Regulating platform competition in markets with network externalities: Will predatory pricing restrictions increase social welfare? *

Ohad Atad[†] and Yaron Yehezkel [‡]

Abstract

We consider an infinitely repeated platform competition in a market with network externalities. The platform that dominated the market in the previous period becomes the incumbent in the current period. We examine the effect of an antitrust policy that prohibits both platforms (symmetric regulation), or just the incumbent (asymmetric regulation) from charging predatory prices. We show that symmetric regulation decreases consumer surplus and does not affect efficiency. Asymmetric regulation increases consumer surplus and improves welfare when the size of the market remains constant over time. Yet, when market size varies over time, this policy may lead to inefficient entry.

JEL Classification: L1, L4, L5.

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1 Introduction

When platforms compete in a market with network externalities, consumers' decision as to which platform to join is based not only on the intrinsic quality of the platforms' services or

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[†]Coller School of Management, Tel Aviv University (ohadatad@post.tau.ac.il)

[‡]Coller School of Management, Tel Aviv University (yehezkel@tauex.tau.ac.il)

products, but also on the consumer's expectations regarding the platforms' ability to attract other consumers. An incumbent platform can benefit from a *focal* position, in that consumers expect other consumers to join the incumbent, even when an entrant platform offers a superior quality. In a dynamic setting, the entrant can challenge its non-focal position by charging a price below cost. However, the incumbent platform can also charge a price below cost in order to maintain its focal position. This may result in an *inefficient incumbency*, when a low-quality incumbent maintains its dominance in the market due to its focal position: consumers' expectations that other consumers will continue to join it.

The main research question of this paper is how predatory price restrictions – a policy that prohibits both platforms (symmetric regulation) or just the incumbent (asymmetric regulation) from charging a price below cost – affect the efficiency of entry in a market with network externalities. Naturally, asymmetric regulation facilitates entry by a new platform and may reduce the problem of inefficient incumbency in markets that are unable to self-correct. Yet, it may create the opposite problem of *inefficient entry*, when an entrant platform of lower quality than the incumbent enters and dominates the market due to the regulatory restriction on the incumbent's price. This raises the question of what are the market conditions under which asymmetric regulation, which prohibits only the incumbent from charging a predatory price, reduces the problem of inefficient incumbency without creating the problem of inefficient entry.

This paper considers infinitely repeated platform competition in a market with network externalities. The incumbent platform enjoys a focal position due to consumers' favorable expectations that other consumers will join it. The platform that dominated the market in the previous period becomes the incumbent that gains the focal position in the current period. Hence, platforms compete on focality and such competition may involve predatory prices. In every period there is a stochastic realization of the platforms' qualities such that each platform can be of a quality superior or inferior to that of its competitor. Moreover, the market size is also stochastic, and can be higher or lower in each period.

We start with an unregulated market, and solve for a Stochastic Markov Perfect Equilibrium, where in each period, one of the platforms wins with some probability, depending on the realization of the quality gap between the two platforms. We find that the market is characterized by inefficient incumbency: the platform that dominated in the past can maintain its focal position even with a lower quality than the entrant. This requires the dominant platform to adopt a predatory pricing scheme in the form of a negative price.

We consider symmetric regulation, when both the incumbent and the entrant platforms are banned from implementing predatory pricing. We find that such regulation has no effect on total welfare, while it actually decreases consumer surplus because it softens price competition.

We then move to asymmetric regulation, where only the incumbent is prohibited from charging predatory pricing while the entrant receives a "grace period" in which it can charge a negative price for gaining a foothold in the market. We find that when the size of the market remains constant over time, such regulation decreases the problem of inefficient incumbency. As the two platforms become more forward-looking, restricting a predatory price becomes a more efficient tool in reducing the problem of inefficient incumbency and the regulated market approaches the efficient outcome in which the superior quality platform always wins.

However, the main result of our paper is that when market size varies over time, restricting the incumbent's ability to charge a price below cost may lead to inefficient entry. In particular, we find that such inefficient entry occurs in a period of low market size, when market size is expected to grow in future periods, and when platforms are forward-looking.

