Competition with Capacity Constraints and Non-Linear Pricing, with an Application to Mergers*

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Abstract

This paper analyzes a model of competition between capacity-constrained firms. In contrast to existing literature, we consider the case in which firms can set non-linear price schedules. We show that there exists a unique pure-strategy equilibrium, in which each firm obtains profits on its pivotal units (i.e., the part of demand that can only be served by this firm) but prices non-pivotal units at marginal costs. The model is highly tractable and capable of providing policy guidance in complex market environments. We analyze mergers and show that effects are driven by the change in pivotality, thereby providing a micro-foundation for the use of pivotality analyses in antitrust.

JEL classification: D21, D43, , K21, L13, L41

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1 Introduction

Several industries, such as the steel and aluminum production industry as well as several chemical industries, are characterized by firms facing capacity constraints. This usually occurs as production facilities and factories only have a certain capacity, and it is very costly or even impossible for firms to increase production beyond the factory's capacity limit.

The canonical model to analyze competition between firms with capacity constraints is the Bertrand-Edgeworth model in which capacity constrained firms set linear prices to consumers. As has been show by the seminal contributions in this literature (i.e., Beckman, 1965, Levitan and Shubik, 1972, and Dasgupta and Maskin, 1986), the resulting equilibrium is in mixed strategies, which implies that firms randomize their prices over a sizable range. This appears not only to be at odds with real-world market outcomes but also implies that "a firm's sales (and payoff) are a discountinuous function of its strategy (i.e. its price)" (Shapiro, 1986, p. 346), which is a fundamentally unrealistic feature. In addition, due to its discontinuities and the lack of a pure-strategy equilibrium, the model is intractable for many applied purposes, which has severely hampered its use.

An important assumption in the existing literature is that firms set linear prices. While this is a natural assumption to start with, it is nowadays no longer fulfilled due to a rise in automated pricing, made possible by software programs or artificial intelligence. This makes it easy to charge different prices for different units sold. Even before this time, some papers, such as Rust et al. (2004) and Figuerola-Ferretti and Gilbert (2005), document non-linear pricing in the steel and the aluminum market, respectively. The question therefore arises how competition with capacity-constrained firms works in case firms can charge non-linear prices.

The goal of this paper is to analyze this situation, provide a characterization of the equilibrium, and apply it to merger control. We consider a model with a general number of firms, offering a homogeneous good to consumers. Firms can have different capacities but, following the literature, we consider symmetric and constant marginal costs.

Our model differs from the canonical Bertrand-Edgeworth setting in two respects. First, we relax the assumption that firms can only post uniform prices and instead allow for the possibility of price discrimination. In particular, we allow a firm to set a different price for each unit it sells (within its capacity limit), thereby considering full non-linear pricing. This implies that firms compete in price schedules. Second, as prices for different units can differ not only between firms but also within a firm, we need to specify how the the aggregate capacity is allocated to consumers once firms have set their price schedules (i.e., the rationing rule). Here we consider two possibilities: The first is an adaptation of the efficient rationing-rule to our setting, which implies that consumers first buy from the firm that offers the largest consumer surplus for its supplied units. The second is a Walrasian auctioneer, who aggregates purchases on behalf of buyers, which is equivalent to assuming a single purchaser. We show that both possibilities lead to the same equilibrium.

We obtain the following results: First, an equilibrium in pure strategies occurs. This is in sharp contract to most results on Bertrand-Edgeworth competition in which, as mentioned above, only a mixed-strategy equilibrium arises.

Second, in this equilibrium, a firm sets a different price for its pivotal units compared to the non-pivotal units. The pivotal units of a firm are those units that only the firm can supply, that is, the part of the demand that cannot be served by other firms due to the capacity constraints. Instead, the non-pivotal units are the part of the demand that can be served by more than one firm. As the non-pivotal units are therefore subject to competition, in equilibrium firms set the price equal to marginal costs for these units. Instead, they obtain profits on pivotal units. These profits are equal to the willingness-to-pay of the consumers with the lowest valuations.

Third, to implement this equilibrium, firms' price schedules follow quantity discounts. The first units are offered at a high price which falls until all pivotal units are bought. For the remaining non-pivotal units, the price stays constant at marginal costs.

Fourth, we show that the degree of market power in the industry is directly related to the degree of pivotality. In particular, the higher this degree, the larger is share of producer surplus relative to consumer surplus.

These results show that considering non-linear pricing in a model of competition with capacity constraints leads to a highly tractable and intuitive equilibrium that is amenable to comparative statics analysis and policy application. In particular, the model allows evaluating policy questions with a direct focus on the implications of market power. Given the paucity of results in more complex settings, we believe this is a useful step towards a more meaningful analysis of homogenous goods markets with capacity constraints.

Specifically, we apply the model to the question of merger control in homogeneous goods markets. As already recognized by Edgeworth's original contribution, market power in homogeneous goods industries typically stems from the existence of capacity constraints. Due to the complexities of the Bertrand-Edgeworth model, however, relatively little is so far known about merger effects in this setting—common as it may be, as exemplified by recent cases in the European Union in which the European Commission relied the Bertrand-Edgeworth model (e.g., *Outokumpu/Inoxum* in the stainless steel industry and *Novelis/Aleris* in the aluminum industry).[1] The main result of this literature is that there is no merger effect if capacity is either very small or very large relatively to demand. Froeb et al. (2003) showed this in a computational experiment that is applied to a parking merger in the U.S. Chen and Li (2018) show that the result holds more generally in Bertrand-Edgeworth games with symmetric firms.

Our model, by contrast, allows deriving far more specific guidance on the conditions when mergers are likely to increase market power. First, merger effects are substantially driven by the change in pivotality caused by a merger (i.e., the part of demand over which individual suppliers exercise market power because other firms cannot serve it). Our model therefore provides a micro-foundation for the use of pivotality analysis in antitrust and regulation. Second, merger effects are amplified by efficiency differences in off-equilibrium capacity between merging and non-merging firms. Third, we derive simple closed form solutions for merger effects that depend on parameters that are observable by competition authorities (e.g., margins, capacity shares, and capacity utlization). Finally, we provide more specific results on how mergers change pivotality. Our analysis should therefore also prove useful for other models of competition that prominantly feature capacity constraints (and hence pivotality).[2]

Related literature: The seminal contribution in the literature analyzing Bertrand-Edgeworth competition are by Beckman (1965) and Levitan and Shubik (1972). Several papers, including

[1] Case M.6471 *Outokumpu/Inoxum* and Case M.9076 *Novelis/Aleris*, respectively.

