Multi-Attribute Search

Jacopo Gambato

Universität Mannheim, ZEW

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- **RQ1**: Optimal search process with **correlated products** → role of **learning** through shopping
- **RQ2**: Optimal **menu selection** and **pricing strategy** of a multi-product monopolist when search is costly

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- Products as random draws from match value distribution → difficult to introduce correlation
- Instead: products as collection of attributes
- Products with attributes in common: perfectly correlated in that part of their match value
- Allows for realizations to dictate **direction** of search as it unfolds

Results preview

- Optimal search strategy:
  - forward looking: implied search paths matter
  - non-stationary: expectation updating affects stopping rule
  - search path "reveal preferences" → buyer self-selects towards favored options

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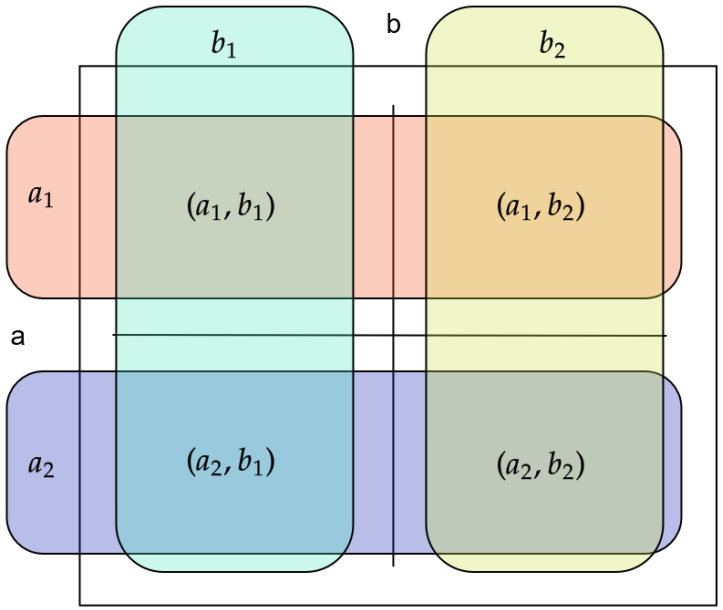
  - search path "reveal preferences" → buyer self-selects towards favored options
- Monopolist optimal strategy:
  - possibly different prices for *ex ante* identical products
  - possibly restriction of supply
  - with menu, prices: monopolist can induce specific order of search for consumer

Products

- **Horizontally** differentiated products defined by attributes: a, b (e.g.: a = color, b = fabric)
- Restriction: $a \in \{a_1, a_2\}$ (e.g. "red", "blue"), $b \in \{b_1, b_2\}$ (e.g. "cotton", "polyester")
- $N = 4$ distinct product: (a_i, b_j), $i, j \in \{1, 2\} \rightarrow$ correlation through shared attributes



The agents

- **Representative consumer**: unit demand; knows products available but not his preferences; observes prices
- Attributes valued independently: $y \sim Bi(V = 1, \alpha)$, $\alpha \in (0, 1)$, $y \in \{a_1, a_2, b_1, b_2\}$; utility:

$$u(a_i, b_j) = a_i + b_j, \quad E[u_{i,j}] = 2\alpha, \qquad i, j \in \{1, 2\}$$

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- **Equilibrium concept**: SPNE

Timing

1. Consumer and monopolist observe α, s,
2. Monopolist selects A, $\mathbf{p}(A)$,
3. Consumer observes A, $\mathbf{p}(A)$, chooses between searching and her outside option (normalized at $u_0 = 0$), and what to search,
4. After each inspection, consumer chooses between stopping and keep searching (and what to search next).

Search dynamic

- All products available ($A \equiv N$), uniform prices normalized at zero,
- Searching (a_1, b_1) (WLOG) lets buyer discover $a_1 \in \{0, 1\}$, $b_1 \in \{0, 1\}$:

$$E[u(a_1, b_2)]|_I = a_1 + E(b_2) \qquad E[u(a_2, b_1)]|_I = E(a_2) + b_1$$

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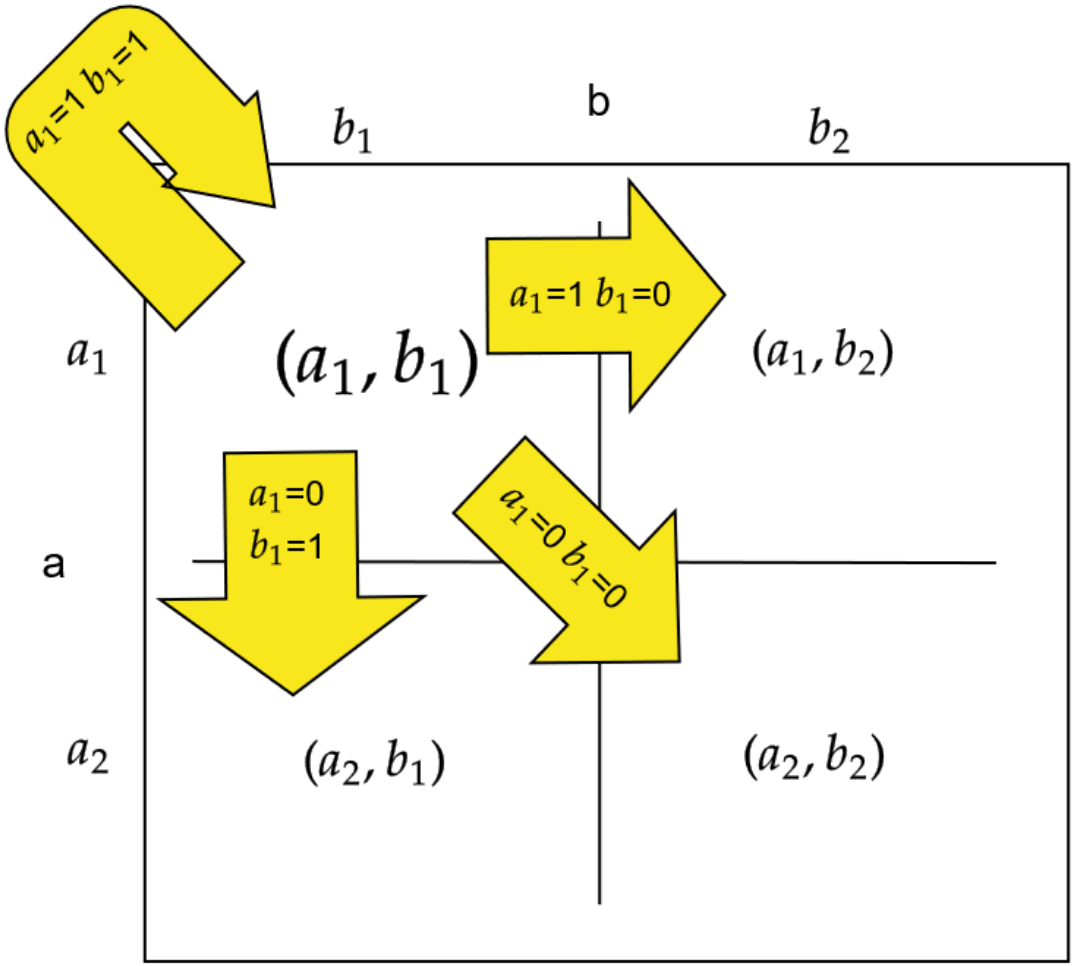
- Going backwards:

$$E[u(a_1, b_1)]|_{I\equiv\varnothing} = \underbrace{2\alpha^2}_{a_1=b_1=1} + \underbrace{(1-\alpha)^2(2\alpha - s)}_{a_1=b_1=0} + \underbrace{2\alpha(1-\alpha)[1 + \max(\alpha - s, 0)]}_{a_1\neq b_1} - s$$

Graphically



Uniform prices trade-off

- Consider uniform price p for all products in $A \equiv N$:

$$E[u(a_1, b_1)]|_{I \equiv \varnothing} = \alpha^2 \max(2 - p, 0) - s$$
$$+ 2\alpha(1 - \alpha) \max(1 - p, \alpha \max(2 - p, 0) + (1 - \alpha) \max(1 - p, 0) - s, 0)$$
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- Higher price: higher revenue if sale takes place, but **discourages** inspection of (a_2, b_2) after bad first realization: $p^D > p^E$

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- p^E and p^D better for different outcomes of the first search; both can be optimal: high probability of trade vs high per-sale revenue

Equilibrium Menu and Prices

Proposition

Consider a multi-product monopolist selecting optimal menu $A \subseteq N$ and pricing $\mathbf{p}(A)$ of multi-attribute products. In equilibrium:

- *Encouraging prices are set for high search costs,*
- *Discouraging prices are set for low search costs and high probability of a match*
- *All products are introduced if and only if prices are not set uniformly,*

Consumer is always steered towards specific search paths through strategic pricing.

