A Simple Globally Consistent Continuous Demand Model for Market Level Data

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17th International Conference on Competition and Regulation

CRESSE, July 2023

Financial support from Fundação para a Ciência e Tecnologia is gratefully acknowledged

(project UIDB/00731/2020)

- We consider a new method of uncovering demand information from market level data on differentiated products.

- Main characteristics:

  1. Representative consumer flexible demand model, following the continuous-choice literature.
  2. Can accommodate the use of data from markets with significant entry and exit of products.
  3. Incorporates a structural error term.
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- Assume a representative consumer that may consume all products.
- Typically use functional forms that allow for flexible substitution patterns and are relatively easy to estimate.
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- Consumers have a set $\Im$ of $J+1$ options available:
  - J inside options ($j = 1, \ldots, J$).
  - One outside option ($j = 0$), which aggregates all other products.
- We follow the continuous-choice literature and define the demand system by specifying an indirect utility function $V(\mathbf{p}, y; \boldsymbol{\theta}, \Im)$ for the representative consumer.
  - $\mathbf{p}$ denotes the vector of $J+1$ prices.
  - y denotes the income of the representative consumer.
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- $V(\mathbf{p}, y; \boldsymbol{\theta}, \Im)$ is assumed to satisfy the properties of an indirect utility function.
  - This implies the demand system can be obtained via Roy's identity.
- $V(\mathbf{p}, y; \boldsymbol{\theta}, \Im)$ is also assumed to satisfy an additional *global consistency property*.
  - Consider two choice sets: $\Im^0$ and $\Im^1$, which are identical with one exception: product k is in set $\Im^0$, but not in set $\Im^1$.
  - Global consistency implies that $V(\mathbf{p}, y; \boldsymbol{\theta}, \Im^1) = \lim_{p_k \to \infty} V(\mathbf{p}, y; \boldsymbol{\theta}, \Im^0)$.

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- Consider again the same two choice sets.
- Let product j be in both sets. Any $V(\mathbf{p}, y; \boldsymbol{\theta}, \Im)$ that satisfies global consistency generates a demand system such that:
  1. $q_k(\mathbf{p}, y; \theta, \Im^1) = 0$.
  2. $q_j(\mathbf{p}, y; \theta, \Im^1) = \lim_{p_k \to \infty} q_j(\mathbf{p}, y; \theta, \Im^0)$.

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- To derive the demand system, we make use, without loss of generality, of a convenient change of variable: $\mathbf{r} = \mathbf{1}./\mathbf{p}$ element-by-element.

- As such, we can restate the indirect utility function $V(\mathbf{p}, y; \boldsymbol{\theta}, \Im)$ as $H(\mathbf{r}, y; \boldsymbol{\theta}, \Im)$ and apply Roy's identify to obtain the Marshallian budget share function of each product j:

$$w_j(\mathbf{r}, y; \boldsymbol{\theta}, \Im) \;=\; \frac{H_j(\mathbf{r}, y; \boldsymbol{\theta}, \Im)\, r_j}{H_y(\mathbf{r}, y; \boldsymbol{\theta}, \Im)\, y}.$$

  - $w_j(\mathbf{r}, y; \boldsymbol{\theta}, \Im) = p_j q_j(\mathbf{p}, y; \boldsymbol{\theta}, \Im)/y$.

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- We follow the continuous-choice literature in approximating $H(\mathbf{r}, y; \boldsymbol{\theta}, \Im)$ with a flexible functional form: a normalized (by the outside option) quadratic function:

$$H(\mathbf{r}, y; \boldsymbol{\theta}, \Im) = a_0 + \sum_j a_j r_j + \frac{1}{2} \sum_j \sum_k b_{jk} r_j r_k + c_0 y + \sum_j c_m r_m y.$$

  - $\mathbf{a}$ denotes a $J \times 1$ vector of parameters, which we will use to match observed shares.
  - $\mathbf{B}$ denotes a $J \times J$ matrix of parameters, which we will use to capture (own and cross) price substitution.
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  - $\mathbf{a}$ denotes a $J \times 1$ vector of parameters, which we will use to match observed shares.
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- This functional form implies the following Marshallian budget share function for each product j:

$$w_j\left(\mathbf{r}, y; \boldsymbol{\theta}, \Im\right) = \frac{a_j r_j + 1/2 \sum_k \left(b_{jk} + b_{kj}\right) r_j r_k + c_j r_j y}{c_0 y + \sum_k c_k r_k y}.$$

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- As such, following the continuous-choice tradition, we may often wish to estimate a more *general* budget share function that does not impose such a restriction *a priori*, as follows:

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- The *general* budget share function implies the following elasticity functions for the budget share of product j with respect to the price of product k and income, respectively:

$$\varepsilon_{jk}\left(\mathbf{r}, y; \boldsymbol{\theta}, \Im\right) = -1\left(j = k\right) - \frac{\sum_k b_{jk} r_k r_j \left(s_j^{true} y\right)^{-1} + c_j r_j}{c_0 + \sum_k c_k r_k}$$

