

How to Detect and Measure Labor Market Collusion?

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Motivation

- Renewed political interest in making labor markets more competitive.
 - DOJ/FTC investigation of merger effects on labor markets.
 - FTC proposal to ban Noncompete Clauses.
 - Investigation of labor market cartels across jurisdictions.
- Need for new (econometric) tools to study competition in labor markets.
- Focuses on most common type of collusion “No Poaching Agreements”.
 - How to **identify** labor market cartels in practice? (detection)
 - How to **measure** their effects? (quantification)

Contribution

This paper proposes:

- Reduced-form wage loss estimation method,
- Structural model of labor market competition used for cartel detection.

Illustration: Collusion in Major League Baseball (1986-88).

Main Results:

- Players loss on average $\approx 27\%$ annual income, but highly heterogeneous.
- Developed and estimate a structural model of labor market competition.
- Model reveals **collusive patterns**:
 1. Decreasing mobility of workers across firms.
 2. Decreasing labor share of income.
 3. Increasing profits from retained employees.

Literature Review

- Models of Labor Market Competition: Card et al. [2018], Anderson and de Palma [1989], Azar et al. [2019], Roussille and Scuderi [2021], d'Haultfoeuille [2010], Lamadon et al. [2019], Dupuy and Galichon [2017], Lamy et al. [2022].
→ **Contribution**: based on Discrete Choice Theory adapted for labor data.
- Measuring Cartel Damages: Laitenberger and Smuda [2015], Boyer and Kotchoni [2015], Gibson [2019], Leamer [2012].
→ **Contribution**: propose a new approach for relevant industry.
- Baseball, Competition, and Antitrust: Berri and Krautmann [2019], Ferguson et al. [2000], Scully [1974], Zimbalist [1992], Pape [2020].
→ **Contribution**: measure of productivity, labor share, and cartel effects.

Collusion in Major League Baseball (MLB)

Why study Major League Baseball?

- Excellent data: typically difficult to access.
 - Linked employer-employee data (1960-2000).
 - Many measures of productivity [Kahn, 2000].
 - Freely available from Baseball-Reference.com

 - Relevant industry: labor is a strategic input in MLB.
 - Lithuanian Basketball League (2021).
 - Portuguese Football League (2022).
 - Polish Basketball League (2022).

 - Well-defined: beginning and end dates following successful litigation.
- [This paper's approach](#): not particular to Baseball but works well nonetheless.

Collusion in Major League Baseball

Peter Ueberroth, MLB's Commissioner (CEO), at team owners' meeting.

- Claimed that bidding wars were a "matcho thing".

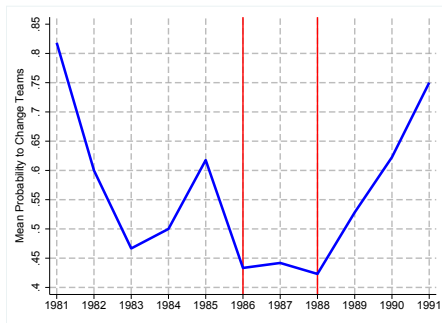
- Polled the owners "eyeball-to-eyeball":

I'm going to go around the room, and you have to tell me if you're going to be signing free agents. You're free to sign a free agent if you want, because we can't agree not to sign free agents. This is in no way binding and you have the right to change your minds. But I want to know those who are and those who aren't.

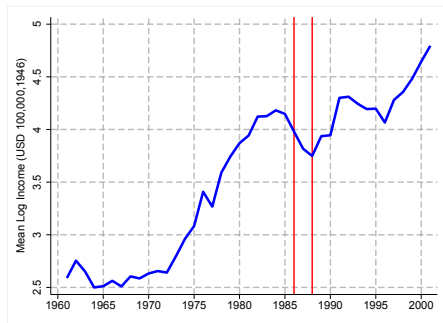
- Result: public salary and contract length discussion, no out-bidding.

- This paper will not focus solely on free-agents.

Effects on Wages and Mobility



(a) Prob. Free-Agent Changes Team



(b) Real Mean Log Wages (\$)

Observations:

- No Poaching Agreement is not fully enforced.
- Industry shocks make wage loss less clear.
- Time necessary before recovery of wages and mobility.