The best of both worlds

- Suppose $p^E \leqslant 1$ is selected; after bad first realization, second search takes place
- If consumer likes an attribute (say, a_1), she searches (a_1, b_2) if:

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- Incentive to increase price of some products to profit off learning of consumer:

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- Consumer would not **start** from (a_1, b_2) (or (a_2, b_1)), but **could inspect it** depending on realizations a_1, b_1:

$$\pi = (1 - (1 - \alpha)^4)p^* + 2\alpha^2(1 - \alpha)(p^{**} - p^*)$$

Consumer adaptation and monopolist response

- For some α, s, consumer can adapt search by ignoring expensive product and gathering more information: from (a_1, b_1) to (a_2, b_2) instead of *e.g.* (a_1, b_2)

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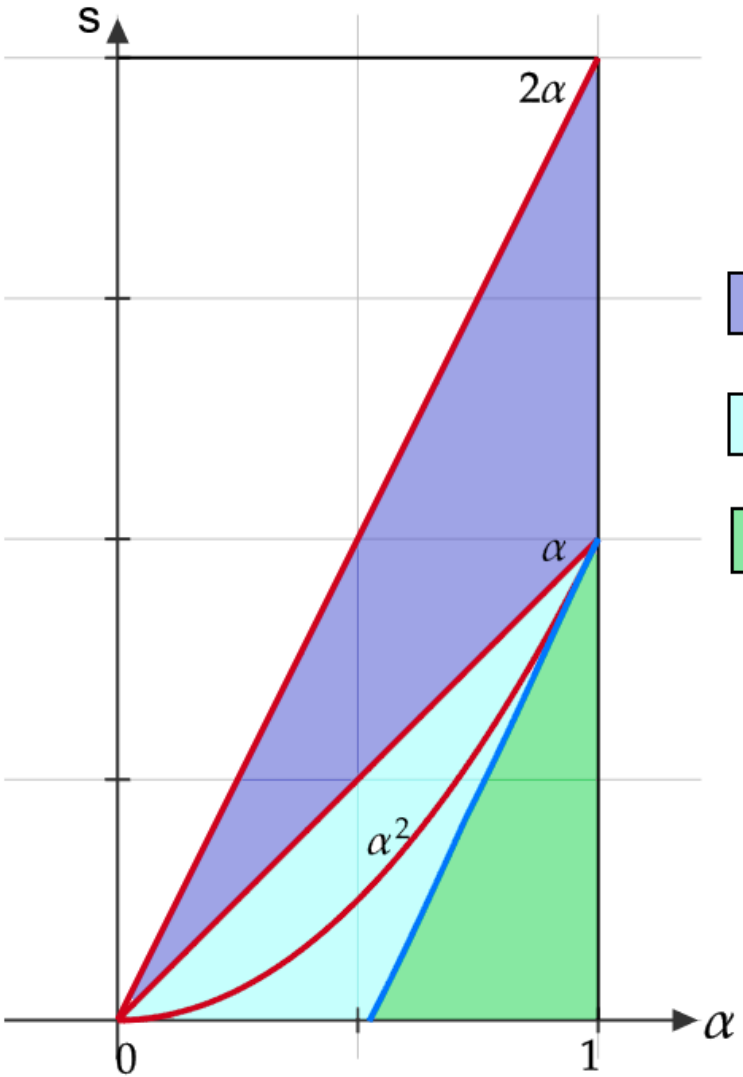
- All can be the best response for different values of α, s; supply restriction if α high, s low; $\widehat{\pi}$ dominated by uniform p^D

Equilibrium menu and prices, graphically



unif. low p | $|A|=2$

diff. low p | $|A|=4$

unif. high p | $|A|=3$

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- Search dynamic matches well recent evidence of "spatial learning" in search (Hodgson, Lewis, 2021)

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Predictability of search process allows monopolist to make buyers self-select based on taste, drives rent extraction

Thank you for your attention