLE's, and no OLE with intermediate consumption, between zero and y, exists:

Proposition 9. *Suppose that the optimal strategy of an individual without a network and a low initial state $s_0 = 0$ is to avoid consumption for all t, while with some initial state s_0 and constant network ξ_0, her optimal action is to consume y. Then for a network comprising of such individuals and the low initial states there exists N' such that if $n > N'$ there could be either a 0-consumption OLE or a y-consumption OLE.*

[34]The online appendix also shows the existence of a maximum-consumption Markov-Perfect equilibrium.

Accordingly, interventions to reduce consumption of the addictive product should be aimed, *inter- alia*, at a reduction of the number of network members, in a way that reduces the prospects of a maximum-consumption equilibrium, and second, improving the prospects of a consumption-less equilibrium, by preventing individuals with high initial stock, high influence over others, or highly influenced by others, from joining the network. We elaborate further on the latter implication in Section (4.3). The damage from a maximum-consumption equilibrium can be mitigated by interventions that are aimed to reduce the magnitude of maximum consumption. Take, for example, Facebook, Instagram and TikTok's infinite scrolling feature discussed in the introduction. This feature facilitates and enhances maximum consumption. Regulatory intervention that limits the use of this feature can help alleviate the damage, by placing a cap on per-period consumption, at least when it comes to adolescents.

In the next section, we consider the general existence of an OLE, not only in the special cases of no consumption or maximum consumption, and we study the effect of changes in the model's parameters and network members on such equilibria.

4.3 Internal equilibria and comparative statics

Sections 4.1 and 4.2 have shown the conditions for zero consumption and maximum-consumption OLE's. In what follows we establish, using Tarski's fixed point theorem, that for any vector of initial stock and for any influence matrix, a pure strategy OLE always exists. Hence when these extreme OLE's of zero and maximum consumption do not exist, an OLE with intermediate consumption always exists.

We start by proving the existence of best responses, in the following lemma, and then prove the existence of internal OLE's, in the following proposition:

Lemma 5. *Let $\underline{s}_0$ be some initial state and fix $\underline{a}^{-i} = (a_t^{-i})_{t \in N}$ to be the consumption strategy of all individuals except individual i (each a_t^{-i} is a vector with $n-1$ elements). Then there exists a best-response for individual i and the correspondence that assigns the set of best-replies to each $\underline{a}^{-i}$ is non-empty, compact valued, and upper hemicontinuous.*

Proposition 10. *For any vector of initial stock $\underline{s}_0 = [s_0^1, s_0^2, ..., s_0^n]^T$ and for any influence matrix Γ a pure strategy OLE always exists.*

Proposition 10 guarantees the existence of an OLE. This implies that when the network is not in one of the corner solutions discussed in the previous sections, of zero or full consumption, there always exists an OLE with intermediate consumption (where at least one individual consumes between 0 and y in some period).

Let us now study the effect of changes in the number of network members, of the influence matrix or in network members' initial states on the OLE. To do this, consider a particular intermediate-consumption OLE. Individual i's utility and state evolution is:

$$u_i(\underline{a}) = \sum_{t=0}^{\infty} \beta^t w(a_t^i, s_t^i) \tag{10}$$

$$s_t^i = \delta s_{t-1}^i + \sum_{k=1}^{n} \gamma_{ik} a_{t-1}^k \tag{11}$$

Her marginal utility of additional consumption at time τ is:

$$\frac{\partial u_i(\underline{a})}{\partial a_\tau^i} = \beta^\tau w_1(a_\tau^i, s_\tau^i) + \sum_{t=\tau+1}^{\infty} \beta^t \delta^{t-\tau} w_2(a_t^i, s_t^i) \tag{12}$$

The next lemma shows that in any OLE, any small parameter change that induces some individuals in the network to consume more causes the network to switch to a new OLE in which all individuals consume more, and vice versa: any small parameter change that induces some individuals in the network to consume less causes the network to switch to a new OLE where all individuals consume less:

Lemma 6. *Let μ be some parameter, and σ_μ some OLE in the game with this parameter. Consider a new identical situation with μ' instead of μ, such that $\mu' > \mu$, and let $\emptyset \neq I \subset N$ be the set of individuals who have a profitable deviation from σ_μ when the parameter is in-fact μ'. If all individuals in I now want to consume [weakly] more (less) than in σ_μ then there exists an OLE $\sigma_{\mu'}$ where all individuals in the network want to consume [weakly] more (less) than in σ_μ.*

Note that for an individual with positive consumption in the original OLE, the change from μ to μ' causes consumption to strictly (and not only weakly) increase. The only case where the change could have no effect is where $a_t^i = 0$ or $a_t^i = y$ in some OLE. Here, although individual i's marginal utility increases by the change in μ, when $a_t^i = 0$ this may not be enough to overcome the future harm of increased consumption, so that i's optimal action remains $a_t^i = 0$, and conversely she cannot consume more than y.

We now use the result in Lemma 6 to study changes in the influence matrix, the addition or omission of network members, and changes in individuals' initial stocks of consumption;

Corollary 2. *Consider an intermediate-consumption OLE. All of the following changes yield a new OLE in which consumption of all network members (weakly) increases (decreases):*

(i) Any increase (decrease) in the influence parameter γ_{ij} $(i = 1, ..., n,\ i \neq j)$*.*

(ii) Any addition (omission) of a network member i*.*

(iii) Any increase (decrease) in an individual's initial stock of consumption.

The first part of Corollary 2 shows that for any intermediate consumption OLE, if the influence of one network-member on another network member is increased, this has detrimental repercussions for the whole network and it is not only this other network member who became more influenced that consumes more in the new OLE. Due to the peer pressure inflicted by each network member on the others via the influence matrix Γ, an increase in the particular influence of individual j on individual i (γ_{ij}) causes all network members to consume more of the addictive and harmful product. Conversely, a decrease in the influence of individual j on individual i causes a decrease of such harmful consumption by all network members. This implies that it is all the more important from a welfare perspective to try to intervene so as to reduce such influence. For example, a teacher or psychologist could try to separate two adolescents that grew to be particularly close to each other and it is suspected that they, together with others, use drugs or alcohol. Such efforts would reduce overall drug or alcohol consumption in the whole group. Facebook, for example, provides popular users with a "Top Fan Badge" that enables the user " ... to more easily identify your most engaged followers and to encourage them to engage more on your Page."[35] Translated to our framework, this intensifies the already large influence parameters this popular user has on her most influenced followers. A related example concerns AI features in social media platforms such as Instagram's "explore" feature, which are designed to expose the user to the type of content she is expected to engage with the most.[36] Here, rather than raising the

[35]See https://www.facebook.com/gpa/blog/top-fan-badge.

[36]See https://help.instagram.com/140491076362332/?helpref=hc_fnav.

influence parameter itself, Instagram exposes the user to content involving the largest influence parameters. TikTok, for its part, uses the "for you" feature, which automatically includes in the user's "feed" she is exposed to upon entry into the app the content she is expected to engage with the most, i.e., the content released by network members with the strongest influence over the individual.[37] Facebook's algorithm pushing a notification to a friend whose photo appears in a photo posted by another friend, encouraging her to respond, is similarly analogous to artificially boosting the second friend's influence over to first friend.

Similarly, the second part of Corollary 2 implies that when an individual who influences others, or is influenced by others, is added to the group, the whole group consumes more of the addictive product. This result is consistent, for example, with Facebook executive Lars Backstrom's statement that use of Facebook increases with the number of the user's friends.[38] Moreover, this individual herself will consume more of the addictive product after joining the network than what she would have consumed on her own. This implies that in our framework, any addition of individuals to the network is harmful, regardless of how well they behaved before they had joined the network. For example, an individual may abstain from the addictive product when alone. Yet in our framework, it is never sound policy to add her to the network with the hope that she would have a good influence on others. Since what matters is the accumulated stock of consumption, the network's original consumption will have a bad influence on the new network-member: she may start consuming the addictive product herself, and this, in turn, will have a bad influence on the original members of the group. They too will consume more of the addictive product in the new OLE.

Part (iii) of the corollary implies that the higher (lower) is the initial stock of consumption by an individual, the larger (smaller) is consumption of the addictive product by the whole network. This can be combined with the result in parts (i) and (ii) to imply that if a new member must be added to the network, it is better to add one with the smallest initial stock of consumption and the smallest influence on the other members and smallest possible influence of the other members on the new member. The converse is true with respect to the question who is it best to remove from the network: it is

[37] See https://later.com/blog/tiktok-algorithm/.

[38] See http://www.graphanalysis.org/SIAM-AN10/01_Backstrom.pdf.

the individual who dominates all others with respect to her own stock, influence over others, and influence by others. Part (iii) easily extends also to an increase in the initial multi-dimensional state: for two initial states $\underline{s}_0 < \underline{s}_0'$ (element-wise comparable), for each OLE starting from $\underline{s}_0$ there is an OLE starting from $\underline{s}_0'$ where consumption of the addictive product for all t is higher and all of the states are higher along the equilibrium path compared to $\underline{s}_0$.

Interestingly, Facebook and Instagram have designed their platforms in a way that even a user who has deactivated her account but later decided to reconnect automatically retains all of the friends and content that she was exposed to when disconnected.[39] This inflates her initial stock upon her return to the platform and, by Part (iii) of Corollary 2, enhances equilibrium consumption of the whole network. Practices of Facebook, Instagram and TikTok that encourage new users to offer links to their profile in other networks in which they have already shared content are also a form of inflation of the initial stock. For example, a new TikTok user who has been using Facebook and Instagram is encouraged to provide links to those following her on TikTok to all of her content in the other two networks.[40]

The above-mentioned comparative statics hinge on our assumption that harm is concave in aggregate stock. When harm is sufficiently convex, these comparative statics are reversed, because then individuals' reaction functions are downward sloping rather than upward sloping. Hence, while our OLE existence result remains intact, any increase in an influence parameter, any addition of a new member to the network, and any increase of an individual's initial stock, cause equilibrium consumption of the addictive product by all network members to decrease.

Another type of change we wish to study is that of addition of a new member to the network where, prior to this addition, the network enjoyed a consumption-less equilibrium. In other words, we wish to examine whether a "rotten apple", with high enough initial stock, can "spoil the barrel" in the sense that this rotten apple causes other network members to start consuming. We can gain more insight about individuals' equilibrium behavior in such scenarios by establishing next that any individual's optimal reaction to

[39] See https://www.facebook.com/help/250563911970368; https://help.instagram.com/370452623149242; https://www.guidingtech.com/instagram-delete-vs-deactivate-difference/.