Damage Estimation

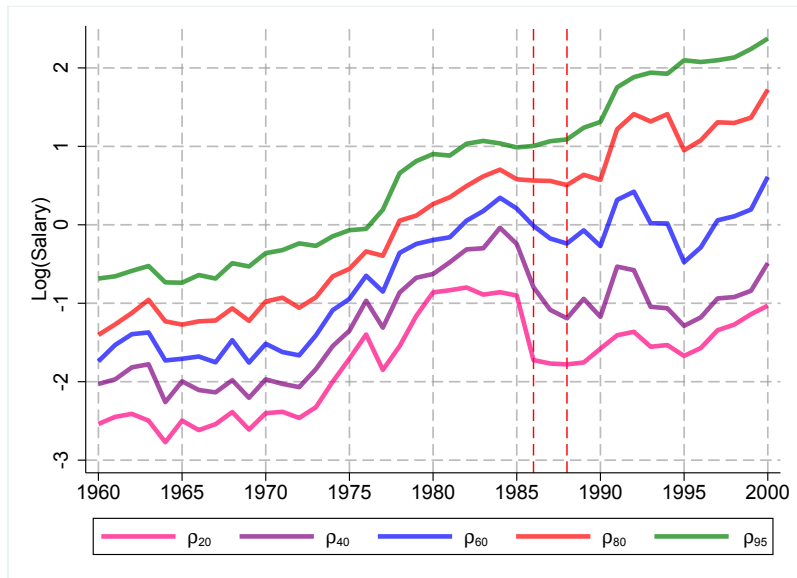
Goal: Measure income loss resulting from collusion.

- Necessary in class lawsuit [Leamer, 2012].
- Must be simple to explain to non-economist.
- Ideally should provide heterogeneous effects with testable robustness.
- Computationally tractable and require no special data.

Proposal: Inter-percentile Differences-in-Differences.

- Does all this !
- Method can be used outside of this context.

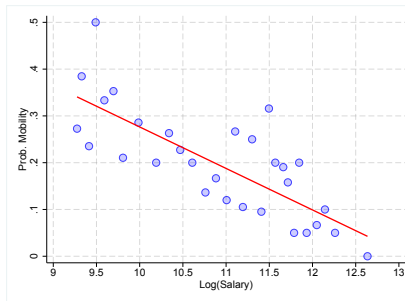
Inter-Percentile Differences-in-Differences



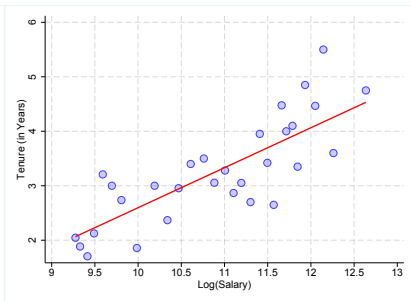
Why use the top 5% of earners?

At least three main reasons:

1. Statistics: Parallel trends seems reasonable (lower bound?).
2. Mobility: Players with *ex ante* limited mobility (not losing much).
3. Tenure: Players with greater tenure (can resist renegotiations).



(a) Income & Mobility (1985)



(b) Income & Tenure (1985)

Unconditional Quantile Regression

■ UQR [Firpo et al., 2009]:

- Measures how observation X_i changes statistic $iqr(X_1, \dots, X_N)$.
- Approximated by the Recentered Influence Function (RIF).
- Can calculate a sample counter-part used in a regression model.

■ Model: use in Diff-in-Diff framework for all percentiles $\rho \in \{1, 2, \dots, 95\}$:

$$\text{RIF} \left[\log(w)_{i,t}, iqr_{\log(w)}(\rho, 95) \right] = \mu_\rho + \mathbb{1}(\text{Cartel}_t = 1)\tau_\rho + \mathbb{1}(\text{Post-Cartel}_t = 1)\gamma_\rho + \zeta_{i,t}^\rho$$

■ Estimation: run OLS for each selected percentile (cluster bootstrap).