Our results have policy implications on the regulation of platforms, and in particular on banning predatory pricing. In its *Brook Group* decision almost three decades ago,[1] the United States Supreme Court laid the current framework for evaluating predatory pricing claims, by requiring the plaintiff to prove two key elements: first, that the defendant priced below its own costs; and second, that the defendant had a reasonable probability of recouping its losses during the predation period, by increasing prices once it was able to deter the entrant. Since then, there have been several cases in which platforms allegedly adopted predatory pricing. In a 2020 US House of Representatives (HoR) subcommittee on Antitrust report determined that "Predatory pricing is a particular risk in digital markets, where winner-takes-all dynamics incentivize the pursuit of growth over profits".[2] The US HoR report specified

[1] Brooke Grp. Ltd. v. Brown &Williamson Tobacco Corp., 509 U.S. 209 (1993)

[2] US House of Representatives subcommittee on Antitrust - Investigation of Competition in Digital Markets, pp. 397. Available at:

several examples of allegedly predatory pricing strategies by a platform against its rivals. One example is the case of Amazon and Diapers.com: "Prior to buying it, Amazon identified Diapers.com as its "largest and fastest growing competitor in the on-line diaper and baby care space......and, in 2010, Amazon hatched a plot to go after Diapers.com and take it out. Specifically, Amazon's documents show that the firm entered into an aggressive price war..... willing to bleed out $200 millions in losses on diapers in one month" (pp. 263). As another example in the HoR report, Amazon's membership program, Amazon Prime, suggested a strategy in which the platform would charge low prices in order to secure market dominance, and raise prices in the future: "as part of its business strategy .... Amazon has adopted a predatory-pricing strategy across multiple business lines at various stages in the company's history....Because of the nature of its marketplace business, Amazon's below-cost prices on products and services tend to lock customers into Amazon's full marketplace ecosystem.... " (pp. 297).

In a *Washington Post* article, Oremus (2021) notes that platforms such as Google, Amazon, Apple or Facebook launch new products free of charge, or at money-losing costs.[3] For example, in 2015, Google launched Google Photos with unlimited free storage and no ads, a business strategy that resulted in losing money in the short-run. Likewise, Apple launched Apple Music and Apple TV Plus with free or discounted introductory offers, with the goal of enhancing the exposure of its software ecosystem and, by doing so, increased the sales of iPhones, iPads and Apple Watches. Apple's rivals such as Spotify, whose primary business is streaming music, cannot match such discounts. Another example is Facebook, which launched a competing service to Substack, the fast-growing newsletter platform that connects writers directly with subscribers. Facebook didn't charge for subscriptions on its newsletter platform and paid best-selling authors such as Malcolm Gladwell and Mitch Albom to join the platform.

In some countries, platforms faced antitrust scrutiny for engaging in predatory pricing. Behringer and Filistrucchi (2015) analyze two legal cases of predatory pricing in the market

https://judiciary.house.gov/uploadedfiles/competition_in_digital_markets.pdf?utm_campaign=4493-519. Last visited in 3.2022.

[3]https://www.washingtonpost.com/technology/2021/07/06/facebook-bulletin-antitrust/. Last visited in 3/2022.

for newspapers in the UK. Bhattacharjea (2018) studies legal cases in India against the taxi platforms Uber and Ola for allegedly engaging in predatory pricing. Conor (2022) reports that a new Portuguese law prohibits online travel platforms from using most favored nation clauses or offering below-cost pricing.

Our paper contributes to the understanding of predatory pricing by highlighting the role of predatory pricing in the context of platform competition. In our model, a firm sells at a loss only because of the presence of network effects, that is, because firms in our model are "platforms" and because of consumers' coordination on the focal platform. Hence, we can evaluate the effects of focality on the motivation to adopt predatory pricing and its effect on welfare. Furthermore, we study both the short and long-run effects of predatory pricing. In recent years, antitrust policy has adopted a somewhat tolerant approach to the practice of predatory pricing. As Oremus (2021) notes, this is because in the short run, consumers obviously benefit from low prices. In the long run, should the firm raise its price once the competitor is driven out of the market, new competitors will enter anyway, once again triggering a price war. In our paper, the platform that dominates the market by using predatory pricing will face competition in the next period, once the platform attempts to recover the loses from its former predatory practice. This feature of our model enables us to study the long-term effects of predatory pricing.

As for policy conclusions, the report by the HoR recommends a stricter policy against predatory pricing. In its conclusion regarding predatory prices,[4] the subcommittee addresses the second requirement of *Brooke Group* decision: *"The Subcommittee recommends clarifying that proof of recoupment is not necessary to prove predatory pricing or predatory buying, overriding the Supreme Court's decisions ....".* Our paper implies the following recommendations. First, a symmetric ban of predatory pricing imposed on both the incumbent and the entrant platforms has a negative effect on consumers, without mitigating the inefficient incumbency problem. As for asymmetric regulation only on the incumbent, such a policy can reduce the problem of inefficient incumbency, without creating the problem of inefficient entry, when the market size is expected to remain constant over time, and when platforms are

[4]subsection VI.B.3.c - Recomendations\Strengthening the Antitrust Law\Rehabilitate Monopolization Law\Predatory Pricing

not too forward-looking. Yet, when market size varies over time and platforms are forward-looking, asymmetric regulation can be welfare reducing because it creates the problem of inefficient entry.