[2] E.g., both in Bertrand-Edgeworth competition and in supply function equilibria, whether firms can exercise market power depends on their degree of pivotality (see Vives, 1986, and Genc and Williams, 2011).

Dasgupta and Maskin (1986), Maskin (1986), Allen and Hellwig (1986), Osborne and Pitchik (1986), and Vives (1986), extended and generalized these analysis in several ways e.g., by proving existence and uniqueness of the mixed strategy equilibrium, considering product differentiation, and studying limit pricing. However, non of these papers considers non-linear prices.

Subsequent literature has attempted to vary the Bertrand-Edgeworth setup to obtain cleaner results. E.g., Dudey (1992), Martínez-de-Albéniz and Talluri (2011), Anton et al. (2014) and Cattaneo et al. (2021) consider dynamic Betrand-Edgeworth models. In stylized settings, these papers show that some of the issues plaguing Bertrand-Edgeworth equilibria can be alleviated. As a result, this literature has advanced our understanding of dynamic pricing policies. Yet, these dynamic models also add new technical requirements, that make them difficult to apply in other contexts. While these papers allow for different prices at different time periods, they still consider uniform pricing within a time period.

Finally, there is a small literature on mergers in Bertrand-Egdeworth games with uniform pricing, i.e., Chen and Li (2018) and Hirata (2009). However, they were able to make progress indetermining the competitive effects of mergers only under restrictive assumptions, such as firms being symmetric (Chen and Li, 2018) or the number of firms being limited to three (Hirata, 2009).

The paper is structured as follows. Section 2 describes the model, analyzes the case with inelastic demand, and studies merger effects and shows that they are driven by changes in pivotality. Section 4 goes on to allow for general demand functions. This makes the analysis somewhat more involved, but retains the decisive role of pivotality. Section 4, finally, concludes.

2 Rectangular demand

Consider a market for a homogeneous product. The demand function is inelastic and given $p(x) = v$ for $x \leq \theta$ and $p(x) = 0$ for $x > \theta$. There are $n \geq 2$ firms with individual capacity $k_i > 0$ and total capacity $k = \sum_i k_i$. All firms have constant marginal cost $c \geq 0$ until their capacity limit binds.

In contrast to the standard Bertrand-Edgeworth model, we consider the situation in which firms can not only set linear prices but price schedules. In particular, for each unit $u_i \in [0, k_i]$, firm i can set $p_i(u_i)$, which is the price p_i as a function of the purchased unit u_i, that is, a firm is allowed to set a different price for each unit it sells and can explicitly distinguish between the ranking of the units (e.g., charge a different price for the first units it sells compared to later units). We do not restrict $p_i(u_i)$ in any way other than having to fulfill the capacity constraint ($u_i \leq k_i$). Our model therefore allows for a large variety of non-linear pricing schemes (including volume rebates and fully individualized pricing).[3]

As firms are capacity constrained, we need to describe how demand is allocated to firms. We allow for two possibilities (and show below that they lead to the same equilibrium outcomes). The first is a modified efficient rationing rule. Following e.g. Kreps and Scheinkman (1983) and Vives (1986), with linear pricing, efficient rationing implies that the firm with the lower price serves consumers first. We adapt this rule to our case in which firms offer price schedules, by supposing that the firm that offers the higher consumer surplus serves consumers first. Second, we consider a "Walrasian" sales process, where firms set price schedules to a Walrasian auctioneer, who aggregates purchases on behalf of buyers. This is equivalent to assuming a single purchaser.

Firms compete by simultaneously setting price schedules and then consumers decide if and where to buy, according to the rule specified above. We solve for the Nash equilibrium of the game.

To make the problem interesting, we assume that the aggregate capacity in the market exceeds θ, that is $k \equiv \sum_i k_i \geq \theta$. If $k < \theta$, demand exceeds aggregate supply, leading to monopoly pricing irrespective of other market characteristics, i.e., each firm charges a price of v for all units.

2.1 Pre-merger equilibrium

Before solving the game, it is useful to define a notion of pivotality of a firm, that is, the part of market demand that would not be served in the absence of the firm. Define $\tilde{k}_i = \max\{\theta - k_{-i}, 0\}$ as the pivotal capacity of firm i, where $k_{-i} = \sum_{j \neq i} k_j$. Similarly, $\tilde{k} = \sum_i \tilde{k}_i$ denotes the pivotal capacity of the market. It will often be useful to express pivotality in relative terms. To this end,

[3] For instance, a volume rebate could be of the form

$$p_i(u_i) = \begin{cases} \bar{p}_i & \text{for} \quad u_i \in [0, \underline{k}], \\ \underline{p}_i & \text{for} \quad u_i \in (\underline{k}, k_i], \end{cases}$$

with $\underline{k} < k_i$ and $\underline{p}_i < \bar{p}_i$.

let $\mu_i = \tilde{k}_i/\theta$ denote pivotality as a percentage of demand, with $\mu = \sum_i \mu_i \in [0,1]$.[4] Similarly, let $\gamma_i = \tilde{k}_i/k$ denote pivotality as a percentage of supply, with $\gamma = \sum_i \gamma_i \in [0,1]$. In what follows, we will mostly use μ, which expresses the percentage of sales that is effectively subject to monopoly power. Even so, it will sometimes be convenient to use γ to derive specific results.