Summary Estimates

- Assume top 5% of earners are unaffected & parallel trends (control group).
- Estimate for each percentile:

	Mean	S.E.	25th perc.	Median	75th perc.	Min.
% Income Loss	-0.27	0.24	-0.51	-0.27	-0.01	-0.61

- In a before-after analysis, estimate of -34% ($\approx 2x$ those awarded).
- Advantages:
 - No additional data compared to before-after, runs with OLS.
 - Heterogeneous effects robust to industry shocks (DiD).
 - Observable selection (covariates) and event-study.

Structural Model of Labor Market Competition

Why a structural model?

- Cartel limits firms' ability to hire workers from other firms.
→ as if firms devalue the productivity of other workers.
- **Goal: model and measure devaluation.**
- Material evidence in antitrust case.

Labor Supply

Worker i with wage $w_t^{i \rightarrow j}$ from firm j at time t obtains mean utility

$$U_t^{i \rightarrow j} = X_t^{i \rightarrow j} \beta + \alpha w_t^{i \rightarrow j} + \xi_t^{i \rightarrow j} \quad (1)$$

If unemployed, the mean utility is $U_t^{i \rightarrow U}$. The total utility is

$$V_t^{i \rightarrow j} = \begin{cases} U_t^{i \rightarrow j} + \epsilon_t^{i \rightarrow j} & \text{if employed,} \\ U_t^{i \rightarrow U} + \epsilon_t^{i \rightarrow U} & \text{if unemployed,} \end{cases} \quad (2)$$

Assumption (Single Index)

The probability of player i joining firm j at time t , $s_t^{i \rightarrow j}$ can be represented as a **single index function** $\mathcal{S}(\cdot) : \mathbb{R} \rightarrow [0, 1]$ which is at least twice differentiable, always increasing ($\mathcal{S}'(x) > 0$), and which depends on the expected surplus

$$\Delta_t^{i \rightarrow j} = U_t^{i \rightarrow j} - \mathcal{O}_t^{i \rightarrow j}, \quad (3)$$

where $\mathcal{O}_t^{i \rightarrow j}$ represents the value (in utility) of the worker's outside-option.

Logit Competition [Anderson and de Palma, 1989]

$$s_t^{i \rightarrow j} = \frac{\exp(U_t^{i \rightarrow j})}{\exp(U_t^{i \rightarrow U}) + \sum_{k \in \mathcal{J}} \exp(U_t^{i \rightarrow k})} = \left[1 + \exp \left(- \left(U_t^{i \rightarrow j} - \mathcal{O}_t^{i \rightarrow j} \right) \right) \right]^{-1} \quad (4)$$

$$= \mathcal{S} \left(U_t^{i \rightarrow j} - \mathcal{O}_t^{i \rightarrow j} \right) \quad (5)$$

where $\mathcal{O}_t^{i \rightarrow j}$ takes the form of the expected maximum utility,

$$\mathcal{O}_t^{i \rightarrow j} = \ln \left[\exp(U_t^{i \rightarrow U}) + \sum_{k \in \mathcal{J} - \{j\}} \exp(U_t^{i \rightarrow k}) \right]. \quad (6)$$

Labor Demand

Firms profits given by:

$$\Pi_t^{i \rightarrow j} (w_t^{i \rightarrow j}) = \left(\underbrace{\exp(Y_t^{i \rightarrow j} \delta)}_{\text{Marginal Product}} - w_t^{i \rightarrow j} \right) \underbrace{\mathcal{S} \left(X_t^{i \rightarrow j} \beta + \alpha w_t^{i \rightarrow j} + \xi_t^{i \rightarrow j} - \mathcal{O}_t^{i \rightarrow j} \right)}_{\text{Pr. Accept Offer}} \quad (7)$$

Wage set by first-order condition:

$$w_t^{i \rightarrow j} = \exp(Y_t^{i \rightarrow j} \delta) - \frac{\mathcal{S} \left(X_t^{i \rightarrow j} \beta + \alpha w_t^{i \rightarrow j} + \xi_t^{i \rightarrow j} - \mathcal{O}_t^{i \rightarrow j} \right)}{\underbrace{\alpha \mathcal{S}' \left(X_t^{i \rightarrow j} \beta + \alpha w_t^{i \rightarrow j} + \xi_t^{i \rightarrow j} - \mathcal{O}_t^{i \rightarrow j} \right)}_{\text{Markdown}}} \quad (8)$$