In many markets, the task of adequately assessing the quality of an entrant (or even the incumbent), especially in the case of the highly innovative markets for platforms, is a challenging task for both regulators and policy makers. The results of this paper suggest that compared to an assessment of the intrinsic quality of the platforms, the expected aggregated number of consumers in a certain market is an attribute a policy maker can more accurately assess. Therefore, an analysis based on the observed number of consumers in a market may lead to a better prediction, and a higher probability of efficient policy outcomes.

We should note that when we consider regulating the incumbent's negative price, we define the identity of the incumbent platform not necessarily by chronological order. That is, an "incumbent" platform in a certain market is not necessarily the platform that was the first to enter. Instead, incumbency in our model is defined by focality. Asymmetric regulation should be imposed on the dominant platform – the platform that earned the market expectations that consumers would join it.

As a final remark, the "price" in our model does not necessarily need to be a monetary transfer. For example, consumers in our model can "pay" the platform by giving data that the platforms can monetize, while such data reduces the consumers' utility by reducing their privacy (e.g., Jullien, Lefouili, and Riordan (2020), Ichihashi and Smolin (2022) and Markovich and Yehezkel (2022)). Hence, banning predatory pricing in our model is equivalent to restricting a practice in which a focal platform subsidizes consumers in order to attract them, and in future periods collects their data.

The rest of the paper is organized as follows: Section 2 surveys related literature. Section 3 describes the model, the concept of *focality* and the benchmark equilibrium in a static game. Section 4 describes the features of the Stochastic Markov Perfect Equilibrium of an infinitely repeated game with stochastic qualities and market size. Section 5 studies the competitive implications of an unregulated market. In Section 6, we consider a symmetrically regulated market, when both the incumbent and the entrant are prohibited from charging predatory pricing. Section 7 studies the implications of an asymmetrically regulated market.

In Section 8 we compare platform profits, consumer surplus and total welfare, in unregulated and asymmetrically regulated markets. We conclude in section 9. Appendix A contains technical proofs.

2 Related Literature

Our paper combines and contributes to the legal literature and economic literature on predatory pricing, and to the literature on platform competitions.

In the context of platform competition, Behringer and Filistrucchi (2015) study the potential for predatory pricing in a two-sided market. They show that even a monopoly may have an incentive to charge a price below-cost to some consumers (on one side of the market). They then develop a rule to identify predatory pricing in the context of a two-sided market and apply it to two cases of alleged predatory pricing in the market for daily newspapers. Our paper differs in its focus. Behringer and Filistrucchi focus on a two-sided market, where a below-cost price on one side can be compensated by an above-cost price on the other side. We contribute to their paper by considering predatory pricing in the context of a dynamic game. Predatory pricing in our model is inter-temporal: a platform charges a price below cost in one period, to gain focality that enables the platform to charge a high price in future periods. Moreover, we consider platform competition and coordination problem.

Our paper adopts the approach that a platform's dominance emerges from its focality: consumers expect other consumers to join it, which makes it difficult for new platforms to enter the market. Caillaud and Jullien (2001, 2003) consider platform competition when one of the platforms enjoys a favorable bias in consumer beliefs (or focality). They show that the equilibrium strategy for the non-focal platform is to subsidize one side of the market, and charge a high price from the other. Hagiu (2006) extends the focality approach to a sequential game, when platforms compete on one side of the market and then on the other side. Jullien (2011) assumes a multi-sided market where one of the platforms offers a higher intrinsic quality than its competitor, and finds that when the focality outweighs quality, a focal platform can dominate the market even when competing against a higher quality platform. Halaburda and Yehezkel (2013) study the advantages of focality when consumers have asymmetric information concerning their benefits from joining the platforms. Halaburda and Yehezkel (2016; 2019) extend the concept of focality to a partial degree of focality. Markovich and Yehezkel (2022) show that when platforms compete on a large user and small consumers, they may charge the large user a negative fee, when attracting the large user results in attracting the small consumers.