We first characterize the equilibrium of the game. Consider the following equilibrium candidate: For all $i = 1, ..., n$,

$$p_i(u_i) = \begin{cases} v & \text{for} \quad u_i \in \left[0, \tilde{k}_i\right], \\ c & \text{for} \quad u_i \in \left(\tilde{k}_i, k_i\right], \end{cases}$$

that is, each firm's price schedule is a quantity discount or volume rebate in which the firm charges a price of v for its first $\tilde{k}_i$ units—i.e., its pivotal units—and a price of c for the remaining $k_i - \tilde{k}_i$ units—i.e., its non-pivotal units. It is easy to see that no firm has a profitable deviation from this price schedule. For its pivotal units, a firm achieves full extraction of consumer surplus; hence, it cannot do better. Conversely, for non-pivotal units, there is competition against a price of c, which cannot be profitably undercut. A deviation to increase the price for non-pivotal units above c could only be profitable if a firm can ensure that, given the equilibrium candidate, it offers a larger consumer surplus than other firms, which implies that consumers will buy from the firm first, even if it increases prices above c. However, the number of non-pivotal firms is the same for all firms that make profits and equal to $k - \theta$.[5] Therefore, if a firm would raise its price of non-pivotal units above c, it would be last firms that consumers buy from, which implies that it does not sell its non-pivotal units. Hence, such a price increase cannot be profitable. Therefore, the candidate equilibrium is indeed an equilibrium. This holds regardless of whether there is a single purchaser or whether consumers are served according to the modified rationing rule. The equilibrium outcome is also unique, because non-pivotal units are subject to Bertrand competition, whereas pivotal units are subject to monopoly pricing.[6]

[4] In electricity markets, $1 - \mu_i$ is often called the "residual supply index" (RSI) of firm i. It is usually applied to the largest firm in the market as a measure of dominance. As we will see below, however, market pivotality μ is generally a more accurate predictor of market power than the RSI of the largest firm – not least because in industries with capacity constraints, market power can easily arise also with fragmented suppliers (in particular, in situations of high capacity utilization).

[5] This is due to the fact that $k_i - \tilde{k}_i = k_i - \theta - \sum_{j \neq i} k_j = k - \theta$.

[6] The equilibrium itself is not necessarily unique, as e.g. in case of $n \geq 3$, only two firms charging a price of c for their non-pivotal units, whereas the others charging a price above c also constitutes an equilibrium. However, these equilibria are outcome equivalent.

The following proposition summarizes this result:

Proposition 1 *In the unique equilibrium outcome, each firm i obtains a profit of $v - c$ for its pivotal units $\tilde{k}_i$, and a profit of zero for its non-pivotal units $k_i - \tilde{k}_i$.*

As an example consider a situation with $\theta = 10$ and $n = 3$. The capacity constraints are $k_1 = 5.5$, $k_2 = 3.5$, and $k_3 = 2$. It follows that $\tilde{k}_1 = 4.5$, $\tilde{k}_2 = 2.5$, and $\tilde{k}_3 = 1$. Therefore, consumers buy 8 units at a price of v and 2 units at a price of 2. The average price in the market $\bar{p}$ is thus $\bar{p} = (8v + 2c)/10$.

It is easy to see, and also explained above, that all firms offer the same consumer surplus as each firm prices only its last unit at c but all other units at v, thereby leaving a consumer surplus of $v - c$. Therefore, regardless of whether applying the modified efficient rationing rule or whether there is a single purchaser, consumers will buy the entire capacity from two of the three firms and only the pivotal units from the third firm. The identity of the firms does not matter, as non-pivotal units are priced at marginal costs.

Generalizing the formula for the average market price from the example above, it is evident that it is determined by the market's degree of pivotality μ, which captures the percentage of sales that are subject to monopoly pricing. Specifically,

$$\bar{p} = \mu v + (1 - \mu) c.$$

It follows that industry profit is given by $\pi = \mu (v - c) \theta$, whereas consumer surplus is given by $CS = (1 - \mu)(v - c) \theta$. Thus, total surplus is split between producers and consumers according the degree of pivotality μ.

As a consequence, pivotality $\mu \in [0, 1]$ is a conduct parameter that measures the degree of market power prevailing in a market. It therefore makes sense to inspect the notion of pivotality in some more detail. Defining $s_i = k_i/k$ as the capacity share of firm i and $e = (k - \theta)/k > 0$ as the market's excess capacity as a percentage of total capacity, we obtain that, for pivotal firms, $\gamma_i = \tilde{k}_i/k = (\theta - k_{-i})/k = \theta/k - (1 - s_i) = s_i - (k - \theta)/k = s_i - e$. As $\gamma_i = 0$ for non-pivotal firms,

$$\gamma_i = \max\{s_i - e, 0\} \tag{1}$$

Using that $k/\theta = (k-\theta)/\theta + 1 = ek/\theta + 1 = 1/(1-e)$, where the last equality follows from $k/\theta = 1/(1-e)$, yields

$$\mu = \frac{\gamma}{1-e}.$$

Therefore,

$$\mu_i = \max\left\{\frac{s_i - e}{1-e}, 0\right\}. \tag{2}$$

This formula shows that pivotality can be viewed as a adjusted market share analysis that additionally accounts for the aggregate degree of capacity utilization. When firms' plants are fully utilized ($e = 0$), pivotality levels are identical to capacity shares ($\gamma_i = \mu_i = s_i$). If capacity utilization decreases from that level, however, pivotality levels uniformly decrease below capacity shares. Once excess capacity in the market is larger than an individual firm's capacity share, that firm ceases to be pivotal altogether (and hence $\gamma_i = \mu_i = 0$). The following lemma shows more formally the close connection between pivotality, market concentration, and capacity utilization.[7]

Lemma 1 *For a given capacity allocation $\{k_1, ..., k_n\}$: (i) Pivotality measures μ and γ are (weakly) decreasing in the amount of excess supply e in the market. (ii) Pivotality measures μ and γ are (weakly) increasing in market concentration H, where $H = \sum_i s_i^2$ is the market's (capacity-based) HHI.*

As we will see in the next section, also merger effects are driven by these two factors (concentration and capacity utilization), but not necessarily in the way one might expect.

2.2 Merger analysis

Assume now that firms 1 and 2 merge. Without loss of generality, let firm 1 be the larger merging firm, so $k_1 \geq k_2$. As customary, we consider the percentage change in the average price $\Delta\bar{p}/\bar{p}$ caused by a merger. Since $\bar{p} = \mu v + (1-\mu)c$, any change in price must come about by the change in pivotality caused by a merger. From (2), a merger between firms 1 and 2 does not change the pivotality of non-merging firms. Even so, the pivotality of the merging firms will typically increase through a merger. Proposition 2 shows how this affects post-merger prices.

[7] The proof of this and all following results can be found in the appendix.

Proposition 2 *With rectangular demand, for all $\mu > 0$, the price effect of a merger is given by:*

$$\frac{\Delta \bar{p}}{\bar{p}} = \frac{\Delta \mu}{\mu} m \tag{3}$$

where $m = (\bar{p} - c) / \bar{p}$ is the average margin in the market.