Modelling Collusion

Bidding cost paid to hire worker:

$$\Phi \left(C_t^{i \rightarrow j'} \gamma \right) \exp \left(Y_t^{i \rightarrow j'} \delta \right) \quad (9)$$

Wage given by:

$$w_t^{i \rightarrow j} = \exp \left(Y_t^{i \rightarrow j'} \delta \right) \underbrace{\left\{ 1 - \Phi \left(C_t^{i \rightarrow j'} \gamma \right) \right\}}_{\text{Effective Discount}} - \frac{\mathcal{S} \left(X_t^{i \rightarrow j'} \beta + \alpha w_t^{i \rightarrow j} + \xi_t^{i \rightarrow j} - \mathcal{O}_t^{i \rightarrow j} \right)}{\alpha \mathcal{S}' \left(X_t^{i \rightarrow j'} \beta + \alpha w_t^{i \rightarrow j} + \xi_t^{i \rightarrow j} - \mathcal{O}_t^{i \rightarrow j} \right)} \quad (10)$$

Goal: model how cartel increases bidding costs at beginning of collusive era.

- C1. Should increase for workers in competitors' firms.
- C2. Results in decreased mobility of these players.
- C3. Lowers labor share of income and increases profits.

Econometric Strategy

Mobility: a match of worker i with firm j in year t is denoted by

$$\mathbb{1}_t(i \rightarrow j) = \begin{cases} 1 & \text{if } i \text{ joined firm } j \text{ in year } t, \\ 0 & \text{otherwise.} \end{cases}$$

Data: we observe (a) the worker-firm match, (b) the associated wage, measures of (c) productivity, (d) bidding costs, and (e) firm desirability.

$$\mathcal{D}_i := \left\{ \mathbb{1}_t(i \rightarrow j), w_t^{i \rightarrow j} \times \mathbb{1}_t(i \rightarrow j), Y_t^{i \rightarrow j}, C_t^{i \rightarrow j}, X_t^{i \rightarrow j} \right\}_{t \in \mathcal{T}}^{j \in \mathcal{J}}. \quad (11)$$

Problem: cannot estimate labor supply by multinomial choice model.

1. Unobserved wages: who made what offer?
2. Unobserved attributes: what are the terms of contracts?

Cannot write the choice model of workers choosing among job offers.

Proposed Solution

Solution: labor demand equation is **invertible** under **supermodularity** assumption.

Remark (Offer Acceptance Probability)

$$s_t^{i \rightarrow j} = \mathcal{S} \left(\Gamma^{-1} \left[\alpha \left(\exp \left(Y_t^{i \rightarrow j'} \delta \right) \{ 1 - \Phi \left(C_t^{i \rightarrow j'} \gamma \right) \} - w_t^{i \rightarrow j} \right) \right] \right) \quad (12)$$

Problem: Cannot use MLE because $s_t^{i \rightarrow j}(w_t^{i \rightarrow j})$ is unknown if no match.

Solution: d'Haultfoeuille [2010] shows identification with instruments.

Assumption (Identifying Restriction)

There exists *preference shifting instruments* $X_t^{i \rightarrow j}$ such that

$$\mathbb{1}_t(i \rightarrow j) \perp\!\!\!\perp X_t^{i \rightarrow j'} \mid w_t^{i \rightarrow j}, Y_t^{i \rightarrow j}, C_t^{i \rightarrow j}. \quad (13)$$

Estimation Generates unconditional moments used in GMM.

$$\mathcal{G}(\alpha, \delta, \gamma) = \mathcal{N}^{-1} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}_t} \sum_{j \in \mathcal{J}_t} \left[\frac{\mathbb{1}_t(i \rightarrow j)}{s_t^{i \rightarrow j}} - 1 \right]' \left(Y_t^{i \rightarrow j'}, C_t^{i \rightarrow j'}, X_t^{i \rightarrow j'} \right). \quad (14)$$

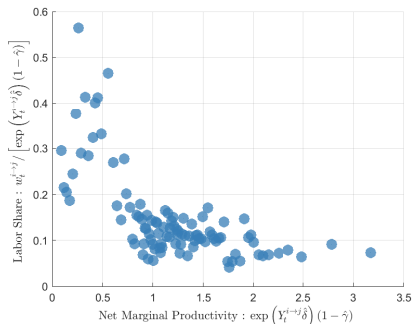
Application

- Model: Use a Gumbel probability model for $\mathcal{S}(\cdot)$ and logistic for $\Phi(\cdot)$.