In the context of a dynamic game, Halaburda, Jullien and Yehezkel (2020) consider a repeated competition between a high quality platform and a low-quality platform, when the platform that won the previous period is focal in the current period. Biglaiser and Cŕemer (2020) study repeated platform competition with heterogeneous consumers. Bourreau and Kraemer (2022) consider a dynamic game between a focal incumbent and a non-focal entrant when consumers can multi-home and study the effects of Interoperability. We contribute to this literature by considering the effects of regulation that prohibits both platforms, or just the focal platform, from charging a negative price. Moreover, we extend the analysis of a dynamic game by introducing a stochastic market size, where the market has the potential to grow in the next period, and show that it is in fact a key feature to competitive market outcomes. Hagiu and Wright (2020) consider infinitely repeated competition when firms improve their products along time through learning from customer data. In their model, dominating the market in previous periods provides a competitive advantage in the current period through learning. They study how such a learning process affects an incumbent's competitive advantage. In our paper, the competitive advantage from dominating the market in the past is driven by consumers' favorable expectations. We contribute to this paper by studying the regulation of banning a negative price.

Our paper also contributes to the legal and economic literature of predatory pricing. Predatory pricing in competition law deems prices as "excessive" when a dominant firm is able to abuse its market position, in order to charge "unfair" prices.[5] While excessive pricing schemes are considered unlawful and result in regulatory interventions in many countries and

[5]"Unfair" prices can be broadly divided into two cases, both of which may require regulatory intervention. In the first case, when prices are "too high" compared to the economic value provided, the dominant firm is able to extract consumer surplus, and therefore to harm consumer welfare in the market. In the second case, prices are "too low" (predatory prices), and a dominant firm can prevent competition and maintain its market position by setting price levels that are low enough to ensure that an entrant will not be able to secure non-negative total profits ex-post. Our main focus in this paper is on the second case.

jurisdiction,[6] both the theory of the harm of excessive pricing in competition law, and its implementation through regulatory prohibition of excessive prices, remain highly controversial. Edlin (2002) argues that predation cases should not be restricted to below-cost pricing, and that it is possible to successfully deter competition by implementing "above-cost predation" pricing techniques. Evans and Padilla (2005) argue that the *Brook Groop* framework is vulnerable to an inherent difficulty of assessing the welfare effects of such pricing schemes, which leads to "great variation" in court decisions and a "hardly satisfying outcome", in which courts reach different conclusions where "differences are seldom justified in economic terms". Hemphill and Weiser (2018) argue that the *Brook Groop* framework led to a reality in which (since the *Brook* decision) "antitrust claims alleging a predatory price cut have fallen into disuse". Gilo and Spiegel (2018) develop two formal models of excessive price prohibition benchmarks (a retrospective benchmark, and a contemporaneous benchmark), and show that while those may benefit consumers in the market by reducing the price charged by the dominant firm (when the dominant firm is a monopoly), it may also relax the competitive response of an incumbent firm, which may facilitate an inefficient entry to the market. O'Donoghue and Padilla (2019) argue that economic theory lacks a generally accepted definition of what is an excessive price, and that the current definition of excessive prices adopted by EU courts is "imprecise and difficult to administer in practice". Rey, Spiegel and Stahl (2022) consider predation in the context of an infinitely repeated game, where in each period an incumbent can accommodate or predate a new entrant. We contribute to this literature by studying the competitive implications of predatory price restrictions on platforms in a market with network externalities.

The closest paper to ours is by Farrel and Katz (2005) who study the competitive implications of predatory price restrictions in a market with network externalities, in a two-period environment. We make two contributions to this paper: first, by solving for a Stochastic Markov Perfect Equilibrium, we are able to identify and highlight the role of market size variations along time in an efficient market outcome of such a regulatory intervention. While Farrel and Katz (2005) finds that asymmetric regulation is welfare enhancing, we shows that

[6] See the OECD Competition Committee debate on Excessive Prices. Available at: http://www.oecd.org/competition/abuse/49604207.pdf

when the market size is stochastic between periods, asymmetric regulation can be welfare reducing. Second, we consider an infinitely repeated game in which the winning platform in each period becomes the focal in the next, which in turn means that in our setup, the entrant internalizes that winning the market in the current period will subject it to price restrictions in the next period, hence we are able to evaluate the long-term effect of predatory price restrictions.

3 The Model

Consider an infinitely repeated game. The market consists of two competing platforms $i = A, B$, and a homogeneous consumer population of size N_t in period $t = 1, ..., \infty$. The two platforms have equal cost (normalized to 0), and platform $i \in \{A, B\}$ offers the consumers joining it in period t a base value of $q_{it} > 0$, which can be referred to as quality. In addition, consumers derive a utility from network effects. The utility of a consumer from joining platform i in period t is

$$U_{it} = q_{it} + \beta n_t - p_{it}, \tag{1}$$

where n_t is the total number of consumers[7] platform i is able to attract in period t, β represents the strength of the network effects in the market and p_{it} is the price. Notice that p_{At} and p_{Bt} can be interpreted not just as a monetary transfer, but as any means of monetizing the consumers' utility. For example, p_{At} and p_{Bt} can represent the amount of data that each platform collects from consumers and commercializes, when commercializing data inflicts disutility on consumers because of the breach of their privacy. A "negative" price can represent bonuses, coupons or additional services that a platform provides consumers that join it.