Proposition 2 shows that merger effects have a remarkably simple structure in our base model. Specifically, price effects are driven by two variables that are directly observable by competition authorities: firms' pre-merger market power (m) and the change in pivotality caused by a merger ($\Delta\mu/\mu$) (rather than its level).[8]

Since the merger-induced change in pivotality plays such an important role in markets with capacity constraints, it is useful to explore its underlying driving forces. Proposition 3 shows that $\Delta\mu/\mu$ follows a clean and parsimonious rule.

Proposition 3 *For all $\mu > 0$, the change in pivotality caused by a merger is given by:*

$$\frac{\Delta \mu}{\mu} = \frac{\Delta \gamma}{\gamma} = \begin{cases} e/\gamma & \text{if } e \in [0, s_2) \\ s_2/\gamma & \text{if } e \in [s_2, s_1) \\ (s_1 + s_2 - e)/\gamma & \text{if } e \in [s_1, s_1 + s_2) \\ 0 & \text{if } e \geq s_1 + s_2 \end{cases}$$

Proposition 3 implies that merger effects strongly depend on how many of the merging firms are pivotal. If neither the merging firms nor the merged entity is pivotal ($e \geq s_1 + s_2$), then a merger causes no price effects. If the merged entity is pivotal but not the merging firms separately ($e \in [s_1, s_1 + s_2)$), then merger effects are increasing in the share of both merging parties ($\Delta\mu/\mu = (s_1 + s_2 - e)/\gamma$). If only one the merging firms is pivotal pre-merger ($e \in [s_2, s_1)$), then merger effects are increasing in the share increment of the smaller firm alone ($\Delta\mu/\mu = s_2/\gamma$). Finally, if both merging firms are pivotal pre-merger ($e \in [0, s_2)$), then merger effects are independent of the shares of either party ($\Delta\mu/\mu = e/\gamma$).[9]

[8] The absolute price change $\Delta\bar{p}$ can be found by replacing the percentage margin m in (3) with the absolute margin $\bar{p} - c$.

[9] Note that this comparison considers different merger possibilities within a given market, that is, we undertake a ceteris paribus comparison that takes the distribution of capacities $\{k_1, ..., k_n\}$, and hence γ, as given.

An immediate consequence of the result that $\Delta\mu/\mu = e/\gamma$ in case no firm is pivotal is that regardless of the capacity allocation $\{k_1, ..., k_n\}$, any merger between two pivotal firms increases market pivotality by the same amount, and thus also generates the same price effect.

Consider for example the industry described in the example below Proposition 1, in which all firms are pivotal. A merger between firms 1 and 2 to a new firm denoted by $1+2$ implies that capacity constraints are $k_{1+2} = 9$ and $k_3 = 2$. In the resulting equilibrium, $\tilde{k}_{1+2} = 8$ and $\tilde{k}_3 = 1$, leading to an average price of $\bar{p}$ is thus $\bar{p} = (9v + c)/10$, and therefore to a average (percentage) price increase of $\Delta\bar{p}/\bar{p} = (v - c)/(8v + 2c)$. The same result occurs for a merger between any two other firms.

Besides determining a closed-form solution for merger effects, Proposition 3 also addresses the more general role of concentration analysis in mergers with capacity constraints. In real world merger proceedings, market shares continue to be by far the most important variable used by competition authorities and courts. A recent literature has therefore analyzed the predictive value of market shares in different settings (e.g., see Nocke and Whinston, 2022, and Reisinger and Zenger, 2022). Proposition 3 shows that in markets with capacity constraints, the role of capacity shares crucially depends on how many of the merging firms are pivotal. Merger effects are driven either by both firms's share (if none of them is pivotal), by the smaller merging firm's share (if one firm is pivotal), or by neither of the two (if both firms are pivotal).

We next consider the relationship between capacity utilization and merger effects. From Lemma 1, higher capacity utilization implies a larger degree of pre-merger market power. One might therefore suspect that higher capacity utlization also tends to amplify merger effects (much like higher market shares). Yet, in general this is not so. This is easiest to see by considering the asboute change in pivotality $\Delta\mu$, which denotes the percentage point increase in market power that a merger causes. Using (2) yields:

$$\Delta\mu = \begin{cases} e/(1-e) & \text{if } e \in [0, s_2) \\ s_2/(1-e) & \text{if } e \in [s_2, s_1) \\ (s_1 + s_2 - e)/(1-e) & \text{if } e \in [s_1, s_1 + s_2) \\ 0 & \text{if } e \geq s_1 + s_2 \end{cases}$$

Now consider a given capacity allocation $\{k_1, ..., k_n\}$ and vary the degree of excess capacity e by changing θ. This yields the following result:

Corollary 1 *(i) The impact of capacity utilization on the change in pivotality $\Delta\mu$ caused by a merger follows an inverted U-shape with a maximum at $e = s_1$. (ii) In markets with large excess supply ($e \geq s_1 + s_2$) and in markets with excess demand ($e \leq 0$), mergers do not change pivotality ($\Delta\mu = 0$).*

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Figure 1 about here

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Figure 1[10] displays inverted U-shape relation of excess capacity on $\Delta\mu$ (and hence merger effects).[11] While higher capacity utilization increases merger effects for low levels of utilization, the opposite is true for high levels of utilization. The intuition is as follows: When demand is large relative to supply pre-merger, so utilization is high ($e \to 0$ from above), then even fragmented firms have substantial market power pre-merger, as most of their capacity is already pivotal. A merger will therefore not increase market power much further, even if it involves significant shares. Conversely, when supply is large relative to demand pre-merger, so utilization is low ($e \to s_1 + s_2$ from below), then competition will be intense even after a substantial increase in concentration, because the merged entity's rivals hold so much competitive spare capacity.

The latter argument is often the main argument raised by merging parties in merger control proceedings. Since goods are homogeneous and rivals have some spare capacity, it is argued, any attempt by the merged entity to raise prices will be undercut by competition from rivals. As a closer inspection of Corollary 2 shows, however, the argument has rather narrow limits. Because

[10] This figure and the following ones can be found at the end of the appendix.

[11] Note that $\Delta\mu$ is proportional to the price change $\Delta\bar{p}$ caused by a merger as $\Delta\bar{p} = (\Delta\mu/\mu)(\bar{p} - c) = \Delta\mu(v - c)$, where v and c are constants. Hence, $\Delta\bar{p}$ changes with e in the same way as $\Delta\mu$.

unless excess capacities in the market are truly significant ($e > s_1 + s_2$), spare capacities will not be sufficient to offset the increased pivotality caused by a merger (in particular, if the merging firms' capacity shares are large).