[40] https://support.tiktok.com/en/getting-started/setting-up-your-profile/linking-another-social-media-account.

a changing network is between her reaction to a constant network that is smaller than the changing network and her reaction to a constant network that is larger. This will enable us to use our results from the constant network case, in which we have shown that a large enough constant network can cause an abstainer to start consuming the addictive product, to the strategic case:

Lemma 7. *Let $\xi_i(t)$ be the aggregate stock per period of a network that changes with time such that $\underline{\xi} \leq \xi_i(t) \leq \overline{\xi}$ for all t and let $\underline{\xi}$ and $\overline{\xi}$ be two constant networks. Let a_t^i be individual i's optimal reaction given the changing network, $\underline{a}_t^i$ her optimal reaction given $\underline{\xi}$ and $\overline{a}_t^i$ her optimal reaction given $\overline{\xi}$. Then $\underline{a}_t^i \leq a_t^i \leq \overline{a}_t^i$.*

The intuition for Lemma 7 is that if the individual consumes in response to the lower constant network, she surely consumes in response to the higher changing network. This follows from the game being supermodular, as we show in the proof of Proposition 10. By the same reasoning, the individual's response to a changing network cannot involve more consumption than her response to a constant network binding the changing network from above.

Lemma 7 has important implications for the formation and expansion of networks that may involve a cascade of harmful addiction. In particular, we know from Lemma 7 that when an individual prone to consuming the addictive product (the "rotten apple") is added to a network of abstainers, the reaction of these abstainers to the changing aggregate stock caused by the rotten apple is at least as strong as the abstainer's reaction to a constant network binding this changing aggregate stock from below. To illustrate, suppose that individual j was in a consumption-less OLE corresponding to Proposition 7. Suppose now that some new individual, $i \neq j$ is added to the network, and her initial stock is such that she consumes the addictive product. The minimum stock individual j is exposed to each period due to individual i's consumption is $L_j \equiv \gamma_{ji} \inf a_t^i$. By Lemma 7, individual j's best-response to the changing network caused by individual i is larger than individual j's best-response to a constant network of L_j. Our results on constant networks (Section 3) imply two mechanisms from which we can deduce individual j's consequent consumption of the addictive product. First, if, given a constant network of L_j, the critical level is reduced such that $s_j^0 > s_c(L_j)$, where s_j^0 is individual j's initial stock, and $s_c(L_j)$ is the critical level caused by a constant network of L_j,[41] then j's best

[41] Recall that by Lemma 3 and figure 3, critical levels are non-increasing in network size and, at least

response to such a constant network is to consume. Alternatively, by Proposition 3, if $L_j > (1-\delta)s_c$, where s_c is the critical level absent a network between an individual's zero-consumption steady state and her lowest positive-consumption steady state absent a network, individual j will surely consume due to the rotten apple's consumption (even ignoring j's initial state). Consequently, by Lemma 7, positive consumption is j's best response to the changing network caused by the rotten apple as well. Now consider the aggregate stock contributed by these two consuming individuals, i and j. By a similar reasoning, their consumption forms a changing network that the remaining individuals in the network are exposed to. This changing network too is bounded from below by some constant network that, by the results in Section 3, induce consumption of other network members, and so forth. By the existence result of Proposition 10, we know that the positive consumption induced by this cascade forms an OLE.

To make things more concrete, the following algorithm characterizes the set of individuals who end up consuming the addictive product in such a cascade. Let a single individual i's initial state be s^i and suppose that when this individual is exposed to a constant network ξ, $f(s^i,\xi) \equiv inf_t a_t$ is her lowest consumption along the optimal path. Denote our "rotten apple" w.l.o.g as individual 1. Suppose she has a high enough initial state so that $f(s^1,0) > 0$. Suppose further that the other $n-1$ individuals in the network have sufficiently low initial states such that they would not consume when disconnected from the network. The following algorithm characterizes which network members will consume in equilibrium due to the addition of individual 1 to the network and derive a lower bound for the level of equilibrium consumption for each network member:

Step 1: set $C_1 = \{1\}$, $L_1^1 = f(s^1,0)$. If the other network members abstain, individual 1 consumes alone. If other network members consume, individual 1's equilibrium consumption can only increase. Thus, the other network members are exposed to a changing network due to individual 1's consumption and the set $\{j \notin C_1 | f(s^j, \gamma_{j1} L_1^1) > 0\}$ represents the set of individuals that consume because of individual 1's consumption. Set $C_2 = C_1 \cup \{j \neq 1 | f(s^j, \gamma_{j1} L_1^1) > 0\}$, and for each $j \in C_2$, set $L_j^2 = f(s^j, \gamma_{j1} L_1^1)$.

Step k: If the rest of the individuals act as described in step k-1, each individual j in the set C_{k-1} consumes at least L_j^{k-1}, and the sum of these (times the appropriate γ's) is the updated changing network everyone is exposed to. We therefore set $C_k = C_{k-1} \cup \{j \notin$

in certain cases, they are strictly decreasing.

$C_{k-1}|f(s^j, \sum_{i\in C_{k-1}} \gamma_{ji}L_i^{k-1}) > 0\}$ and for each $j \in C_k$, set $L_j^k = f(s^j, \sum_{i\in C_{k-1}} \gamma_{ji}L_i^{k-1})$. If, at some point, C_k includes all network members, then they end up in an equilibrium in which all of them consume the addictive product (i.e., the rotten apple, individual 1, spoiled the whole barrel – all remaining $n-1$ individuals). Otherwise, the rotten apple might spoil part of the barrel: all of the functions L_j^k are increasing and bounded by some limit (otherwise, an unbounded level of aggregate stock would have induced all network members to consume). When this limit is (almost) reached, the L_j^k functions stop changing and the set C_∞ is the minimum set of individuals who are guaranteed to consume the addictive product in equilibrium.[42]

Consider now the possibility of rehabilitation, in the form of some external intervention that disconnects the individual from the network, when absent rehabilitation an OLE with consumption is predicted. We summarize the result in the next corollary:

Corollary 3. *(condition for timeliness of rehab) Suppose that without a network the individual's lowest steady state is* 0 *(accompanied by a constant consumption of* 0*) and the second lowest steady state is* $\overline{s}$ *(supported by a constant consumption of* $\overline{a} = (1-\delta)\overline{s}$*), and let* s_c *be the critical level between these two steady states. If rehabilitation of the individual, via an intervention that disconnects the individual from the network, is implemented before period* $t_c = \frac{\ln\left(1-\frac{s_c(1-\delta)}{y\sum_{j=1}^{n}\gamma_{ji}}\right)}{\ln\delta} - 1$*, it is effective.*

Corollary 3 follows from a calculation of the number of periods that it takes a network consuming y each period to reach the critical level that prevails absent the network s_c. If the aggregate stock contributed by such a network is larger than $(1-\delta)s_c$, it eliminates zero consumption as a steady state, but if disconnection from the network is implemented before the time indicated in Corollary 3 it is surely effective, as in any consumption profile, network members cannot consume above y per period. Note that Corollary 3 does not hinge on the type of equilibrium the network is in, or on whether the network is in equilibrium at all. It also applies to any network which is, for instance, in a Markov Perfect Equilibrium or is off equilibrium.

[42] Another (straightforward) implication of Lemma 7 is that if in a particular OLE one individual abstains from the addictive product despite the fact that all of her peers engage in maximum consumption, of y, each period, this individual will abstain in any OLE.

5 The case of sufficiently convex harm functions with large networks

We have seen throughout our results that when the harm function is sufficiently convex in aggregate stock, the OLE is affected in an way opposite to the case where harm is concave: Addition of a consuming individual to the network reduces consumption of others, as well as any increase in an influence parameter or the initial stock of a network member. In this section we consider the combination of sufficiently convex harm and a large enough network. Here we show that when harm is sufficiently convex, a sufficiently large network eliminates consumption of the addictive product completely, in the constant network case, and causes overall consumption to be bounded, in any strategic game, regardless of the equilibrium solution concept or whether the network is on or off equilibrium. We summarize this result in the following proposition.

Proposition 11. *Assume* $w_{22} << 0$. *(i) With a constant network* $\xi \to \infty$*, the individual abstains from the addictive product for all periods.*

(ii) If $n \to \infty$ *in the strategic network case the sum of individuals' consumption in each period is bounded.*

Proposition 11 implies that very large networks could eliminate consumption of the addictive product when the harm they inflict on the individual is sufficiently convex. Take, for example, a smoker potentially exposed to a very large network of smokers in some public place. Taking the smoking of others as given, the aggregate harm from passive smoking could cause the individual to abstain from smoking altogether, so as to counteract the harm from the network, if marginal harm sufficiently grows with the size of the network and the network is large enough. A similar logic applies if all network members are assumed to react strategically, but to a lesser extent, since given that all individuals abstain, each of them may be induced to smoke. Yet as shown in Proposition 11, aggregate consumption is bounded, even as the network of smokers grows indefinitely.

6 Policy implications

Our results from sections 3 and 4 have direct policy implications with respect to networks the characteristics of which are influenced by certain entities, as in the case of social media platforms such as Facebook, Instagram and TikTok. In particular, these social media platforms intensively encourage individuals to expand their network and its usage. Facebook, Instagram and TikTok have allowed generous ceilings on the number of friends or those followed, and they consistently encourage individuals to expand their networks within these high ceilings. In particular, Facebook's "people you may know" feature – a list of potential friends suggested by Facebook[43]– is easily confused by users, due to an identically designed interface, with the list of friend requests the user receives, in a way that encourages expansion of the user's network beyond her prior intentions. Furthermore, the constant alerts and notifications with unanswered friend requests can exploit human vulnerabilities of needing to reciprocate social gestures (Neyman (2017); Turel and Osatuyi (2017)). The notifications are intentionally vague (e.g., "you have a new friend suggestion", rather than specifying the potential friend's name) to tempt the user to enter the app and engage (Fraser and Conlan (2022)). Also, Facebook is pre-installed on most android phones, via deals reached between Facebook and phone manufacturers.[44] This too is a potential driver of network expansion and use. The fact that Facebook makes an individual's number of friends public information was found to encourage individuals to expand their network to improve their reputation (Kim and Lee (2011)). Even Facebook's generous 5000 friend limit can be passed in the sense that Facebook encourages a user reaching this threshold to turn her profile into a "page", with an unlimited number of followers.[45] Hence this popular user can continue exposing her content to as many followers as she wishes. The same is true for TikTok and Instagram: The number of followers a user can have is unlimited in these networks too. Additionally, TikTok encourages the individual to import her phone contacts and friends on Facebook and Instagram to connect with her via TikTok, suggests the user's account to her phone contacts and suggests her account to people who sent her links or who opened links sent by her via other apps.[46] TikTok also encourages enlargement of one's network by allowing only people with more than 1000 followers to upload live videos,[47] and

[43]See https://www.facebook.com/help/336320879782850.