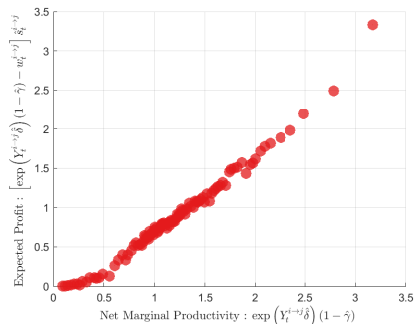
$$s_t^{i \rightarrow j} = \exp \left(- \left[\alpha \left\{ \exp \left(Y_t^{i \rightarrow j} \delta \right) \left\{ 1 + \exp \left(C_t^{i \rightarrow j} \gamma \right) \right\}^{-1} - W_t^{i \rightarrow j} \right\} \right]^{-1} \right)$$

- Marginal Productivity: year and team fixed effects, with performance variables specific to pitchers and hitters.
- Bidding Costs: experience, free-agency, and past-employer dummy.
 - Separate coefficients for pre-cartel, cartel, and post-cartel.
- Instrument by Home Bias: is player born in team's state + Share of "locals".

Model used to learn about labor market.



Labor Share

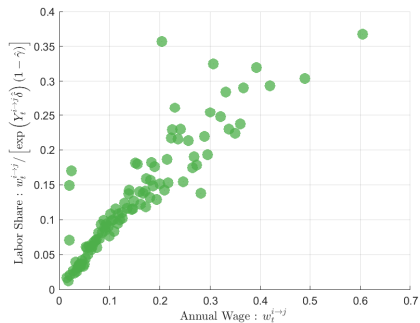


Expected Profits

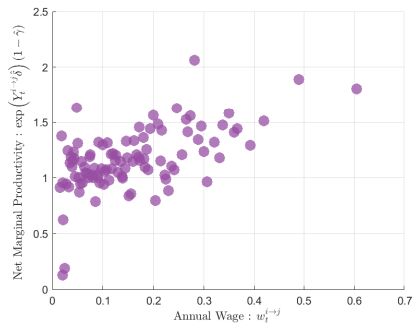
Figure: Labor Share and Profits v. Productivity (1982-5)

- Best players are not paid proportionately: most profitable for firms.
- High wage does not imply higher productivity.

Model used to learn about labor market.



Labor Share



Marginal Productivity

Figure: Labor Share and Productivity v. Wage (1982-5)

- Most well paid have high labor share.
- Correlation between wage and productivity is weak.

Collusive Evidence: Bidding Costs

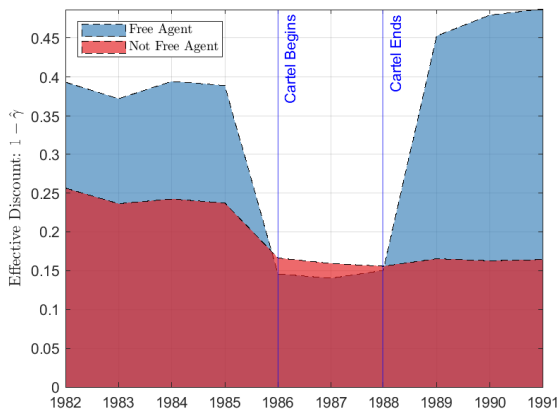


Figure: Bidding Costs of Free-Agents

- Effectively discounting the valuation of both free and non-free agents.

Collusive Evidence: Bidding Costs

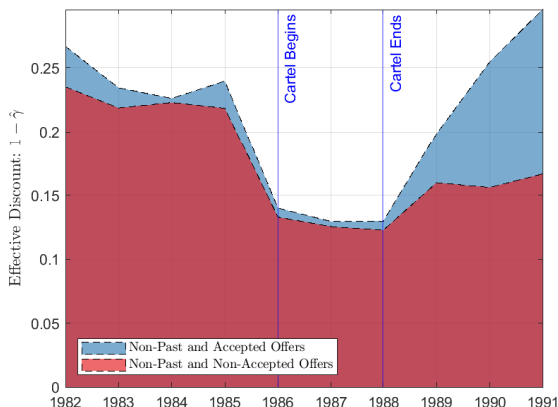


Figure: Bidding Costs of Accepted and Rejected Offers

- Effectively discounting non-past employees when bidding.