The timing of each period is as follows. First, nature selects realizations of N_t, q_{At} and q_{Bt}. Let $q_t \equiv q_{Bt} - q_{At}$ denote the quality advantage of platform B in period t. The two platforms continuously innovate and in every period, one of the platforms can be of higher

[7]We assume that the market is fully covered, and allow the market size to vary over time: N_t denotes the total number of consumers in the market at $t = 1, 2$

quality than the other. Suppose that q_t is a stochastic variable which is drawn in each period from a uniform distribution with a support $[-\sigma, +\sigma]$. The distribution function of q_t is $f(q_t) = 1/(2\sigma)$ and the cumulative distribution is $F(q_t) = (q_t + \sigma)/(2\sigma)$. The realization of q_t is independently drawn between periods. Our results only depend on the quality gap. Hence, the model fits cases were the base quality of both platforms improve over time. As we show, because the quality gap is stochastic, the equilibrium has the feature that each platform has a positive probability of winning the market at each period. The number of consumers in each period, N_t, is also a stochastic variable between periods: $N_t = 1$ with probability $1 - \rho$ and $N_t = n$ with probability ρ, where $n \geq 1$ and $0 < \rho < 1$. As we show, the stochastic market size results, under asymmetric regulation, in inefficient entry, when a low-quality entrant dominates the market. Suppose that the distribution of q_t is sufficiently wide such that $\sigma > \beta n$.

At the second stage of each period, platforms compete by setting prices p_{At} and p_{Bt} simultaneously. Third, consumers observe these prices, and decide simultaneously and non-cooperatively whether to join platform A, B, or not to join either platform and obtain a utility of 0. We assume that consumers cannot multi-home (i.e., consumers can join only one platform) and they make a new decision on which platform to join in each period. Platforms discount future profits by δ $(0 < \delta < 1)$. Because consumers re-join a platform in each period, expectations concerning the future do not affect their decisions.

As is usual in markets with network effects, the stage where consumers decide which platform to join may result in multiple equilibria, depending on consumers' beliefs. To address this issue, we follow Halaburda, Jullien and Yehezkel (2020) by adopting the concept of focality:

Focality advantage

Consider a certain period. Assume both platforms set their respective prices, p_{At} and p_{Bt}, and in turn, consumers decide on the platform that maximizes their utility, which is determined by the price gap $p_{Bt} - p_{At}$. An equilibrium in which all consumers join platform A exists when $q_{At} - p_{At} + \beta N_t \geq q_{Bt} - p_{Bt}$, or $p_{Bt} - p_{At} \geq q_t - \beta N$, where recall that $q_t = q_{Bt} - q_{At}$. An equilibrium in which all consumers join platform B exists when $q_{Bt} - p_{Bt} + \beta N_t \geq q_{At} - p_{At}$

or $p_{Bt} - p_{At} < q_t + \beta N$. As the price ranges overlap, both equilibria exist if:

$$q_t - \beta N_t < p_{Bt} - p_{At} < q_t + \beta N_t.$$

The concept of focality represents consumers' beliefs concerning the equilibrium that will be played in this case. Consumers may expect one of the platforms to attract other consumers, and will be reluctant to join its competitor. Focality means that when both equilibria are possible, consumers will join the focal platform, expecting all other consumers to follow suit. Therefore, if platform A is *focal*, and both equilibria are possible in a price range $p_{Bt} - p_{At}$, all consumers will join platform A for all $p_{Bt} - p_{At}$.[8] Notice that a non-focal platform B can still win the market (i.e., attract all consumers). Yet, to do so the non-focal platform needs to set a price that ensures not only that there is an equilibrium in which all consumers join it, but that there is no other equilibrium in which consumers join the focal platform A. When, given the prices, the equilibrium in which consumers join platform B is unique, it is indeed rational for consumers to expect that other consumers will join B, as doing so becomes a dominant strategy for each consumer.

We assume that in the first period of the game, platform A is the focal platform. In any other period, the platform that dominated in period $t-1$ is focal at time t. As consumers make a new decision in each period on which platform to join, dynamics is meaningful at time t because the winning platform is focal at time $t+1$.