[44]See, e.g., https://www.digitalinformationworld.com/2020/10/heres-why-android-phones-in-some.html; https://www.searchenginejournal.com/facebook-cannot-be-deleted-from-certain-android-phones/285713/#close.

[45]See https://www.facebook.com/help/116067818477568.

[46]See https://www.wired.com/story/tiktok-friends-contacts-people-you-may-know/.

[47]See https://www.adobe.com/creativecloud/video/hub/guides/how-to-go-live-on-tiktok#:~:text=First%2C%20you%20must%20be%20at,the%20capability%20to%20go%20Live.

by enabling users to monetize their use by reaching usage and follower thresholds.[48] Facebook and Instagram's practice of sharing their advertising revenue with particularly popular users who have many followers is another vehicle they use for expanding networks and increasing user engagement.[49] These popular users are encouraged to increase the number of their followers and increase their followers' exposure to the network, via monetary incentives.[50] Instagram similarly encourages expansion of the user's network. It also enables users to share their posted content with their friends on Instagram and Facebook simultaneously.[51]

These social media platforms also impose no limit on the quantity of peer content an individual is exposed to. Much to the contrary, these three networks are designed to induce excessive exposure to content. The infinite scrolling feature encourages users to engage with content posted by other network members excessively. Neyman (2017) documents this feature as "addictive software design" in the sense that it enables endless and effortless scrolling through content. Aza Raskin, who originally designed the infinite scroll feature, was quoted as admitting that features such as infinite scrolling are designed so that they "don't give [the user's] brain time to catch up with [his or her] impulses." He added that " ... many designers were driven to create addictive app features by the business models of the big companies that employed them ... you're going to start trying to invent new ways of getting people to stay hooked."[52] The Facebook algorithm arranges the content each user is exposed to by default in a way that would attract the individual's attention the most, to encourage her to engage with this content, and react with her own content, for as long as possible.[53] In Instagram, especially popular users are encouraged, via monetary rewards, to place excessive content in their profile so that their many followers will engage excessively with this content. If such an influencer places content in her "story", it expires after 24 hours. This, on one hand, reduces the aggregate stock followers are exposed to, but on the other, it causes followers to

[48] See https://www.tiktok.com/creators/creator-portal/en-us/getting-paid-to-create/creator-next/.

[49] See https://www.facebook.com/business/help/1884527914934148?id=1200580480150259. See also https://www.facebook.com/creators/getting-started-with-fan-subscriptions for a popular user's ability to collect fan subscriptions on Facebook. See https://creators.instagram.com/earn-money/subscriptions for parallel features in Instagram.

[50] See https://www.facebook.com/business/help/300444652164185?id=2514811085399429; https://creators.instagram.com/earn-money/badges.

[51] See https://help.instagram.com/1936968516554161/?helpref=uf_share

[52] Andersson (2018); Griffin (2022)

[53] See https://www.facebook.com/help/1155510281178725.

enter the app more often so as not to miss content from this influencer that expires quickly (Belanche et al (2019)). The frequent entry into the app in itself raises users' aggregate stock of use. Also, expiration after 24 hours in the "story" feature encourages posting of additional content that otherwise may not have been shared.[54] Conversely, the Instagram influencer can maximize her revenue by placing some of the content in her "highlights" so that it permanently remains and enhances the aggregate stock followers are exposed to in the future as well. "Likes" too, and other such gratifying rewards for engagement, stimulate individuals to excessively expand aggregate stock. Social media platforms such as Facebook, Instagram and Twitter employ features with a "variable reward component", such as "pull to refresh" buttons that encourage engagement in a manor similar to slot machines (Langvardt (2019)). Other stimulators of engagement are via direct encouragement, such as Facebook's notifications saying "what's on your mind" or "help your friend celebrate her birthday" or "say hi to your new Facebook friend" or "congratulate your friend" on some event (Hristova et al (2020); Langvardt (2019)). Facebook also leverages a user's content to invoke engagement by her friends. As documented by Langvardt (2019), if one user posts a photo with one of her friends in it, Facebook pushes a notification to the friend, who is encouraged to respond, and incentivized to keep checking the app to see how others have responded. Instagram and Facebook also launched a "candid stories" feature, encouraging users to mutually share live photos following randomized daily alerts and notifications (Barker (2023)), and TikTok introduced similar encouragement with its "TikTok now" feature.[55] Snapchat encourages users to communicate with each other daily through its "streak" feature. If either friend misses a day, then the streak is lost, seen by teens as breaching a social obligation, and losing the opportunity to receive access to unique emojies (Langvardt (2019)). Sandy Parakilas, a former Facebook employee, was quoted saying that "social media is very similar to a slot machine, ... There was definitely an awareness of the fact that the product was habit-forming and addictive, ... You have a business model designed to engage you and get you to basically suck as much time out of your life as possible and then selling that attention to advertisers."[56]

[54] See https://www.facebook.com/business/news/insights/how-do-people-perceive-and-use-instagram-stories-and-feed#What%E2%80%99s-the-appeal-of-stories?.

[55] See https://newsroom.tiktok.com/en-us/introducing-tiktok-now.

[56] Andersson (2018).

Our results imply that such behavior by social media platforms can expand the aggregate stock affecting the individual in a way that induces her to excessively consume the addictive product, possibly leading her to harmful addiction, and also intensifies harmful addiction. In particular, the increase in the network and aggregate stock brought about by the platform's tactics can eliminate the individual's zero-consumption steady state (Proposition 3), and reduce her critical level below her initial stock, in a way that turns an abstainer into an addict. The increase in aggregate stock caused by the platform also increases the individual's consumption in any steady state (Proposition 6). In the strategic network case, such features used by platforms increase the size of the network, the influence matrix, and the maximum per-period consumption, all of which we show tend to exacerbate addiction and its prospects (Corollary 2). As demonstrated in the discussion following Corollary 2, Facebook Instagram and Tiktok have also designed their platforms in a way that increases the initial stock of consumption of a returning user, in a way that we show stimulates excessive consumption.

Our results imply that deterring social media platforms from encouraging individuals to expand their networks and their use, at least with respect to particularly vulnerable individuals, such as adolescents, can help prevent harmful addiction. For example, one can think of regulation obligating social media platforms to reduce the size of an adolescent's network only to a certain smaller core or limit the quantity of content users may post.

In addition to ex ante regulation of social media platforms, ex-post liability, at least in extreme cases of harmful addiction, can be considered. For example, social media platforms could be sued, under an appropriate tort rule, by an addicted individual that was significantly harmed by the expansion of her network and its use. Such liability is allegedly similar to the case of individuals harmed by their addiction to nicotine, leading to suits against tobacco companies for the harm caused by such addiction. Indeed, as mentioned in the introduction, dozens of suits against social media platforms were brought in the U.S. alleging that they have knowingly designed their platforms in a way that encourages harmful addiction to the network. Similarly, ex-post antitrust liability could be considered, when the social media platform is dominant in the relevant market. As mentioned in the introduction, such antitrust liability could hinge on the claim that encouraging addiction exacerbates a network effect that excludes rivals, or on direct exploitation of consumers, in antitrust jurisdictions in which such exploitation is a violation, such as the EU. In such cases, the allegation could be that just as a firm's dominance enables it to charge an excessive price that could entail antitrust liability, a social media platform's dominance enables it to design a service that stimulates harmful addiction. Had there been viable competition over users, competing networks could have offered safer networks that do not encourage harmful addiction.[57]

The response of social media platforms such as Facebook, Instagram and TikTok to such legal intervention could be that the expansion of networks and their use caused by these companies can actually reduce consumption if harm is sufficiently convex in aggregate stock, as we show in Section 5 and in the discussion following Corollary 2. It may not be known to the regulator or court whether the harm function is sufficiently convex to an extent outweighing the peer pressure to consume. Yet in the case of social media platforms, we can allegedly infer the answer to this question from the behavior of market actors. For example, the fact that social media platforms consistently push individuals to expand their network and its usage implies that larger aggregate stock increases overall use rather than decreasing it. The aim of social media platforms is to maximize the time users engage with the network, so as to enhance the platform's profits from advertising. Industry executives have often testified that their main marketing efforts are focused on expanding an individual's network and her time of exposure to the network's content as much as they can, so as to maximize advertising revenue and collection of the individual's valuable data (Griffin 2022). Indeed, Meta Platforms, Inc.'s annual 2022 report reveals that Meta "generates substantially all of [its] revenue from advertising," and any loss in user engagement is "likely to have a material and adverse impact" on the revenue Facebook generates. Meta further disclosed that its "financial performance has been and will continue to be significantly determined by our success in adding, retaining, and engaging active users." (Lemert 2022). Consequently, as Lemert (2022) highlights, Facebook's object is not to maximize users' well being, but rather the length of time they spend using the network. This, in turn, through a simple revealed preference argument, implies that social media platforms believe the harm function to be

[57] Rosenquist et al (2022)'s legal policy paper too claims, in the context of merger control and exclusionary conduct, that the fact Facebook, for example, had designed a network that encourages harmful addiction raises serious antitrust implications. They also highlight that, with addictive products such as the use of social media platforms, the level of output (i.e., consumers' use of the social media platform) is not a good proxy for consumer welfare.

concave, or at least not very convex, so that the peer pressure caused by larger aggregate stock induces more (rather than less) consumption by users. Consequently, policy-makers should feel relatively confident that such social media platforms are characterized by a concave harm function, or one that is at least not too convex. This can help justify regulatory interventions against social media platforms.

7 Conclusion

We have shown that rational addiction to a harmful product can occur even when the individual is informed about the harm that the addictive product will cause her. This happens in the prevalent case where the individual is attached to a network of other individuals, who exert direct or indirect peer pressure on the individual to consume the addictive product. Consumption by the individual's peers accumulates and contributes to the aggregate stock affecting the individual's utility. Even though the individual is aware of the future negative results of her current consumption, a large enough network nevertheless induces her to become addicted, as long as harm is concave, or not too convex, in aggregate stock. The larger is consumption by the rest of the network, the stronger the peer pressure on the individual to consume the addictive product, which reduces the individual's marginal utility from consuming an alternative, non-addictive, product, rather than the addictive product. Even when the individual would have abstained from the addictive product without the network, she becomes addicted to it with a large enough network. The individual follows the idiom "if you can't beat them, join them": current consumption mitigates the harm inflicted on the individual by the network, so she initiates consumption, despite the future harm this consumption causes her. If she is particularly prone to addiction, but nevertheless manages to abstain when disconnected from a network, a large enough network causes the individual to dedicate her entire income to the addictive product rather than abstaining. We have shown that for any initial stock and influence matrix, an Open Loop Equilibrium among strategic individuals always exists. In such an equilibrium, any addition of a new member to the network, any increase in the influence matrix and any increase in a network member's initial stock increase equilibrium consumption of the addictive product. It is only when the harm function is sufficiently convex that these comparative statics are reversed.