More specifically, $e > s_1 + s_2$ implies that neither of the merging firms is pivotal pre-merger, and so none of them earns a positive margin. If, instead, at least one of the merging firms earns a positive margin, then this shows that rivals' excess capacities failed to undermine the exercise of pricing power even pre-merger. Much less would they have an incentive to undermine price increases post-merger when pivotality increases even further.

Further, $e > s_1 + s_2$ implies that the merged entity must not become pivotal either. In mergers that involve significant capacity shares, this requires an unusually large imbalance between demand and supply. E.g., if the merging parties have a joint market share of 40%, then at least 40% of production capacities in the market must be idle to prevent a price increase. One should therefore expect the "large spare capacity" argument to hold only in exceptional cases.[12]

3 General demand

In the previous section, we have assumed that demand is inelastic. In this section, we consider a general downward-sloping demand $x(p)$. The largest demand that firms can profitable serve is then $x(c)$.[13]

In line with the definitions of the previous section, we denote firm i's pivotal capacity by $\tilde{k}_i = \max\{x(c) - k_{-i}, 0\}$, where $k_{-i} = \sum_{j \neq i} k_j$. In addition, $\mu_i = \tilde{k}_i / x(c)$ denotes firm i's pivotality as a percentage of demand, with $\mu = \sum_i \mu_i \in [0, 1]$. As above, we assume that aggregate supply exceeds demand at a price of c, that is $\sum_i k_i \geq x(c)$. All other assumptions are as above.

As demand is downward-sloping, firms will no longer charge the same price for its pivotal

[12]It is interesting that the opposite "small spare capacity" argument is much less common. For obvious reasons, merging parties tend to be reluctant to argue that their merger is harmless because they possess so much market power already. Moreover, when capacities are tight, competition authorities are more likely to take a dynamic perspective on merger effects: Either by exploring a theory of harm with respect to capacity competition (á la Cournot); or by accounting for demand cyclicality (which may imply that capacity tightness is merely a temporary phenomenon, whereas the merger is a permanent change in the structure of the market).

[13]At prices below c, firms would not be willing to serve incremental demand anymore, even if they have the capacity to do so.

units. However, the equilibrium is characterized by similar price schedules of firms as above. Specifically, the price schedules that constitute an equilibrium are[14]

$$p_i(u_i) = \begin{cases} p\left(x(c) - \tilde{k}_i + u_i\right) & \text{for} \quad u_i \in \left[0, \tilde{k}_i\right], \\ c & \text{for} \quad u_i \in \left(\tilde{k}_i, k_i\right], \end{cases}$$

for all $i = 1, ..., n$. This implies that firms again offer a downward-sloping price schedule, but this time in a continuous way. For all non-pivotal units, firms are in Bertrand competition and charge a price of c. For all pivotal units, a firm charges unit prices that are equal to the surplus of consumers with valuations between $p\left(x(c) - \tilde{k}_i\right)$ and $p\left(x(c)\right)$, that is, for consumers with the lowest valuations. This occurs because, given that consumers' buying behavior is either characterized by the adjusted efficient rationing rule or by behaving as a single purchaser, they buy first from firms which offer the highest consumer surplus. It follows that in the equilibrium pricing schedules, the unit price falls gradually down to marginal costs for pivotal units and is equal to marginal costs for non-pivotal units. An implication of the result is that firms obtain profits only at the level of the consumers with the lowest valuations and according to their pivotality.

Akin the Proposition 1, the next proposition states this equilibrium outcome:

Proposition 4 *In the unique equilibrium outcome, each firm i obtains a profit of $\int_{x(c)-\tilde{k}_i}^{x(c)} (p(z) - c)\, dz$ for its pivotal units $\tilde{k}_i$, and a profit of zero for its non-pivotal units $k_i - \tilde{k}_i$.*

As in the last section, we can illustrate the result of the proposition with an example. Consider a market with a demand function of $p(x) = 10 - x$ and 3 firms. All firms have constant marginal cost $c = 2$. The capacities are $k_1 = 3.5$, $k_2 = 3$, and $k_3 = 2$. With these numbers $x(c) = x(2) = 8$. The resulting pivotal capacity of each firm is $\tilde{k}_1 = 3$, $\tilde{k}_2 = 2.5$, and $\tilde{k}_3 = 1.5$. The following price schedules therefore constitute an equilibrium of the game. For its first 3 units, firm 1 charges price of $p(5+u_1)$, with $u_1 \in [0, 3]$, and a price of 2 for its final 0.5 units. Similarly, firm 2 charges price of $p(5.5+u_2)$, with $u_2 \in [0, 2.5]$, for its first 2.5 units and a price of 2 for its final 0.5 units, and firm 3 a price of $p(6.5 + u_3)$, with $u_3 \in [0, 1.5]$, for its first 1.5 units and a price of 2 for

[14] We show this in the proof of Proposition 4.

its final 0.5 units. Two out of the three firms sell all their capacity, while one firr only sells its pivotal units. The average price is $(1.5 \times 2.75 + 2.5 \times 3.25 + 3 \times 3.5 + 2)/8 = 99/32 \approx 3.1$.

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Figure 2 about here

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The equilibrium is depicted in Figure 2 (in which assume that firms 2 and 3 sell their entire capacity). In this figure, the gray-shaded areas are the profits of the firms. Each firm obtains a profit with its pivotal units given by $\tilde{k}_i$, with $i = 1, 2, 3$. Given our definitions, $\tilde{k}_i$ can be written as $(1 - \mu_i)x(c)$, which explains the formulas at the x-axis.

Using these insights, we can provide a formula for the average price. It is given by

$$\frac{1}{x(c)} \left[\sum_i \int_0^{\tilde{k}_i} p(x(c) - u)du + \left(x(c) - \tilde{k} \right) c \right],$$

which can be written as

$$\frac{1}{x(c)} \left(\sum_i \int_{x(c)(1-\mu_i)}^{x(c)} p(z)dz \right) + (1 - \mu)\, c. \tag{4}$$

It is evident from (4), that the degree of pivotality is important for the split between consumer and producer surplus also in case of general demand. However, the formula is more involved compared to the case with rectangular demand and now does not only depend on the degree of pivotality in the industry (i.e., μ), but also how it is distributed between firms (i.e., μ_i is now part of the average price). This is because, holding aggregate capacity fixed, shifting more and more capacity to a firm with already a large amount of capacity, allows the firm to benefit over-proportionally compared to the loss of those firms with less capacity. This can also be seen in Figure 2. If capacity was shifted from firm 3 to firm 1 in the example, the average price would increase, although aggregate capacity stays unchanged.