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- The parameterization above assumes that consumer preferences are defined directly over products. However, it also allows them to be mapped onto a space of characteristics, e.g. as follows:

$$\begin{aligned} a_j &= g_0\left(\mathbf{x}_j, \xi_j; \beta\right) \\ b_{jj} &= g_1\left(\mathbf{x}_j^1; \alpha_1\right) \\ b_{jk} &= g_2\left(d_{jk}\left(\mathbf{x}_j^2, \mathbf{x}_k^2\right); \alpha_2\right) \\ c_j &= g_3\left(\mathbf{x}_j^3; \alpha_2\right) \end{aligned}$$

- $\mathbf{x}_j$ and ξ_j denote observed and unobserved characteristics of product j.
- $\mathbf{x}_j^1$, $\mathbf{x}_j^2$, and $\mathbf{x}_j^3$ denotes subsets of $\mathbf{x}_j$ assumed to influence own-price elasticities, cross-price elasticities, and income elasticities, respectively.
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- The general budget share function is homogeneous of degree zero in the parameters and hence the parameters are identified up to a scalar.
  - Without loss of generality, we normalize c_0.
- Given this normalization, the identification of the remaining parameters is standard given a large enough sample.
  - The $J \times 1$ vector of parameters $\mathbf{a}$ are identified from variation in the budget shares across the different products.
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- The estimation algorithm that we propose includes four steps:

1. Set initial values for parameters in $\mathbf{B}$ and $\mathbf{c}$.
2. Conditional on $\mathbf{B}$ and $\mathbf{c}$, solve (analytically) for the parameters $\mathbf{a}$ that match the observed budget shares:

$$a_j = \left(s_j^{true}/r_j\right)\left(c_0 y + \sum_k c_k r_k y\right) - \sum_k b_{jk} r_k - c_j y.$$

3. Conditional on $\mathbf{B}$ and $\mathbf{c}$, estimate the unobserved characteristics ξ_j, by estimating $a_j = g_0\left(\mathbf{x}_j, \xi_j; \beta\right)$.
4. Estimate $\mathbf{B}$ and $\mathbf{c}$ by GMM, assuming the true unobserved characteristics are mean independent of a set of instruments $\mathbf{Z}$:

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- We run a Monte Carlo experiment to analyze the convergence properties of our proposed demand model.
- We consider a setting with $J = 120$ products and $T = 10$ markets.
- The data-generating process (500 replications) is as follows for each market:
  - Draw 6 observed characteristics for each product (uniform distribution): $x_{1j}, \ldots, x_{6j}$
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- We consider the following mapping of consumer preferences onto the space of characteristics:

$$\begin{aligned} a_j &= \sum_l \beta_l x_{lj} + \xi_j \\ b_{jj} &= \alpha_{11} x_{1j} + \alpha_{12} x_{2j} \\ b_{jk} &= \alpha_{21} \left| x_{3j} - x_{3k} \right| + \alpha_{22} \left| x_{4j} - x_{4k} \right| \\ c_j &= \alpha_{31} x_{5j} + \alpha_{32} x_{6j}. \end{aligned}$$

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- We consider the following set of instruments:
  1. BLP instruments.
  2. The sum of the distances in observed characteristics between each product and its rivals.
  3. The interaction of BLP instruments with the sum of the distances in observed characteristics between each product and its rivals.
  4. The interaction of BLP instruments and the sum of the distances in observed characteristics between each product and its rivals with income.
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Non-Linear Parameters					
	True	*Mean*	*Std Dev*	*Median Bias*	*Median Abs. Error*
α_{11}	20	19.98	0.67	0.02	0.44
α_{12}	20	20.03	0.65	0.04	0.45
α_{21}	-0.1	-0.10	0.00	-0.00	0.00
α_{22}	-0.1	-0.10	0.00	0.00	0.00
α_{31}	0.1	0.10	0.00	-0.00	0.00
α_{32}	0.1	0.10	0.00	-0.00	0.00
Linear Parameters					
β_0	1	1.00	0.07	0.00	0.05
β_1	1	1.00	0.24	0.00	0.17
β_2	1	0.99	0.23	-0.01	0.16
β_3	1	1.00	0.03	-0.00	0.02
β_4	1	1.00	0.03	-0.00	0.02
β_5	1	1.00	0.08	0.01	0.05
β_6	1	1.00	0.08	-0.00	0.05

- The results suggest that the algorithm performs reasonably well, with all parameters converging to the true values.

- In what the price parameters are concerned, a bit surprisingly it seems to be easier to identify cross-price than own-prices parameters.

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- We propose a new method of uncovering demand information from market level data on differentiated products.
  - Flexible in the sense of Diewert (1974) and globally consistent in the sense it can deal with entry and exit of products over time.
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- We are now in the process of expanding the Monte Carlo experiments and also of applying the model to the carbonated soft drink industry.
    - An industry characterized by a large number of products and by consumer purchases typically involving more than one unit of product.