Our results imply that merely making the individual fully aware of the risks of an addictive product, or of her excessive use, or even changing defaults so that the individual would need to opt-in for features encouraging addiction, may not be enough. They imply that more attention should be dedicated to addressing the individual's network of other users when trying to rehabilitate her from addiction. Expansion of the network adds insult to injury: it raises the individual's chances of becoming addicted, and above and beyond this effect causes addiction to be more severe. Intervention helping the individual to disconnect from the network (e.g., in the case of alcohol or drug addiction in a group of adolescents) could fully rehabilitate her if exercised in a timely manner, before aggregate stock has passed the individual's critical level. Even short of complete disconnection from the network, we show that any decrease in the size of the network, or limitation of the time or content the individual is exposed to the network, can help the individual rehabilitate, or at least reduce consumption of the addictive product.

Our results imply that regulation limiting or deterring social media platforms from aggressively inducing young individuals to be exposed to larger networks with excessive peer content should be considered. In extreme cases of addiction and harm, ex-post tort or antitrust liability could be considered. Such liability can be supported by the causal relation we identify between the platform's efforts inducing the individual to expand her network and her exposure to content and the individual's addiction and harm.

Appendix

The proofs of Propositions 1-2 and Lemma **1** replicate Orphanides and Zervos (1994, 1995) to incorporate a constant network affecting the individual. Hence we relegate them to the online appendix.

Proof of Lemma 2:

Let $\xi_1 < \xi_2$ be two possible networks, and define by a_t the optimal strategy when starting from s_0 with the network ξ_2. Define by s_t^a the state at time t when taking into account only the actions and the initial state. The true state is therefore $s_t = s_t^a + \kappa_t \xi_2$. By definition:

$$\begin{aligned} V(s_0;\xi_2) &= (1-\beta)\sum_{t=0}^{\infty}\beta^t w(a_t, s_t) = (1-\beta)\sum_{t=0}^{\infty}\beta^t w(a_t, s_t^a + \kappa_t\xi_2) \\ &< (1-\beta)\sum_{t=0}^{\infty}\beta^t w(a_t, s_t^a + \kappa_t\xi_1) \leq V(s_0;\xi_1) \end{aligned} \tag{13}$$

Where the first inequality is true since $w_2 < 0$ and the second inequality is due to the fact that a_t is not necessarily the optimal strategy when the network is in fact ξ_1.

Similarly, let $s^1 < s^2$ be two initial states and fix ξ. If a_t is the optimal strategy when starting at s^2, then

$$\begin{aligned} V(s^2;\xi) &= (1-\beta)\sum_{t=0}^{\infty}\beta^t w(a_t, s_t) = (1-\beta)\sum_{t=0}^{\infty}\beta^t w(a_t, s_t^a + \delta^{t-1}s^2) \\ &< (1-\beta)\sum_{t=0}^{\infty}\beta^t w(a_t, s_t^a + \delta^{t-1}s^1) \leq V(s^1;\xi) \end{aligned} \tag{14}$$

where here s_t^a also represents the effect of the network but not the discounted initial state which is written explicitly.■

Proof of Lemma 3:

We prove by contradiction. Assume that the optimal path with $\hat{\xi}$ is increasing (the actions are $\hat{a}_t$ and the states are $\hat{s}_t$) and the optimal path with ξ is decreasing (the actions are a_t and the states are s_t). The path with $\hat{\xi}$ increases, hence $\hat{s}_{t+1} = \delta\hat{s}_t + \hat{a}_t + \hat{\xi} > \hat{s}_t$ which implies $\hat{a}_t > (1-\delta)\hat{s}_t - \hat{\xi}$. Similarly, the path with ξ decreases, hence $s_{t+1} = \delta s_t + a_t + \xi < s_t$ which implies $a_t < (1-\delta)s_t - \xi$. Since $\hat{s}_t > s_0 > s_t$ and $\hat{\xi} < \xi$ (i.e. $-\hat{\xi} > -\xi$) we get $(1-\delta)\hat{s}_t - \hat{\xi} > (1-\delta)s_t - \xi$ which ultimatly implies $\hat{a}_t > a_t$. In addition, using the wrong actions is sub-optimal, so the following two inequalities hold (recall that s_t^a and $s_t^{\hat{a}}$ represent the state evolution due to the actions only, without the network):

$$\sum_{t=0}^{\infty} \beta^t w(\hat{a}_t, \hat{s}_t^{\hat{a}} + \kappa_t \hat{\xi}) > \sum_{t=0}^{\infty} \beta^t w(a_t, s_t^a + \kappa_t \hat{\xi}) \quad (15)$$

$$\sum_{t=0}^{\infty} \beta^t w(a_t, s_t^a + \kappa_t \xi) > \sum_{t=0}^{\infty} \beta^t w(\hat{a}_t, \hat{s}_t^{\hat{a}} + \kappa_t \xi) \quad (16)$$

Summing and rearranging yields:

$$\sum_{t=0}^{\infty} \beta^t [w(\hat{a}_t, \hat{s}_t^{\hat{a}} + \kappa_t \hat{\xi}) - w(\hat{a}_t, \hat{s}_t^{\hat{a}} + \kappa_t \xi)] > \sum_{t=0}^{\infty} \beta^t [w(a_t, s_t^a + \kappa_t \hat{\xi}) - w(a_t, s_t^a + \kappa_t \xi)] \quad (17)$$

Since $\hat{a}_t > a_t$ for all t, $\hat{s}_t^{\hat{a}} > s_t^a$ as well. In addition, $w_{12} > 0$ and $w_{22} > 0$ so $w_2(a_t, s_t^a + x) < w_2(\hat{a}_t, \hat{s}_t^{\hat{a}} + x)$ for every $x \in [\kappa_t \hat{\xi}, \kappa_t \xi]$. Integrating both sides

$$\int_{\kappa_t \hat{\xi}}^{\kappa_t \xi} w_2(a_t, s_t^a + x) dx < \int_{\kappa_t \hat{\xi}}^{\kappa_t \xi} w_2(\hat{a}_t, \hat{s}_t^{\hat{a}} + x) dx \quad (18)$$

results in

$$w(a_t, s_t^a + \kappa_t \xi) - w(a_t, s_t^a + \kappa_t \hat{\xi}) < w(\hat{a}_t, \hat{s}_t^{\hat{a}} + \kappa_t \xi) - w(\hat{a}_t, \hat{s}_t^{\hat{a}} + \kappa_t \hat{\xi}) \quad (19)$$

or equivalently

$$w(\hat{a}_t, \hat{s}_t^{\hat{a}} + \kappa_t \hat{\xi}) - w(\hat{a}_t, \hat{s}_t^{\hat{a}} + \kappa_t \xi) < w(a_t, s_t^a + \kappa_t \hat{\xi}) - w(a_t, s_t^a + \kappa_t \xi) \quad (20)$$

Multiplying by β^t and summing over all t we get a contradiction to (17).

The "Hence" part easily follows. If the critical level would rise with ξ, all the initial states between the old and the new critical level would have to converge downward with the higher ξ, while they converged upward with the lower ξ, a contradiction.■

Proof of Lemma 4:

Fix $\xi \geq 0$. Let $\bar{s}$ be an internal steady state (i.e. a steady state that is supported by a non-corner consumption, $\bar{s} \neq \frac{\xi}{1-\delta}, \frac{y+\xi}{1-\delta}$). Then

$$(1 - \beta\delta) w_1((1-\delta)\bar{s} - \xi, \bar{s}) + \beta w_2((1-\delta)\bar{s} - \xi, \bar{s}) = 0. \quad (21)$$

and for every $n \geq 1$, $(-1)^n M_n > 0$ where

$$M_n = \begin{vmatrix} A & \beta B & \delta\beta^2 B & \dots & \delta^{n-2}\beta^{n-1}B \\ \beta B & \beta A & \beta^2 B & \dots & \delta^{n-3}\beta^{n-1}B \\ \delta\beta^2 B & \beta^2 B & \beta^2 A & \dots & \delta^{n-4}\beta^{n-1}B \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta^{n-2}\beta^{n-1}B & \delta^{n-3}\beta^{n-1}B & \delta^{n-4}\beta^{n-1}B & \dots & \beta^{n-1}A \end{vmatrix}, \tag{22}$$

$A = w_{11} + \frac{\beta}{1-\beta\delta^2} w_{22}$, $B = w_{12} + \frac{\beta\delta}{1-\beta\delta^2} w_{22}$ and the second derivatives are calculated at $((1-\delta)\bar{s} - \xi, \bar{s})$. We use a standard analysis of variation approach. Suppose that the initial state is $\bar{s}$ and consider a consumption policy $a_t = \bar{a} + \epsilon b_t$ where ϵ is small enough and b_t is some bounded series. Clearly, $s_1 = \delta s_0 + \bar{a} + \xi + \epsilon b_0 = \bar{s} + \epsilon b_0$, $s_2 = \delta s_1 + \bar{a} + \xi + \epsilon b_1 = \bar{s} + \epsilon\delta b_0 + \epsilon b_1$ and in general $s_t = \bar{s} + \epsilon \sum_{k=0}^{t-1} \delta^{t-k-1} b_k$. The overall payoff (upto $(1-\beta)$) when using this policy is $f(\epsilon) = \sum_{t=0}^{\infty} \beta^t w(\bar{a} + \epsilon b_t, \bar{s} + \epsilon \sum_{k=0}^{t-1} \delta^{t-k-1} b_k)$. According to intermediate Lemma 2 in the proof of Proposition 2, the only optimal strategy that starts at state $\bar{s}$ is $\bar{a}$, hence the function $f(\epsilon)$ attains its maximum at 0, which means that $f'(0) = 0$ and $f''(0) < 0$.