We now turn to the effects of a merger between firms 1 and 2. As just described, the profit of a firm i with a share μ_i of pivotal units can be written as

$$\psi_i(\mu_i) := \int\limits_{(1-\mu_i)x(c)}^{x(c)} \frac{p(z) - c}{x(c)} dz \tag{5}$$

Since demand is downward-sloping, ψ is increasing in μ_i. This implies that price effects are larger than in the case with rectangular demand. Indeed, with general demand, mergers not only increase the pivotality μ of the merging firms, but also the average profit margin ψ of pivotal sales.

The price change of the merger is therefore given by

$$\frac{1}{x(c)} \left(\int_{x(c)(1-\mu_{1+2})}^{x(c)} p(z)dz - \int_{x(c)(1-\mu_1)}^{x(c)} p(z)dz - \int_{x(c)(1-\mu_2)}^{x(c)} p(z)dz \right) - (\mu_{1+2} - \mu_1 - \mu_2)\, c. \tag{6}$$

As in case of rectangular demand, a merger increases the level of pivotality, that is $\mu_{1+2} > \mu_1 + \mu_2$, which leads to a price increase. In addition, as just described, the newly merged firm now also sets a price schedule which involves higher prices than without the merger, due to the fact that $p(x(c)(1 - \mu_{1+2})) > p(x(c)(1 - \mu_1)) + p(x(c)(1 - \mu_2))$. This effect is absent with rectangular demand and leads to a higher increase in the average price.

We illustrate the price increase with the example used above. If in this example firm 1 and firm 2 merge, they have a total capacity of $k_{1+2} = 6$ and firm 3's capacity remains at $k_3 = 2$. The pivotal capacity of the firms is then $\tilde{k}_{1+2} = 6$ and $\tilde{k}_3 = 1.5$. It follows that the newly merged firm 1+2 charges prices of $p(2 + u_1)$, with $u_1 \in [0, 6]$, and a price of 2 for its final 0.5 units and firm 3 charges a price of $p(6.5 + u_3)$, with $u_3 \in [0, 1.5]$, for its first 1.5 units and a price of 2 for its final 0.5 units. The average price is $(1.5 \times 2.75 + 6 \times 5 + 0.5 \times 2)/8 = 281/64 \approx 4.4$. Therefore, the price increase is approximately 1.3, which is around 42%. This is displayed in Figure 3.

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Figure 3 about here

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4 Conclusion

This paper provides an analysis of competition between capacity-constrained firms that are able to charge non-linear price schedules. We show that, in contrast to existing models that focus on uniform prices by firms, the equilibrium with non-linear pricing is simple, tractable, and intuitive. It underscores the notion of pivotality of a firm. In particular, in the equilibrium, firms make profits on their pivotal units and are able to extract the full consumer surplus for these units, although only the consumer surplus from low valuation consumers. Instead, non-pivotal units are priced at marginal costs. We apply our model to mergers and show the change in pivotality is important indicator for merger effects.

We note that our results on pivotality are not restricted to our specific model of homogeneous good competition. Indeed, it is applicable to any model of competition where the degree of capacity utilization (and hence pivotality) plays a major role. For instance, both in Bertrand-Edgeworth competition and in supply function equilibria, the degree of market power is predominantly determined by firms' pivotality (see Vives, 1986, and Genc and Reynolds, 2011, respectively).

Our results are generally in line with results of other merger models. Merger effects are always driven by a specific factor determining the competitive interaction between the parties. This latter element depends on the type of competition in the specific model at hand. In differentiated Bertrand competition, it is given by the diversion ratios between the merging parties (Werden, 1996). In Cournot competition, it is given by the market share increment caused by the merger (Reisinger and Zenger, 2022). In price competition with capacity constraints, it is given by the increase in pivotality caused by a merger.

Appendix

Proof of Lemma 1

(i) Note that, for a given asset allocation, an increase in e is equivalent to a decrease in θ. From (2), it follows that

$$\mu = \frac{k}{\theta}\sum_{i=1}^{n} \max\left\{s_i - e, 0\right\} \tag{7}$$

Since $k/\theta = 1/(1-e)$ as shown in the text, this is equivalent to

$$\mu = \sum_{i=1}^{n} \max\left\{\frac{s_i - e}{1-e}, 0\right\}.$$

Defining $\tilde{n}$ as the number of pivotal firms and $\tilde{s}$ as the combined capacity share of pivotal firms, we thus have:

$$\frac{\partial \mu}{\partial e} = \frac{-\tilde{n}(1-e) + \tilde{s} - \tilde{n}e}{(1-e)^2} = \frac{\tilde{s} - \tilde{n}}{(1-e)^2} \leq 0$$

This inequality continues to hold as the number of pivotal firm decreases with a rise in e, that is, even for the last firm j that remains pivotal, the numerator reads $\tilde{s}_j - 1 \leq 0$. And once there is no pivotal firm anymore, the numerator equals zero. The proof for γ is similar. Hence, part (i) follows.

With respect to part (ii), consider some given capacity allocation. Now move some capacity share Δ from firm i to firm j. This change in capacity shares changes the HHI by $(s_i - \Delta)^2 + (s_j + \Delta)^2 - s_i^2 - s_j^2 = 2(s_j - s_i)\Delta$. This expression is larger than zero if and only if $s_j > s_i$. Therefore, moving capacity share from smaller (larger) to larger (smaller) firms is equivalent to increasing (decreasing) the HHI.