Static benchmark

Consider a one-period benchmark. Solving for the equilibrium prices, in an equilibrium in which the focal platform A wins, platform B sets $p_B = 0$ and platform A sets $p_A = \beta N - (q_B - q_A)$ (for simplicity, in what follows we drop the subscript "t" unless needed). In an equilibrium in which the non-focal platform B wins, platform A sets $p_A = 0$ and platform B charges a price that rules out the equilibrium in which all consumers join platform A: $-p_A + q_A + \beta N < -p_B + q_B$, or: $p_B = (q_B - q_A) - \beta N$. It will only be worthwhile for the platforms to win the game when they earn positive profit: $\pi_i = p_i N \geq 0$. Notice that

[8] Halaburda and Yehezkel (2019) allow for a partial degree of focality. Throughout this paper, we assume platforms are "fully focal", which means that only one of the platforms is focal in a given period.

$\pi_A = -\pi_B = (\beta N - (q_B - q_A))\, N$. Let $q^{static} = \beta N$ denote the quality gap, $q_B - q_A$, that results in $\pi_A = \pi_B = 0$. This implies that platform A wins the market if $q_B - q_A \leq q^{static}$, and platform B wins the market if $q_B - q_A > q^{static}$. We therefore have the following result (the proof follows directly from the text above).

Lemma 1. ***(The equilibrium in a static game when platform*** A ***is focal)*** *Suppose that platform* A *is focal. Then, platform* A *wins the market if*

$$(q_B - q_A) \leq q^{static} \equiv \beta N,$$

and platform B *wins otherwise.*

In a static game, platform A wins the market if the network externalities are greater than the quality gap, indicating that because of consumer beliefs, platform A may still win the market even if it offers inferior levels of services or products. Intuitively, this condition means that the network externalities are of higher importance to the consumers in this market, compared to the quality gap between the services or products provided by the two platforms. This is not an unreasonable condition: Consider a new social network platform with new technology that offers a service of a higher quality, by providing a service identical to that of its rival incumbent platform, only with better data security, but loses the market, because consumers' expectations are biased towards the lower-quality incumbent. We refer to this problem as *inefficient incumbency*: the incumbent platform can overcome entrants of superior quality, due to the incumbent's focal position. The efficient outcome is for platform A to win when $q \leq 0$ and for platform B to win otherwise.We therefore ask how dynamics affect the identity of the winning platform, and when and if prohibiting negative prices can help the market to implement the efficient outcome

4 The Stochastic Markov Perfect Equilibrium

In what follows consider an infinitely repeated game with stochastic qualities and market size. We follow Halaburda, Jullien and Yehezkel (2020) by solving for a *Stochastic Markov Perfect Equilibrium* with the following features. Figure 1 illustrates the equilibrium. In each period,

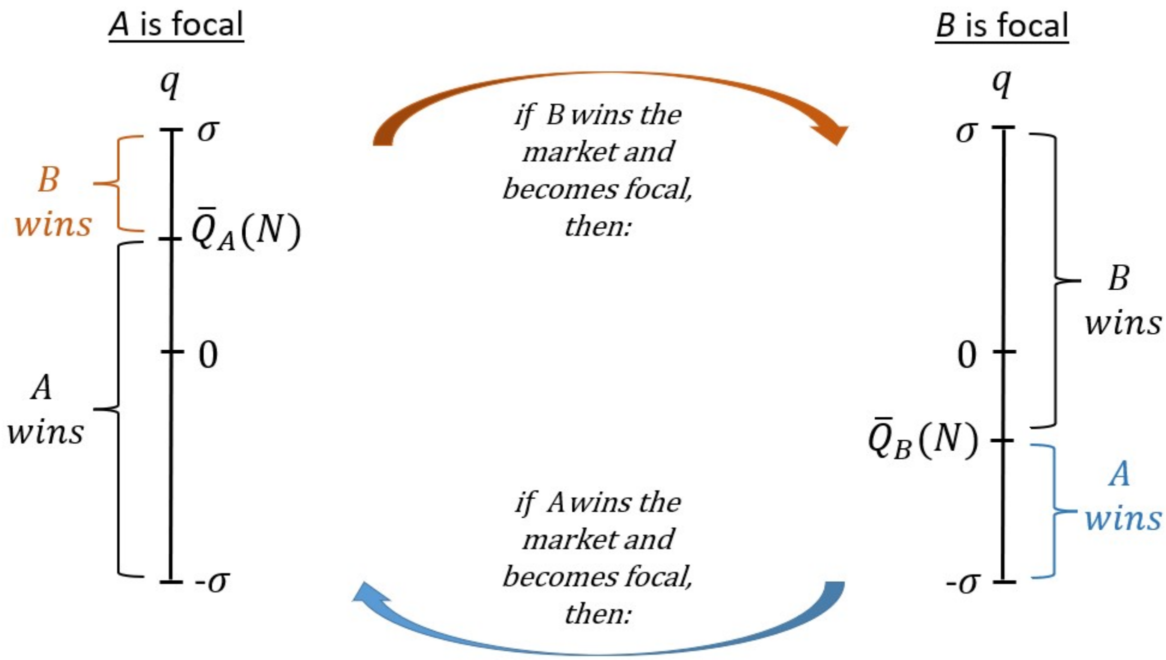

Figure 1: The threshold quality gap q for which a focal platform i wins the market in the current period.