We start with the first-order condition.

$$f'(\epsilon) = \sum_{t=0}^{\infty} \beta^t [b_t w_1(\bar{a}+\epsilon b_t, \bar{s}+\epsilon \sum_{k=0}^{t-1} \delta^{t-k-1} b_k) + (\sum_{k=0}^{t-1} \delta^{t-k-1} b_k) w_2(\bar{a}+\epsilon b_t, \bar{s}+\epsilon \sum_{k=0}^{t-1} \delta^{t-k-1} b_k)] \tag{23}$$

For every t, b_t appears in this summation once when multiplied by w_1 and an additional time for every $T > t$ multiplied by w_2 with a proper discount. Changing the order of summation to account for that, we get:

$$f'(\epsilon) = \sum_{t=0}^{\infty} \beta^t b_t \cdot \tag{24}$$

$$\left[w_1 \left(\bar{a} + \epsilon b_t, \bar{s} + \epsilon \sum_{k=0}^{t-1} \delta^{t-k-1} b_k \right) + \sum_{T=t+1}^{\infty} \beta^{T-t} \delta^{T-t-1} w_2 \left(\bar{a} + \epsilon b_T, \bar{s} + \epsilon \sum_{k=0}^{T-1} \delta^{T-k-1} b_k \right) \right]$$

Set $\epsilon = 0$:

$$f'(0) = \sum_{t=0}^{\infty} \beta^t b_t [w_1(\bar{a}, \bar{s}) + \sum_{T=t+1}^{\infty} \beta^{T-t} \delta^{T-t-1} w_2(\bar{a}, \bar{s})] = \sum_{t=0}^{\infty} \beta^t b_t [w_1(\bar{a}, \bar{s}) + \tfrac{\beta}{1-\beta\delta} w_2(\bar{a}, \bar{s})] \tag{25}$$

Note that the summation is only on $\beta^t b_t$ as the $[\dots]$ term is fixed. It should hold that $f'(0) = 0$ regardless of the series b_t, so the term $[\dots]$ should be zero, which is exactly (21).

Note that this condition can be established in a simpler manner, by setting $a_{t+1} = a_t = \bar{a}$ and $s_{t+1} = s_t = \bar{s}$ in the individual's first order condition for a constant network (equation (8)). To derive the second-order condition, we differentiate ((24)) once again:

$$\begin{aligned} f''(\epsilon) &= \sum_{t=0}^{\infty} \beta^t b_t \Big[b_t w_{11}(\bar{a} + \epsilon b_t, \bar{s} + \epsilon \sum_{k=0}^{t-1} \delta^{t-k-1} b_k) \\ &+ (\sum_{k=0}^{t-1} \delta^{t-k-1} b_k) w_{12}(\bar{a} + \epsilon b_t, \bar{s} + \epsilon \sum_{k=0}^{t-1} \delta^{t-k-1} b_k) \\ &+ \sum_{T=t+1}^{\infty} \beta^{T-t} \delta^{T-t-1} [b_T w_{12}(\bar{a} + \epsilon b_T, \bar{s} + \epsilon \sum_{k=0}^{T-1} \delta^{T-k-1} b_k) \\ &+ (\sum_{k=0}^{T-1} \delta^{T-k-1} b_k) w_{22}(\bar{a} + \epsilon b_T, \bar{s} + \epsilon \sum_{k=0}^{T-1} \delta^{T-k-1} b_k)] \Big] \end{aligned}$$

Set $\epsilon = 0$ and, to avoid cumbersome notation, we drop the brackets after the partial derivatives. They are all evaluated at $(\bar{a}, \bar{s})$ and $f''(0)$ is

$$\sum_{t=0}^{\infty} \beta^t b_t \left[b_t w_{11} + \left(\sum_{k=0}^{t-1} \delta^{t-k-1} b_k \right) w_{12} + \sum_{T=t+1}^{\infty} \beta^{T-t} \delta^{T-t-1} \left[b_T w_{12} + \left(\sum_{k=0}^{T-1} \delta^{T-k-1} b_k \right) w_{22} \right] \right]$$

For every t, the term b_t^2 appears twice in the summation: once multiplied by w_{11} and once for every T when $k = t$. Hence, in total, we have $\sum\limits_{t=0}^{\infty} \beta^t b_t^2 [w_{11} + w_{22} \sum\limits_{T=t+1}^{\infty} \beta^{T-t} \delta^{T-t-1} \delta^{T-t-1}] = \sum\limits_{t=0}^{\infty} \beta^t b_t^2 [w_{11} + w_{22} \frac{\beta}{1-\beta\delta^2}]$

For every $i > j$, the term $b_i b_j$ appears in the following cases:

- $t = i$: as part of the sum $(\sum\limits_{k=0}^{t-1} \delta^{t-k-1} b_k) w_{12}$ when $k = j$.
- $t = i$: as part of the sum $(\sum\limits_{k=0}^{T-1} \delta^{T-k-1} b_k) w_{22}$ for every $T \geq i + 1$ whenever $k = j$.
- $t = j$: for $T = i$ in the term $\beta^{T-t} \delta^{T-t-1} b_T w_{12}$.
- $t = j$: for $T \geq i + 1$ and $k = i$, as part of the sum $(\sum\limits_{k=0}^{T-1} \delta^{T-k-1} b_k) w_{22}$

Hence, in the expression of $f''(0)$, for every $i > j$ there should also appear $b_i b_j$ multiplied by

$$\beta^i\delta^{i-j-1}w_{12}+\beta^i\sum_{T=i+1}^{\infty}\beta^{T-i}\delta^{T-i-1}\delta^{T-j-1}w_{22}$$

$$+\beta^j\beta^{i-j}\delta^{i-j-1}w_{12}+\beta^j\sum_{T=i+1}^{\infty}\beta^{T-j}\delta^{T-j-1}\delta^{T-i-1}w_{22}=2w_{12}\beta^i\delta^{i-j-1}+2w_{22}\beta^i\tfrac{\beta\delta^{i-j}}{1-\beta\delta^2}\quad(26)$$

To conclude,

$$f''(0)=\sum_{t=0}^{\infty}\beta^t b_t^2[w_{11}+w_{22}\tfrac{\beta}{1-\beta\delta^2}]+2\sum_{i=0}^{\infty}\sum_{j<i}\beta^i\delta^{i-j-1}b_ib_j[w_{12}+w_{22}\tfrac{\beta\delta}{1-\beta\delta^2}]\qquad(27)$$

Denote $A=w_{11}+w_{22}\frac{\beta}{1-\beta\delta^2}$ and $B=w_{12}+w_{22}\frac{\beta\delta}{1-\beta\delta^2}$. The last expression can be written as $f''(0)=\underline{b}^TM\underline{b}$ where $\underline{b}=(b_0,b_1,\ldots)$ and M is the operator

$$\begin{pmatrix} A & \beta B & \delta\beta^2 B & \ldots \\ \beta B & \beta A & \beta^2 B & \ldots \\ \delta\beta^2 B & \beta^2 B & \beta^2 A & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.\qquad(28)$$

The calculations are done at a steady state, so $f''(0)<0$ for every $\underline{b}\neq\underline{0}$, which means that M is a definite negative operator. By the Sylvester criteria, this is equivalent to $(-1)^nM_n>0$ for all n, where M_n is the n^{th} primary minor of M. For special cases, we can obtain relatively simple conditions that need to hold. In particular, if we consider constant deviations from the optimal strategy ($\underline{b}=(b,b,\ldots)$) then, after some computation, we have:

$$(1-\beta\delta)w_{11}(\bar{a},\bar{s})+2\beta w_{12}(\bar{a},\bar{s})+\tfrac{\beta(1+\beta\delta)}{1-\beta\delta^2}w_{22}(\bar{a},\bar{s})<0.\qquad(29)$$

■

Proof of Proposition 3:

Let s_c be the critical level between the steady state 0 and the steady state $\overline{s}$ for $\xi=0$. Set $\xi=(1-\delta)(s_c+\epsilon)$ for $\epsilon>0$ small enough. Regardless of consumption, after enough time the state passes s_c. By the principle of optimality, we can start our discussion here. The optimal path starting from all initial states in the range $(s_c,\overline{s})$ rises to $\overline{s}$ (for $\xi=0$) so the optimal path must also rise for this ξ. Thus, $\frac{\xi}{1-\delta}=s_c+\epsilon$ (which is in this range for ϵ small enough) cannot be a steady state, since it will force some of the other values in this range to downward converge to it, instead of upward. Alternatively, the proof of Lemma 3 can be repeated to show that since the path starting from $s_c + \epsilon$ converged upward for $\xi = 0$, it cannot be a steady state (converge downward with a $\geq$ sign instead of $>$). Either way, $\frac{\xi}{1-\delta}$ is not a steady state and thus in any steady state the individual consumes more than 0. Moreover, any steady state that existed at $\xi = 0$ is above s_c for all ξ which means that even after rehab (resetting ξ to be 0) the aggregate stock is too large for full recovery.■

Proof of Corollary 1:

By Proposition 3, for any network of size $\xi > (1-\delta)s_c$, ignoring the individual's own consumption, aggregate stock exceeds the critical level absent the network and at time:

$$t_c = \frac{\ln\left(1 - \frac{s_c}{\xi}(1-\delta)\right)}{\ln \delta} - 1. \tag{30}$$

3

If the individual is disconnected from the network after period t_c, the individual's critical level absent the network has been passed so the disconnected individual's optimal consumption path will converge to the higher steady state.■

Proof of Proposition 4:

By Proposition 3, if s_c is the highest critical level before y, then for any $\xi > (1-\delta)s_c$ y remains the only steady state. Since the network consumes at least y each period, at some point aggregate stock becomes at least $\frac{y}{1-\delta} > s_c$ and the individual's lower steady state is eliminated.■

Proof of Proposition 5:

To prove the proposition, we first prove the following intermediate lemma:

Lemma: Fix ξ and s_0. If consuming y for all $t \in N$ is not optimal in this initial state, it is not optimal for all smaller initial states. Thus, the set of initial states for which consuming y for all t is optimal is of the form $[\underline{s}, \infty)$ (or $\emptyset$, if $\frac{y+\xi}{1-\delta}$ is not a steady state).