Now consider some small shift in capacity share from a smaller to a larger firm. This will leave μ and γ unchanged if the shift occurs between two strictly pivotal firms. If it occurs between two strictly non-pivotal firms, it will equally leave μ and γ unchanged. If it occurs from a non-pivotal firm to a pivotal firm however (or from a non-pivotal firm to a weakly non-pivotal firm), μ and γ will strictly increase. Hence, part (ii) follows. ■

Proof of Proposition 2

Let $\Delta\bar{p}/\bar{p} \equiv (\bar{p}^* - \bar{p})/\bar{p}$ (where an asterisk denotes post-merger variables). Then, the change in the average price caused by a merger is given by

$$\frac{\Delta\bar{p}}{\bar{p}} = \frac{\mu^{*}v + (1-\mu^{*})\,c - [\mu v + (1-\mu)\,c]}{\mu v + (1-\mu)\,c} = \frac{(\mu^{*}-\mu)\,(v-c)}{\mu v + (1-\mu)\,c}. \tag{8}$$

The (average) profit margin m is defined by

$$m \equiv \frac{\bar{p}-c}{\bar{p}} = \frac{\mu\,(v-c)}{\mu v + (1-\mu)\,c}$$

Hence, if $\mu > 0$, we have

$$\frac{v-c}{\mu v + (1-\mu)\,c} = \frac{m}{\mu}$$

Substituting this into (8), letting $\Delta\mu/\mu \equiv (\mu^{*}-\mu)\,/\mu$ yields (3). ■

Proof of Proposition 3

Note that a merger does not change the pivotality of non-merging firms j, since k_{-j} is independent of the ownership of capacities controlled by firms other than j. Note, moreover, that the demand- and supply-based changes in pivotality caused by a merger are equivalent ($\Delta\mu/\mu = \Delta\gamma/\gamma$): since $\gamma = \mu k/\theta$, any percentage change in μ implies an equal percentage change in γ.

Assume two pivotal firms merge. In that case, $\gamma_1 = s_1 - e$ and $\gamma_2 = s_2 - e$ pre-merger and $\gamma^{*}_{1+2} = s_1 + s_2 - e$ post-merger from (1). We therefore have $\Delta\mu/\mu = (\gamma_{1+2} - \gamma_1 - \gamma_2)\,/\gamma = e/\gamma$. Similarly, if a pivotal firm 1 merges with a non-pivotal firm 2, then $\gamma_1 = s_1 - e$ and $\gamma_2 = 0$ pre-merger and $\gamma^{*}_{1+2} = s_1 + s_2 - e$ post-merger Hence, $\Delta\mu/\mu = s_2/\gamma$. Finally, if two non-pivotal firms merge, then $\gamma_1 = \gamma_2 = 0$ pre-merger and $\gamma^{*}_{1+2} = \max\{s_1 + s_2 - e, 0\}$. Hence, $\Delta\mu/\mu = \max\{s_1 + s_2 - e, 0\}\,/\gamma$ (for $\gamma > 0$).■

Proof of Corollary 1

For $e \leq 0$, $\mu = 0$ both before and after a merger, since both firms are pivotal with all their capacity already pre-merger. For $e \in (0, s_2]$, $\Delta\mu = e/\,(1-e)$, which is strictly increasing in . For $e \in (s_2, s_1]$, $\Delta\mu = s_2/\,(1-e)$, which is also strictly increasing in e, but with a smaller slope as in the previous case. For $e \in (s_1, s_1 + s_2]$, $\mu = (s_1 + s_2 - e)\,/\,(1-e)$, which is strictly decreasing in e as $s_1 + s_2 < 1$. Finally, for $e \geq s_1 + s_2$, $\mu = 0$ both before and after a merger, since neither the merging firms nor the merged entity are pivotal. This completes the proof. ■

Proof of Proposition 4

Consider the price schedules given in the main text:

$$p_i(u_i) = \begin{cases} p\left(x(c) - \tilde{k}_i + u_i\right) & \text{for} \quad u_i \in \left[0, \tilde{k}_i\right], \\ c & \text{for} \quad u_i \in \left(\tilde{k}_i, k_i\right], \end{cases}$$

It is straightforward to check that the number of non-pivotal units is the same for all firms. Moreover, they all charge a unit price of $p\left(x(c) - \tilde{k}_i + u_i\right)$ for each of its unit u_i that is pivotal, i.e., $0 \leq u_i \leq \tilde{k}_i$. It follows that for any segment on the demand curve, each firm offers the consumers it serves the same surplus. In particular, although a firm with a larger $\tilde{k}_i$ charges higher unit prices, it also serves more consumers, and these effects balance out. As a consequence, if a firm charges a higher price for some of its units, it would be the last from which consumers buy.

However, if it is the last firm, only consumers with the lowest valuations buy from it, given the firm's capacity constraint. Since in the equilibrium candidate, the firm already extracts the entire surplus from these consumers, it cannot benefit by charging a higher price for its pivotal units. Similarly, charging a price above c for non-pivotal units is not profitable either, as the firm would then also be the last to serve consumers and does not sell its non-pivotal units. As a consequence, no firm has a profitable deviation from the price schedule given above. ■