Collusive Evidence: Labor Share

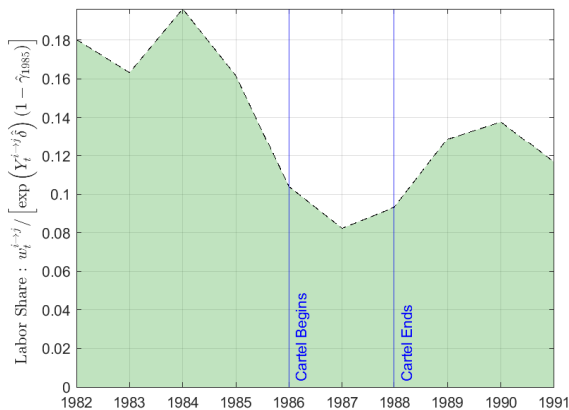


Figure: Labor Share of Income

- Fall in labor share of income : does not recover.

Collusive Evidence: Expected Profits

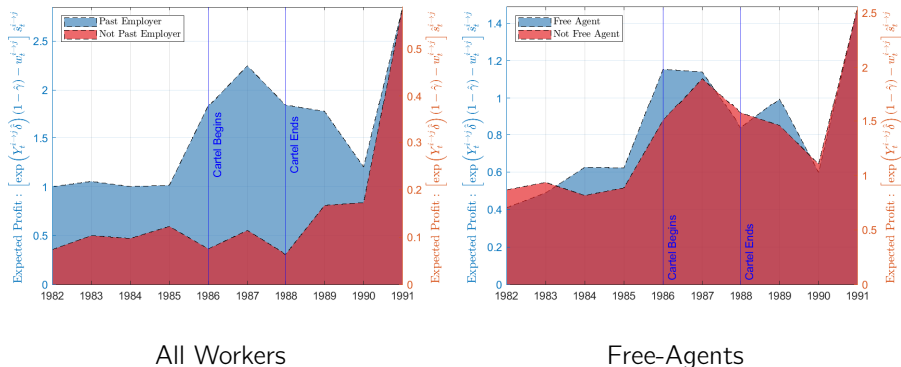


Figure: Expected Profits, Past Employer, and Free-Agents

- Profits mainly rise by hiring past-employees.
- Increase in profits from both free and non-free agents.

Simulating Deviation Wages

Each firm could deviate from collusion: keep outside-option fixed.

- Set wages assuming bidding costs are pre-cartel level.

$$D(w_t^{i \rightarrow j}) = \arg \max_w \left[\exp \left(Y_t^{i \rightarrow j} \delta \right) \left(1 - \Phi_{1985}^{i \rightarrow j} \right) - w \right] \mathcal{S} \left(\hat{\alpha} w + X_t^{i \rightarrow j} \hat{\beta} + \hat{\xi}_t^{i \rightarrow j} - \hat{O}_t^{i \rightarrow j} \right)$$

- Use the structural estimates to measure “gross” outside-option:

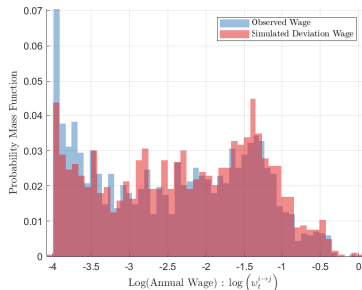
$$X_t^{i \rightarrow j} \hat{\beta} + \hat{\xi}_t^{i \rightarrow j} - \hat{O}_t^{i \rightarrow j} \equiv \hat{\Delta}_t^{i \rightarrow j} - \hat{\alpha} w_t^{i \rightarrow j}. \quad (15)$$

- Intuition: wage firm should have paid if free-rided on collusion of others.
- Note: lower bound wage as outside-option should be higher under competition.