Proof: Let $\hat{s}_0$ be an initial state smaller than s_0. Assume by contradiction that the optimal strategy starting with $\hat{s}_0$ is to consume y whereas the optimal strategy starting with s_0 is some $(a_t)_{t\in N}$. Then

$$\sum_{t=0}^{\infty}\beta^t w(a_t,\delta^t s_0+s_t^a+\kappa_t\xi) > \sum_{t=0}^{\infty}\beta^t w(y,\delta^t s_0+s_t^y+\kappa_t\xi)$$

and

$$\sum_{t=0}^{\infty}\beta^t w(y,\delta^t \hat{s}_0+s_t^y+\kappa_t\xi) > \sum_{t=0}^{\infty}\beta^t w(a_t,\delta^t \hat{s}_0+s_t^a+\kappa_t\xi).$$

Summing these two equations leads to

$$\sum_{t=0}^{\infty}\beta^t\left[w(a_t,\delta^t s_0+s_t^a+\kappa_t\xi)-w(a_t,\delta^t \hat{s}_0+s_t^a+\kappa_t\xi)\right] >$$
$$\sum_{t=0}^{\infty}\beta^t\left[w(y,\delta^t s_0+s_t^y+\kappa_t\xi)-w(y,\delta^t \hat{s}_0+s_t^y+\kappa_t\xi)\right]$$

Note that $a_t\le y$ and $s_t^a\le s_t^y$, so for every x, $w_2(a_t,x+s_t^a+\kappa_t\xi)<w_2(y,x+s_t^y+\kappa_t\xi)$. Hence, performing $\int_{\delta^t\hat{s}_0}^{\delta^t s_0}\cdot dx$ on both sides leads to

$$w(a_t,\delta^t s_0+s_t^a+\kappa_t\xi)-w(a_t,\delta^t \hat{s}_0+s_t^a+\kappa_t\xi) < w(y,\delta^t s_0+s_t^y+\kappa_t\xi)-w(y,\delta^t \hat{s}_0+s_t^y+\kappa_t\xi).$$

Multiplying by β^t and summing leads to a contradiction. Accordingly, if y is not optimal for s_0 it cannot be optimal for $\hat{s}_0<s_0$ and vice versa – if it is optimal for s_0 it is optimal for all initial states $\hat{s}_0>s_0$. Proving that this interval is closed is proven in the same way as in the proof of Proposition 7 below by continuity.

To continue the proof of the proposition, note that $\underline{s}$ is a decreasing function of ξ. One way to see it is to consider

$$f(\xi)=\sum_{t=0}^{\infty}\beta^t w(y,\delta^t \hat{s}_0+s_t^y+\kappa_t\xi)-\sum_{t=0}^{\infty}\beta^t w(a_t,\delta^t \hat{s}_0+s_t^a+\kappa_t\xi), \tag{31}$$

and

$$f'(\xi)=\sum_{t=0}^{\infty}\beta^t\kappa_t w_2(y,\delta^t \hat{s}_0+s_t^y+\kappa_t\xi)-\sum_{t=0}^{\infty}\beta^t\kappa_t w_2(a_t,\delta^t \hat{s}_0+s_t^a+\kappa_t\xi). \tag{32}$$

Since $y\ge a_t$ and $s_t^y\ge s_t^a$, combined with $w_{12},w_{22}>0$, we see that term-by-term the first series is greater than the second, so $f'(\xi)>0$. Thus, if $f(\hat{\xi})>0$ for some initial state $\hat{s}_0$ (meaning, always consuming y is better than the consumption path a_t) it is also true for all $\xi > \hat{\xi}$ (and in-fact, the gain from consuming y compared to the other strategy is even larger).

The only question remaining is how low $\underline{s}$ can go with ξ. We now show that for large enough networks, $\underline{s} = 0$. That is, $\lim_{\xi \to \infty} \underline{s} = 0$, so that for a large enough network, the optimal consumption plan is to consume y for every initial state.

Since consuming y each period is one of the individual's steady states even absent a network, there exists some initial state s' and some network ξ' such that starting from s' with a network ξ', the optimal path is to consume y in every stage. From now on we consider only networks $\xi > \max\{s', \xi'\}$ and the initial state $s_0 = 0$. For such networks, the state after $t = 1$ surpasses s' (regardless of the actions of the individual) and the network is stronger than ξ', so by the principal of optimality, the optimal path starting from $t = 1$ is to consume y. It is left to determine if this is also optimal for $t = 0$ or not for large enough ξ.

If the individual consumes a in the first stage, the payoff can be written as

$$f(a) = w(a, 0) + \sum_{t=1}^{\infty} \beta^t w(y, \delta^{t-1} a + \kappa_t \xi + \kappa_{t-1} y). \tag{33}$$

We first compare the payoff when consuming 0 to the payoff when consuming y:

$$f(y) - f(0) = w(y, 0) - w(0, 0) + \sum_{t=1}^{\infty} \beta^t [w(y, \delta^{t-1} y + \kappa_t \xi + \kappa_{t-1} y) - w(0, \kappa_t \xi + \kappa_{t-1} y)] \tag{34}$$

Using the Lagrange theorem, we can turn the [...] into $\delta^{t-1} y w_2(y, c_t)$ (where $c_t \in [\kappa_t \xi + \kappa_{t-1} y, \kappa_t \xi + \kappa_{t-1} y + \delta^{t-1} y]$). Since $c_t \to \infty$ when $\xi \to \infty$, in this limit $w_2(y, c_t) \to 0$ (since $w_{22} > 0$) and $f(y) - f(0) = w(y, 0) - w(0, 0) > 0$. It follows that for large enough networks, consuming y is better than consuming 0. This ensures that y is the optimal consumption whenever $f'' > 0$ (so that the maximum is not an internal solution).

Suppose now that $f'' < 0$, and the maximum is an internal solution rather than y. The optimal a^* should be chosen such that $f'(a^*) = 0$, i.e.

$$w_1(a^*, 0) + \sum_{t=1}^{\infty} \beta^t \delta^{t-1} w_2(y, \delta^{t-1} a^* + \kappa_t \xi + \kappa_{t-1} y) = 0 \tag{35}$$

Since $w_{22} > 0$, the w_2 part is increasing with ξ, so for higher ξ the equality holds only if

w_1 is smaller, which happens for larger a^* (recall that $w_{11} < 0$). Alternatively, note that by increasing ξ, the derivative of f w.r.t. to a at the optimal action corresponding to the smaller network becomes positive, so optimally the consumption is increased. Either way, $a^*(\xi)$ is an increasing bounded function of ξ and let $\beta = \lim_{\xi \to \infty} a^*(\xi)$. Moreover, following a similar argument, if $a^*(\xi) = y$ then $\forall \xi' > \xi, a^*(\xi') = y$.

Assume that $a^*(\xi) \in (0, y)$ for all ξ large enough. Then the equation $f'(a^*) = 0$ holds for all ξ and it should also hold in the limit $\xi \to \infty$. But since $w_2 \to 0$, in the limit we are left with $w_1(\beta, 0) = 0$, a contradiction to $w_1 > 0$. Hence, $f'(a^*) = 0$ cannot hold for infinitely many ξ, and it is the case that starting from some ξ, the optimal solution is a corner one. As we already established, y is the only candidate and the proof is complete.■

Proof of Proposition 6:

Let ξ be some network and $\overline{s}$ some steady state. For every $\epsilon > 0$ small enough, the optimal path starting from $\overline{s} - \epsilon$ converges upward to $\overline{s}$. Hence, by Lemma 3 it converges upward for every larger ξ, which implies that the steady state cannot decrease with ξ because it would force some of the states between the old and the new steady state to converge downward to the new steady state. Hence $\frac{d\overline{s}}{d\xi} > 0$. We now calculate this expression explicitly.

Let $\overline{s}$ be an internal steady state. Define the LHS of the first order condition in a steady state (this can be derived by setting $a_{t+1} = a_t = \overline{a}$ and $s_{t+1} = s_t = \overline{s}$ in the individual's first order condition along an optimal consumption path (8)) by:

$$R(\overline{s}, \xi) \equiv (1 - \beta\delta) w_1((1 - \delta)\overline{s} - \xi, \overline{s}) + \beta w_2(((1 - \delta)\overline{s} - \xi, \overline{s})) \tag{36}$$

The solutions of the equation $R(\overline{s}, \xi) = 0$ define the steady states for ξ. The response of the steady state to an increase in the network's consumption can be evaluated using implicit differentiation:

$$\frac{d\overline{s}}{d\xi} = -\frac{\frac{\partial R}{\partial \xi}}{\frac{\partial R}{\partial \overline{s}}} = \frac{(1 - \beta\delta) w_{11} + \beta w_{12}}{(1 - \beta\delta)(1 - \delta) w_{11} + (1 - 2\beta\delta + \beta) w_{12} + \beta w_{22}} \tag{37}$$

where all second derivatives are evaluated at $((1 - \delta)\overline{s} - \xi, \overline{s})$ and assuming that $\frac{\partial R}{\partial \overline{s}} \neq 0$.

Let s_0 be some initial state from which the path of aggregate stock converges to $\overline{s}$

when using the optimal time-dependent strategy $a_0, a_1, \ldots$. If the individual keeps this behavior even when ξ increases by ϵ, the path of aggregate stock will converge to $\overline{s} + \frac{\epsilon}{1-\delta}$. Hence, if the new steady state is larger than $\overline{s} + \frac{\epsilon}{1-\delta}$, this cannot be optimal and the individual must consume more along the optimal path (not necessarily for every t, but on average and for $t \to \infty$). Similarly, if the new steady state is smaller than $\overline{s} + \frac{\epsilon}{1-\delta}$, the individual must reduce consumption to reach the steady state. These conditions can be phrased as $\frac{d\overline{s}}{d\xi} \lessgtr \frac{1}{1-\delta}$ where $>$ corresponds to an increase in consumption and $<$ to a decrease.

To conclude, an increase in consumption in response to an increase in the network occurs when

$$\frac{(1-\beta\delta)w_{11} + \beta w_{12}}{(1-\beta\delta)(1-\delta)w_{11} + (1-2\beta\delta+\beta)w_{12} + \beta w_{22}} > \frac{1}{1-\delta} \tag{38}$$

which is equivalent to

$$\frac{(1-\beta\delta)w_{12} + \beta w_{22}}{(1-\beta\delta)(1-\delta)w_{11} + (1-2\beta\delta+\beta)w_{12} + \beta w_{22}} < 0 \tag{39}$$

Note that the nominator is positive, so this expression holds if and only if the denominator is negative. Recall that the second order condition for a constant deviation (9) is

$$(1-\beta\delta)w_{11} + 2\beta w_{12} + \tfrac{\beta(1+\beta\delta)}{1-\beta\delta^2} w_{22} < 0 \tag{40}$$

and the LHS can be re-written as

$$(1-\beta\delta)w_{11} + \beta w_{12} + \beta w_{12} + \tfrac{\beta(1+\beta\delta)}{1-\beta\delta^2} w_{22} \tag{41}$$

Since the right [] are positive, the left [] must be negative, which means that $(1-\beta\delta)w_{11} + \beta w_{12} < 0$ and an increased network encourages increased consumption in a steady state.■

Proof of Proposition 7:

To prove the proposition, let us first prove the two following intermediate lemma's, utilizing our results from Section 3:

Lemma: Suppose that for a constant network $\xi = 0$ the lowest steady state corresponds to zero-consumption. Denote the set of all constant network levels for which $\frac{\xi}{1-\delta}$ is a steady state by I. Then I is a closed interval.