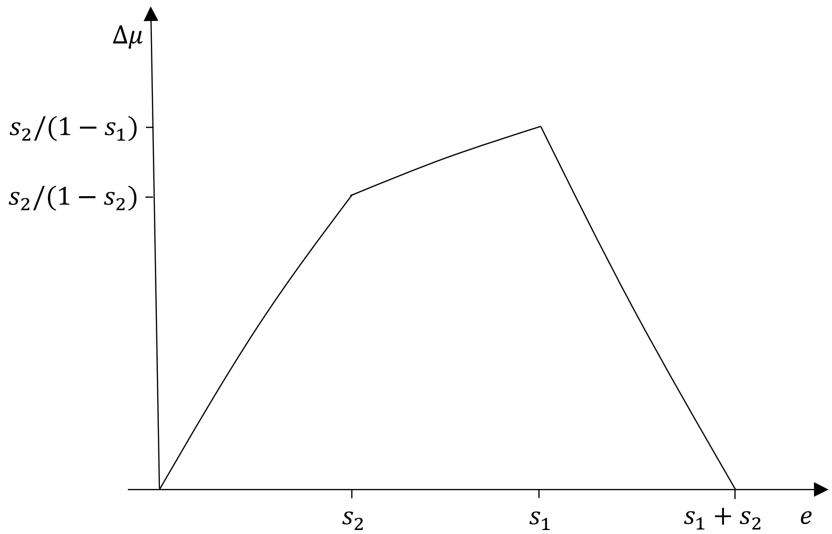

Figure 1: The impact of capacity utilization on merger effects

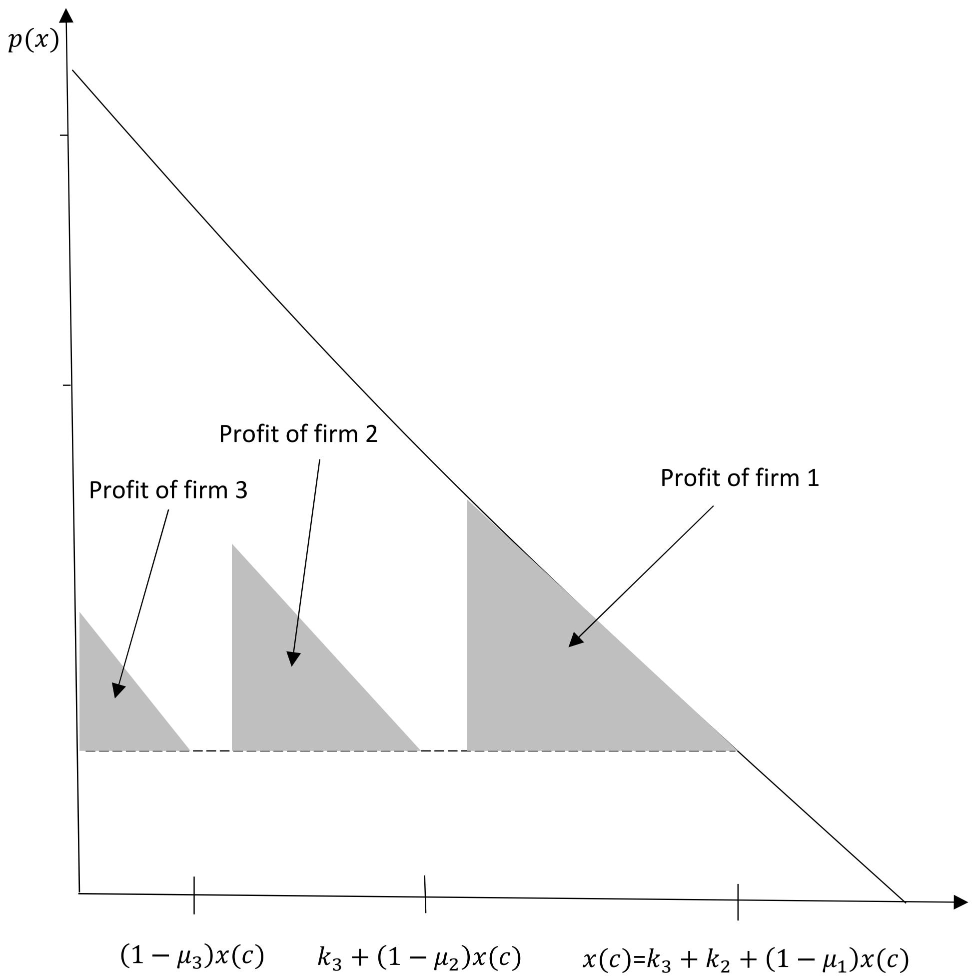

Figure 2: The grey-shaded areas are the profits of the three firms in the pre-merger equilibrium.

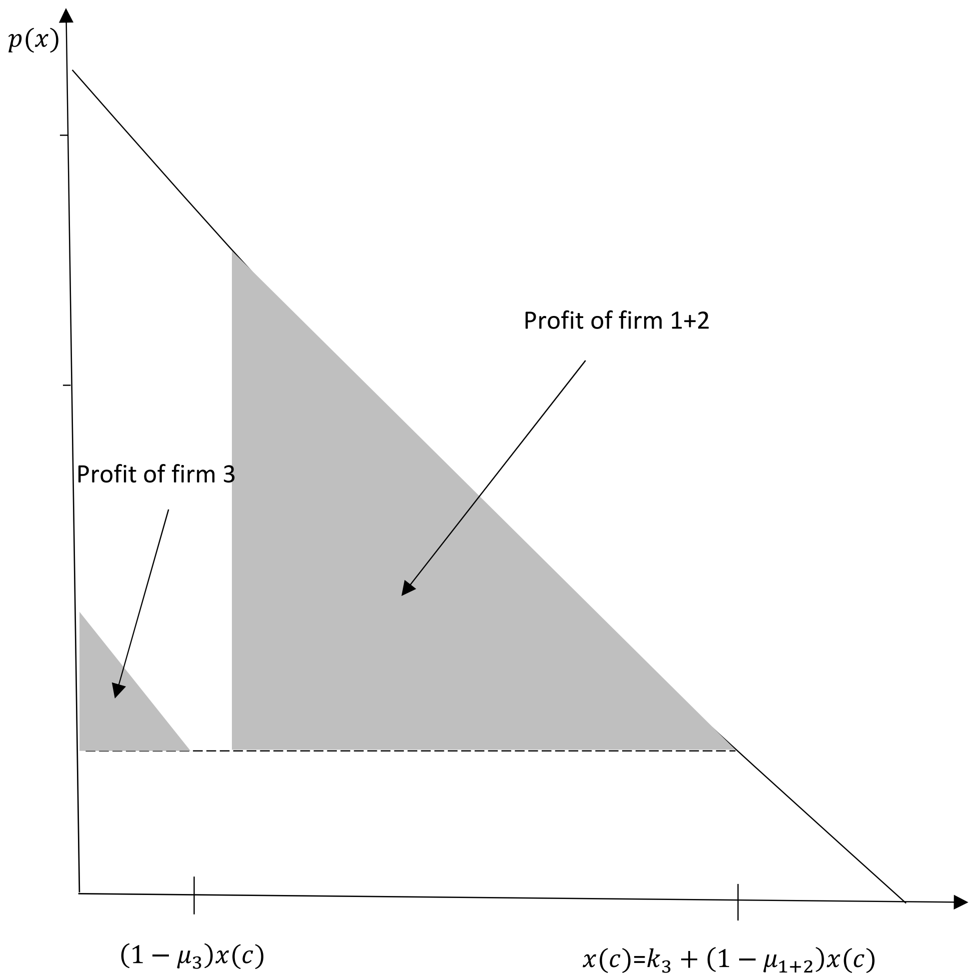

Figure 3: The grey-shaded areas are the profits of the two firms (i.e., the merged firm 1+2 and the non-merged firm 3) in the post-merger equilibrium.

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