there is a focal platform, which is the platform that won the market in the previous period. The equilibrium does not depend on which platform starts the game as the focal platform or on the market size in the first period. Suppose that in the current period platform A is focal and the market size is $N = \{1, n\}$, as described in the left-hand side of Figure 1. In equilibrium, the focal platform A wins the current period if the quality gap in favor of platform B is below an equilibrium threshold: $q \leq \overline{Q}_A(N)$. The focal platform A continues to win the market if future realizations of q are also below $\overline{Q}_A(N)$. When, eventually, there is a high realization of q such that $q > \overline{Q}_A(N)$, platform B "attacks" the market and wins all consumers. Intuitively, this happens when platform B makes an innovative breakthrough in its quality that enables it to overcome its non-focal position and dominate the market. Then, in the next period platform B is focal. Hence, as described on the right-hand side of Figure 1, there is a different equilibrium threshold, $\overline{Q}_B(N)$, such that platform B continues to win the market when its quality gap is higher than this threshold: $q \geq \overline{Q}_B(N)$. Then, once there is a low realization of q such that $q < \overline{Q}_B(N)$ (say, due to an innovative breakthrough by platform A), platform B loses focality back to platform A. When platform A is again focal, platform A maintains its focal position as long as $q \leq \overline{Q}_A(N)$, and loses focality otherwise.

Our analysis extends Halaburda, Jullien and Yehezkel (2020) to the models with both stochastic qualities and market sizes. Moreover, we focus on an asymmetric ban on predatory pricing. The role of the two stochastic variables, q and N, is the following. First, because the quality gap is stochastic, in this equilibrium both platforms compete in the market. The losing platform is a potential competitor that waits for a good opportunity (i.e., when the quality gap is in its favor and sufficient) to win the market. Second, as we show below, because of stochastic market size, asymmetric regulation yields inefficient entry, when a low-quality entrant can obtain a focal position.

Let $V_i^j(N)$ denote the value function: the discounted sum of expected profits of platform i when platform j is focal and the current size of the market is N. The Stochastic Markov Perfect Equilibrium is defined by the vector of 8 value functions, $V_i^j(N)$, and 4 thresholds, $\overline{Q}_i(N)$, for $i = \{A, B\}$ and $N = \{1, n\}$.

5 Unregulated market

We start with the case of an unregulated market: the focal and non-focal platforms can set positive or negative prices. To derive the equilibrium, consider first $V_A^A(1)$. When platform A is focal, the lowest price that platform B is willing to charge in order to win focality satisfies $p_B^A(1) + \delta EV_B^B \geq \delta EV_B^A$, where $p_i^j(N)$ is the price charged by platform i when platform j is focal given the market size N, and $EV_i^j = (1-\rho)V_i^j(1)+\rho V_i^j(n)$ is the expected value function of the next period given that the market size can be either $N = 1$ or $N = n$. Intuitively, platform B is faced with the option of winning focality and gaining EV_B^B in future periods, or losing the market in the current period and waiting for a better opportunity, which results in gaining EV_B^A. Hence, $p_B^A(1) = -\delta(EV_B^B - EV_B^A)$. The highest price that a focal platform A can charge and win the market is $\beta - p_A^A(1) \geq q - p_B^A(1)$ hence $p_A^A(1) = \beta - q - \delta(EV_B^B - EV_B^A)$. Platform A earns $p_A^A(1) + \delta EV_A^A$ from maintaining focality (which occurs when $q \leq \overline{Q}_A(1)$)

and $0+\delta EV_A^B$ when losing focality, (which occurs when $q > \overline{Q}_A(1)$). We therefore have that:

$$V_A^A(1) = \int_{-\sigma}^{\overline{Q}_A(1)} \left[\left(\beta \cdot 1 - q - \delta(EV_B^B - EV_B^A)\right) \cdot 1 + \delta EV_A^A\right] \frac{1}{2\sigma} dq \tag{2}$$

$$+ \int_{\overline{Q}_A(1)}^{\sigma} \left[\delta EV_A^B\right] \frac{1}{2\sigma} dq.$$