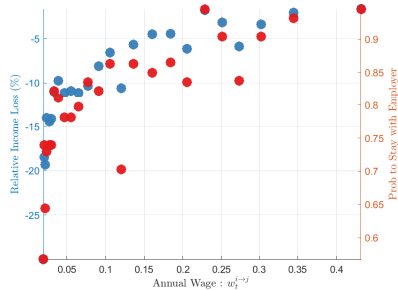
Simulating Deviation Wages: Results

	Mean	Std.	Q1	Med.	Q3	Min.
Relative Income Gain	31.19	102.17	0	0	0	-10.27
Relative Income Loss	-9.57	21.93	0	0	0	-92.52

- Average wage loss of at least 10% < 27% .
- Limited effect for immobile top-income.



Loss Histogram



Loss by Income Level

Conclusion

■ Damage Quantification:

- Inter-Percentile Differences-in-Differences.
- Simple to implement, yet highly informative.
- -27% yearly average loss of income from collusion in the MLB.

■ Structural Model of Labor Market Competition:

- Builds on Discrete Choice Model.
- Learn about labor market (labor supply elasticity, labor share, profits).
- Sudden change in wage, mobility, labor share, and profits indicative of collusion : heterogeneity depending on past-employee status.

Thanks!

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Summary and Discussion

Very general setup:

- Two supply chains with manufacturer (M) and retailer (R) : $M_i - R_i$.
- Asymmetric information: only retailers observe demand state $\theta = \{L, H\}$.
- Joint supply chain profit is non-parametric $\pi_{\theta_i}(q_i, q_j)$:
 - U-shaped in q_i , strictly decreasing in q_j .
 - Regularity assumption : $\pi_{Hi}(q_i, q_j) > \pi_{Li}(q_i, q_j)$
 - Regularity assumption : $\nabla_{q_j} \pi_{Hi}(q_i, q_j) > \nabla_{q_j} \pi_{Li}(q_i, q_j)$
- Remark 1: Model consistent with horizontally differentiated products : can θ be interpreted as *vertical* differentiation?
- Remark 2a: Literature on asymmetric collusion in vertical markets : “A theory of hub-and-spoke collusion” (Sahuguet and Walckiers, IJIO 2017).
- Remark 2b: German coffee cartel (Holler and Rickert, IJIO, 2022): RPM?

Summary and Discussion

Manufacturer offers menu of two-part tariffs: $\{(q_{Hi}, T_{Hi}), (q_{Li}, T_{Li})\}$

- If retailer rejects both contracts, earns zero profit.
- Remark 3: Assumption may drive up the opportunity cost of rejecting the menu. Could assume some “outside-profit” ℓ_i which induces some competition in monopoly manufacturer-retailer relationship (e.g, private label or sell some other product).

Potential industry structure in static (S) world:

- Symmetric Full-Information Monopoly (M): retailers reveal θ to manufacturer, quantities maximize joint-profits.
- Asymmetric Information with Competition: punishment contract such that
 - ★ Retailer accepts low-type contract (IR)
 - ★ Retailer prefers high-type contract over low-type if $\theta = H$.

Summary and Discussion

Collusion and dynamics:

- Collusion: firms can discount future profits at rate δ .
- Requires dynamic incentive contracts:
 - ★ Manufacturer prefers colluding over one-off deviation + punishment contract (IC_M).
 - ★ Retailer prefers to continue colluding in $\theta = L$ rather than get zero + punishment (IR).
 - ★ Retailer does not take L with $\theta = H$ + punishment (IR).
- Corollary 1: if $\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M) > \pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)$
 - ★ Manufacturers benefit from including the retailers in collusion.
- Remark 4: This is relative to playing the asymmetric information competitive equilibria. Could exclusionary collusion with just manufacturers be a better threat? Would be plausible presumably (for $p \gg 0$) and damaging.

Other remarks

Open questions:

- Can the model be reverted with manufacturer observing the market demand but retailers setting menus (e.g, Apple selling iPods through supermarkets).
- Quantity distortions: can something be said for consumer surplus ?
- Cartel deterrence : how does a probability q of cartel detection with fines (F_M, F_R) affect collusive stability?
- Would be nice to see, if possible, a graphical representation of stability v. profitability of collusive agreement.

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