Proof: Let ξ_n be some converging series in I, and let ξ be its limit. To show that I is closed, we need to show that $\xi \in I$. For every n, $\xi_n \in I$, thus $\frac{\xi_n}{1-\delta}$ is a steady state when the network is ξ_n. Let $(a_t)_{t\in N}$ be some strategy which is different from constant zero-consumption. Since $\frac{\xi_n}{1-\delta}$ is a steady state, the unique optimal strategy starting from it is zero-consumption:

$$\sum_{t=0}^{\infty} \beta^t w(0, \tfrac{\xi_n}{1-\delta}) > \sum_{t=0}^{\infty} \beta^t w(a_t, s_t^a + \tfrac{\xi_n}{1-\delta}). \tag{42}$$

Taking the limit $n \to \infty$ on both sides and taking account of the fact that we can change the order of summation and limit since w is continuous on $[0, y] \times [0, \frac{y+\sup_n \xi_n}{1-\delta}]$ and hence bounded, yields:

$$\sum_{t=0}^{\infty} \beta^t w(0, \tfrac{\xi}{1-\delta}) \geq \sum_{t=0}^{\infty} \beta^t w(a_t, s_t^a + \tfrac{\xi}{1-\delta}). \tag{43}$$

Inequality (43) is in fact strong because, as demonstrated in the online appendix (replicating Orphanides and Zervos's (1995) analysis of an individual's optimal paths), if zero consumption is one optimal continuation consumption following zero consumption, it is the only optimal continuation consumption. Hence, the strategy of not consuming when the network is ξ and the initial state is $\frac{\xi}{1-\delta}$ outperforms the strategy $(a_t)_{t\in N}$. This is true for every $(a_t)_{t\in N}$, so not consuming is the optimal strategy, which implies that $\frac{\xi}{1-\delta}$ is a steady state and $\xi \in I$. Therefore, I is closed.

To prove that I is an interval, let $\xi_1, \xi_2 \in I$ and consider the function f that maps each ξ to the lowest steady state with the constant network ξ. Suppose first that f is continuous. f satisfies $f(\xi_1) = \frac{\xi_1}{1-\delta}$ and $f(\xi_2) = \frac{\xi_2}{1-\delta}$. In addition, $f(\xi) = \frac{\xi}{1-\delta}$ for all networks in $[\xi_1, \xi_2]$. This is because f increases at least as fast as the function $\frac{\xi}{1-\delta}$: For 0-consumption steady states, $f(\xi) = \frac{\xi}{1-\delta}$, and if we assume positive consumption, it follows from Proposition 6 that f increases faster than $\frac{\xi}{1-\delta}$. But had there been positive consumption in $[\xi_1, \xi_2]$, f would have departed upwards from the function $\frac{\xi}{1-\delta}$, and then meet $\frac{\xi}{1-\delta}$ again at $\xi = \xi_2$, which contradicts the fact that f increases faster than $\frac{\xi}{1-\delta}$ for all ξ. Consider now discontinuities of f. Denote as $\hat{\xi}$ a point of discontinuity. Upward discontinuity ($\lim_{\xi \to \hat{\xi}^-} f(\xi) < \lim_{\xi \to \hat{\xi}^+} f(\xi)$) is overruled by the fact that, in order to meet $\frac{\xi}{1-\delta}$

again at $\xi = \xi_2$, f needs to increase more slowly than $\frac{\xi}{1-\delta}$. Downward discontinuity is overruled because it would imply the existence of initial states in $[\lim_{\xi\to\hat{\xi}^+} f(\xi), \lim_{\xi\to\hat{\xi}^-} f(\xi)]$ which are upward converging for smaller networks than $\hat{\xi}$ and downward converging for larger networks (in contradiction to Lemma 3). Thus, $f(\xi) = \frac{\xi}{1-\delta}$ for all networks in $[\xi_1, \xi_2]$, so $[\xi_1, \xi_2] \subseteq I$. To conclude, I is a convex closed subset of R and hence it is a closed interval.

Now let I be the interval of all networks for which 0-consumption is a steady state. Denote by I_0 the set of all corresponding steady states, i.e., $I_0 = [0, \frac{\max I}{1-\delta}]$. In the following intermediate lemma we show that if the initial state and network are small enough, the individual abstains from consumption even if the initial state is above the network's consumption-less steady state, and the steady state converges downward to the network's own consumption-less steady state:

Lemma: Suppose that for $\xi = 0$ the lowest steady state corresponds to zero-consumption and let I be the set of all constant network levels for which $\frac{\xi}{1-\delta}$ is a steady state. Then for every $\hat{\xi}, \xi \in I$ s.t. $\hat{\xi} < \xi$, the optimal path that starts at $s_0 = \frac{\xi}{1-\delta}$ when the network is $\hat{\xi}$ is a path without consumption by the individual.

Proof: This is proven in a similar manner to Lemma 3. Assume by contradiction that the optimal consumption path starting at s_0 is $a_t \geq 0$ with a strict inequality for at least one t. when the network is $\hat{\xi}$. In addition, the optimal consumption path when the network is ξ starting from s_0 is 0:

$$\sum_{t=0}^{\infty} \beta^t w(a_t, s_t^a + \kappa_t \hat{\xi}) \;>\; \sum_{t=0}^{\infty} \beta^t w(0, s^0 + \kappa_t \hat{\xi}) \tag{44}$$
$$\sum_{t=0}^{\infty} \beta^t w(0, s^0 + \kappa_t \xi) \;>\; \sum_{t=0}^{\infty} \beta^t w(a_t, s_t^a + \kappa_t \xi) \tag{45}$$

Combining these two inequalities yields:

$$\sum_{t=0}^{\infty} \beta^t w(a_t, s_t^a + \kappa_t \hat{\xi}) - \sum_{t=0}^{\infty} \beta^t w(a_t, s_t^a + \kappa_t \xi) > \sum_{t=0}^{\infty} \beta^t w(0, s^0 + \kappa_t \hat{\xi}) - \sum_{t=0}^{\infty} \beta^t w(0, s^0 + \kappa_t \xi) \tag{46}$$

On the other hand, $a_t \geq 0$ and $s_t^a \geq s_t^0$ so $w_2(a_t, s_t^a + x) > w_2(0, s^0 + x)$. Integrating

$(\int_{\kappa_t\hat{\xi}}^{\kappa_t\xi} \cdot dx)$ both sides results in:

$$w(a_t, s_t^a + \kappa_t\xi) - w(a_t, s_t^a + \kappa_t\hat{\xi}) > w(0, s_t^0 + \kappa_t\xi) - w(0, s_t^0 + \kappa_t\hat{\xi}) \tag{47}$$

i.e:

$$w(0, s_t^0 + \kappa_t\hat{\xi}) - w(0, s_t^0 + \kappa_t\xi) > w(a_t, s_t^a + \kappa_t\hat{\xi}) - w(a_t, s_t^a + \kappa_t\xi) \tag{48}$$

which, after multiplying by β^t and summing over t, contradicts (46).[58]

These two intermediate lemmas show that there exists an interval $I_0 = [0, \overline{\xi}]$ such that for an individual without a network ($\xi = 0$) and initial state $s_0 \in I_0$, the optimal strategy is to consume 0 while converging to the steady state $s = 0$. Let $\underline{s} \in I_0^n$ be the n-dimensional initial state in which for all individuals $1, ..., n$ initial stock $s_0^i \in I_0$, and suppose that all individuals except individual i consume 0 for all t. For individual i, the initial state is within I_0 and the constant network she observes is $\xi^i = 0$, so according to the above-mentioned intermediate lemmas, his best response is to consume 0 for all t. It follows that the best response to 0 consumption of all others is also 0 consumption, so this strategy profile is an OLE. On the equilibrium path, all users avoid consumption and the state monotonically converges to a steady state, $\underline{s} = \delta^t \underline{s} \to 0$.■

Proof of Proposition 8:

This follows from Proposition 5. When all individuals use this strategy, the network a single individual sees is a constant network of $(n-1)y$. Thus, for n large enough, the best response is to consume y too. From proposition 5, as n increases, the $\underline{s}$ decreases and ultimately reaches 0. From this point onward, the equilibrium is to consume y regardless of the state.■

Proof of Proposition 9:

By Proposition 7, the consumption-less OLE exists regardless of n (provided that all individuals' initial states are small enough). Suppose there exists another equilibrium for some n with positive consumption. By Corollary 2 , when adding individuals to the network and for a non-degenerate influence matrix, all network members consume more in equilibrium. Let a_t^i be the consumption of some individual and $\overline{a}_t^i = \lim inf_{n\to\infty} a_t^i > 0$.

[58] Of course, this result also holds for $\hat{\xi} = 0$, so an individual connected to a small enough network with small enough initial stock will abstain also if she is disconnected from the network.

Assume by contradiction that $\overline{a}_t^i < y$. There exists n large enough s.t. the limit is (almost) obtained and the total network seen by each individual is larger than the one causing y-consumption in the constant network model. In such a network, the optimal best-response of each individual is to consume y. Hence, in equilibrium, $\overline{a}_t^i < y$ cannot hold and the limit is y. More generally, if ξ is a constant network where the best response is to consume y in each period, it is also a best response to consume y in each period for any non-constant network $\xi_t > \xi$. This follows because $w_{12} > 0$, so $w_2(a_t, x) < w_2(y, x)$ for all x.)■

Proof of Lemma 5:

Let $\underline{a} = (a_t^1)_{t \in N}$ be some consumption strategy of individual 1 (w.l.o.g.) and $\underline{a}^{-i} = (a_t^{-1})_{t \in N}$ the consumption strategy of all the others. Denote the utility of individual 1 by $f(\underline{a}, \underline{a}^{-i}) = (1-\beta) \sum_{t=0}^{\infty} \beta^t w(a_t^1, \underline{s}_t)$, where $\underline{s}_t$ is the evolution of the states according to (2) with the above mentioned strategy profiles. Each partial sum of the form $f_k(\underline{a}, \underline{a}^{-i}) = (1-\beta) \sum_{t=0}^{k} \beta^t w(a_t^1, \underline{s}_t)$ is continuous in $\underline{a}$ and $\underline{a}^{-i}$ (as a finite sum of continuous functions – w) and $f_k \to f$ uniformly (bounded by a geometric series), so by the uniform limit theorem, f is a continuous function as well. A similar argument shows that f is c^2 with respect to any a_t^i). Fix $\underline{a}^{-i}$. The function $f(\cdot, \underline{a}^{-i}) : [0, y]^\infty \to R$ is continuous over the compact domain, so the maximum is attained. Thus, the best response correspondence $\Phi(\underline{a}^{-i}) = argmax_{\underline{a} \in [0,y]^\infty} f(\underline{a}, \underline{a}^{-i})$ is well defined and $\Phi(\underline{a}^{-i}) \neq \emptyset$ for all $\underline{a}^{-i}$. Moreover, Φ is a closed-valued correspondence. Indeed, let $\underline{a}(k)$ be a converging series[59] whose all elements are in $\Phi(\underline{a}^{-i})$, and denote the limit by $\underline{a}$. Let $\underline{c}$ be some possible policy of individual 1. Since all $\underline{a}(k)$ are in $\Phi(\underline{a}^{-i})$ they are best-replies to $\underline{a}^{-i}$, so $f(\underline{a}(k), \underline{a}^{-i}) \geq f(\underline{a}(c), \underline{a}^{-i})$. By taking the limit $k \to \infty$ and using the continuity of f, we get $f(\underline{a}, \underline{a}^{-i}) \geq f(\underline{a}(c), \underline{a}^{-i})$ which implies $\underline{a} \in \Phi(\underline{a}^{-i})$. Finally, Φ is u.h.c. Indeed, let $\underline{a}^{-i}(k)$ be a series of strategies that converges to $\underline{a}^{-i}$, and $\underline{a}(k)$ a series of best replies ($\underline{a}(k) \in \Phi(\underline{a}^{-i}(k))$ that converges to $\underline{a}$. It follows that for every strategy $\underline{c}$ of individual 1, $f(\underline{a}(k), \underline{a}^{-i}(k)) \geq f(\underline{c}, \underline{a}^{-i}(k))$. Again, this inequality is true in the limit $k \to \infty$, so for every $\underline{c} \in [0, y]^\infty$, $f(\underline{a}, \underline{a}^{-i}(k)) \geq f(\underline{c}, \underline{a}^{-i}(k))$, which implies that $\underline{a}$ is a best reply to $\underline{a}^{-i}$, $\underline{a} \in \Phi(\underline{a}^{-i})$ and Φ is u.h.c. Note that Φ is also a function of $\underline{s}_0$, but since it is fixed for the entire proof, the dependence on the initial state was omitted.■

Proof of Proposition 10:

[59]We can use the sup-norm to measure distance between strategies.

W.l.o.g consider individual 1 and assume each other individual j uses the open-loop strategy $(a_t^j)_{t\in N}$. When individual 1 uses the strategy $(a_t^1)_{t\in N}$, his payoff is

$$f(\underline{a}^1, \underline{a}^{-1}) = \sum_{t=0}^{\infty} \beta^t w(a_t^1, s_t) \tag{49}$$

where $s_{t+1}^1 = \delta s_t^1 + \sum_{j=1}^{n} \gamma_{1j} a_t^j$. Let $t', t'' \in N$ and assume $t' \geq t''$. Fix individual $j \neq 1$. Then

$$\frac{\partial f}{\partial a_{t''}^j} = \sum_{t=t''+1}^{\infty} \beta^t \delta^{t-t''-1} \gamma_{1j} w_2(a_t^1, s_t) \tag{50}$$

and

$$\frac{\partial^2 f}{\partial a_{t''}^j \partial a_{t'}^1} = \sum_{t=t'+1}^{\infty} \beta^t \delta^{t-t''-1} \delta^{t-t'-1} \gamma_{1j} \gamma_{11} w_{22}(a_t^1, s_t) + \beta^{t'} \gamma_{1j} w_{12}(a_{t'}^1, s_{t'}) \delta^{t'-t''-1} \tag{51}$$

Since $w_{12}, w_{22}, \gamma_{11}, \gamma_{1j} > 0$, we get that $\frac{\partial^2 f}{\partial a_{t''}^j \partial a_{t'}^1} > 0$. This is also true when $t' < t''$ (then the w_{12} term drops). To conclude, the game is supermodular, i.e. an increase in the action of j at some time causes player 1 to increase her action in all times. This is true for all players. Now, let $F : ([0,y]^\infty)^n \to ([0,y]^\infty)^n$ be the best response function, i.e. $F(\underline{a}^1, \ldots, \underline{a}^n) = (BR_1(\underline{a}^2, \ldots, \underline{a}^n), \ldots, BR_n(\underline{a}^1, \ldots, \underline{a}^{n-1}))$. From the above argument, this function is order-preserving on a complete lattice, so according to Tarski's fixed point theorem, it has a fixed point. This fixed point is an OLE equilibrium (in pure strategies).

Note that if harm is sufficiently convex in aggregate stock so that (51) is negative, reaction functions are downward sloping. Here too, by similar reasoning, an OLE exists.■

Proof of Lemma 6:

Consider the best response function $F(\sigma_\mu)$ for the case in which the parameter is μ'. We have established that this is an increasing function and in addition it is u.h.c., by Lemma 5. Hence, the series $x^{k+1} = F(x^k)$ converges to a fixed point, which is an OLE equilibrium. σ_μ is some equilibrium for some parameter μ, and it is no longer an equilibrium when the parameter changes to μ', since all individuals in I want to consume more (less). In response, all the other individuals will consume more (less) according to $F(\sigma_\mu)$. In response, all network members will want to consume even more (less) according to $F(F(\sigma_\mu))$, and so forth. This iterated process converges to a new equilibrium $\sigma_{\mu'}$ where all network members consume more (less) than in σ_{μ}.

■

Proof of Corollary (2):

Proof of part (i):

We differentiate ((12)) according to γ_{ij}. It appears implicitly in the equation via the state variable in ((11)):

$$\frac{\partial s_t^i}{\partial \gamma_{ij}} = \delta \frac{\partial s_{t-1}^i}{\partial \gamma_{ij}} + a_{t-1}^j = \ldots = \sum_{k=0}^{t-1} \delta^{t-1-k} a_k^j \tag{52}$$

Thus,

$$\frac{\partial^2 u_i(\underline{a})}{\partial a_\tau^i \partial \gamma_{ij}} = \beta^\tau w_{12}(a_\tau^i, s_\tau^i) \sum_{k=0}^{\tau-1} \delta^{\tau-1-k} a_k^j + \sum_{t=\tau+1}^{\infty} \beta^t \delta^{t-\tau} w_{22}(a_t^i, s_t^i) \sum_{k=0}^{t-1} \delta^{t-1-k} a_k^j > 0 \tag{53}$$

where the inequality follows from $w_{12}, w_{22} > 0$. Thus individual i's marginal utility from additional consumption increases with γ_{ij} (unless individual j abstains for all t in equilibrium), so the current strategy profile is not an equilibrium. individual i now wishes to consume more. By Lemma (6), all network members want to consume more.

Proof of part (ii):

Consider an individual j who is disconnected from the network. This is equivalent to adding j to the network and setting $\gamma_{ij} = \gamma_{ji} = 0 \, \forall i, j = 1, ...n$. If we now increase γ_{ij} and γ_{ji} one by one, by part (i) above, we switch to a new OLE with higher consumption by all network members in each step.

Proof of part (iii):

We differentiate ((12)) according to s_0^i. It appears implicitly in the equation via the state variable in ((11)), and $\frac{\partial s_t^i}{\partial s_0^i} = \delta^t$. Thus,

$$\frac{\partial^2 u_i(\underline{a})}{\partial a_\tau^i \partial s_0^i} = \beta^\tau \delta^\tau w_{12}(a_\tau^i, s_\tau^i) + \sum_{t=\tau+1}^{\infty} \beta^t \delta^{t-\tau} \delta^t w_{22}(a_t^i, s_t^i) > 0 \tag{54}$$

where the inequality follows from $w_{12}, w_{22} > 0$. Hence marginal utility from additional consumption increases with the initial state, so the current strategy is not an equilibrium and individual i wishes to consume more. By Lemma (6), all network members want to consume more.

Note that when harm is sufficiently convex in aggregate stock such that (53) and (54) become negative, and reaction functions (51) are downward sloping, the comparative static results in parts (i)-(iii) are reversed: In the new OLE, any increase in an influence parameter, any addition of a new member, and any increase of initial stock reduces consumption by all network members.■

Proof of Lemma 7:

Consider w.l.o.g a 2-player network comprising of individuals i and j where we focus on individual i's optimal reaction (i.e., a sequence of actions) to the consumption of individual j. Suppose that for all t individual j consumes a fixed quantity such that, given the influence matrix Γ, the network seen by individual i is $\underline{\xi}$. Let $\underline{a}_t$ be the best reply of individual i to this strategy profile. Suppose now that instead individual j consumes so that the network seen by individual i is $\xi_i(t)$, which is bounded from below by $\underline{\xi}$ for all t. Since, by the proof of Proposition 10, the game is supermodular, the additional consumption of Player j causes Player i to consume more. By a similar reasoning, individual i's per period consumption is lower than that of her optimal response to a constant network $\overline{\xi}_i$ which is above $\xi_i(t)$ for all t. In particular, we can assume individual j first consumes according to $\overline{\xi}_i$, and then reduces consumption in the changing network $\xi_i(t)$ and complete the proof analogously.■

Proof of Corollary 3:

The proof is analogous to the proof of Corollary 1, noting that if all network members consume y, it is as if the individual is exposed to a constant network of $\xi = y(n-1)$.■

Proof of Proposition 11:

Proof of part (i):

Assume $(a_t)_{t\in N}$ is some consumption plan, and fix some time t'. The payoff is $f(a_{t'}) = \sum_{t=0}^{\infty} \beta^t w(a_t, s_t{+}\kappa t)$ so the optimal action at time $t' > 0$, if it involves positive consumption, should satisfy:

$$0 = f'(a_{t'}) = \beta^{t'} w_1(a_{t'}, s_{t'} + \kappa_{t'}\xi) + \sum_{t=t'+1}^{\infty} \beta\delta^{t-t'} w_2(a_t, s_t + \kappa_t\xi) \tag{55}$$

(Utility before time t' is not affected by the action at this time. Utility for $t > t'$ is affected by the action via the change of state s_t, while the actions a_t remain unchanged). But for large enough w_{22} and ξ, we get w_2 more negative than w_1 is positive in the $[0, y]$

domain, so this expression is always negative and the optimal action is $a_{t'} = 0$.

Proof of part (ii):

Based on the proof of part (i), in the strategic network case, when $n \to \infty$, and assuming we are not in the zero consumption scenario, the increase in the network creates a pressure on the individual to reduce consumption. To see this, assume that the aggregate stock affecting each individual grows unboundedly. But then the second term in (55) outweighs the first term, so that the individual's optimal consumption goes to zero. This is true for all network members, a contradiction to the fact that aggregate stock increases unboundedly. It follows that the consumption of each individual decreases by 1/n (or faster) with the addition of a new member, so that the total stock is bounded.[60] ■

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