Next we derive $V_A^A(n)$. Again, suppose that platform A is focal and maintains its focal position. The market size is $N = n$. Now, the lowest price that platform B is willing to charge in order to win focality satisfies $np_B^A(n) + \delta EV_B^B \geq \delta EV_B^A$, or: $p_B^A(n) = -\frac{\delta}{n}(EV_B^B - EV_B^A)$. Platform A wins by charging $p_A^A(n)$ such that $\beta n - p_A^A(n) \geq q - p_B^A(n)$ or: $p_A^A(n) = n\beta - q - \frac{\delta}{n}(EV_B^B - EV_B^A)$ and earns $np_A^A(n) + \delta EV_A^A$ from maintaining focality. When platform A loses focality (which occurs when $q > \overline{Q}_A(n)$), platform A earns $0 + EV_A^B$. We therefore have that:

$$V_A^A(n) = \int_{-\sigma}^{\overline{Q}_A(n)} \left[\left(n\beta - q - \frac{\delta}{n}\left(EV_B^B - EV_B^A\right)\right) n + \delta EV_A^A\right] \frac{1}{2\sigma} dq \tag{3}$$

$$+ \int_{\overline{Q}_A(n)}^{\sigma} \left[\delta EV_A^B\right] \frac{1}{2\sigma} dq.$$

Next we move to $V_A^B(1)$. Suppose that the market size in the current period is $N = 1$, platform A is non-focal and the realization of the quality gap enables the non-focal A to win. The lowest price that the focal platform B is willing to charge to maintain focality satisfies $p_B^B(1) + \delta EV_B^B \geq \delta EV_B^A$, or $p_B^B(1) = -\delta(EV_B^B - EV_B^A)$. To win the market, platform A sets $-p_A^B(1) > q + \beta - p_B^B(1)$ or $p_A^B(1) = -q - \beta - \delta(EV_B^B - EV_B^A)$. Hence, when a non-focal platform A wins focality (which occurs when $q < \overline{Q}_B(1)$), it earns in the current period $-q - \beta - \delta(EV_B^B - EV_B^A)$ and gains δEV_A^A in the next period. When $q \geq \overline{Q}_B(1)$, platform B maintains its focal position and platform A earns $0 + \delta EV_A^B$. We therefore have:

$$V_A^B(1) = \int_{-\sigma}^{\overline{Q}_B(1)} \left[\left(-q - \beta \cdot 1 - \delta(EV_B^B - EV_B^A)\right) \cdot 1 + \delta EV_A^A\right] \frac{1}{2\sigma} dq \tag{4}$$

$$+ \int_{\overline{Q}_B(1)}^{\sigma} \left[\delta EV_A^B\right] \frac{1}{2\sigma} dq.$$

Next we move to $V_A^B(n)$. Suppose now that the market size is $N = n$, platform A is non-focal and the realization of the quality gap enables the non-focal A to win. The lowest price that the focal platform B is willing to charge to maintain focality satisfies $np_B^B(n) + \delta EV_B^B \geq \delta EV_B^A$, or $p_B^B(n) = -\frac{\delta}{n}(EV_B^B - EV_B^A)$. To win the market, platform A sets $-p_A^B(n) > q + \beta n - p_B^B(n)$ or $p_A^B(n) = -q - \beta n - \frac{\delta}{n}(EV_B^B - EV_B^A)$. Hence, when a non-focal platform A wins focality (which occurs when $q < \overline{Q}_B(n)$), it earns in the current period $n(-q - \beta n - \frac{\delta}{n}(EV_B^B - EV_B^A))$ and then gains δEV_A^A in the next period. When $q \geq \overline{Q}_B(n)$, platform B maintains its focal position and platform A earns $0 + \delta EV_A^B$. We therefore have:

$$V_A^B(n) = \int_{-\sigma}^{\overline{Q}_B(n)} \left[\left(-q - \beta n - \frac{\delta}{n}(EV_B^B - EV_B^A)\right) n + \delta EV_A^A\right] \frac{1}{2\sigma} dq \tag{5}$$

$$+ \int_{\overline{Q}_B(n)}^{\sigma} \left[\delta EV_A^B\right] \frac{1}{2\sigma} dq.$$

Moving to the value functions of platform B, the two platforms are ex-ante identical in terms of their expected quality. We can therefor apply symmetry and obtain: $V_B^B(N) = V_A^A(N)$ and $V_B^A(N) = V_A^B(N)$, for $N \in \{1, n\}$.

Now we can move to the derivation of $\overline{Q}_A(N)$ and $\overline{Q}_B(N)$. The threshold values of $\overline{Q}_A(1)$ and $\overline{Q}_A(n)$ are the quality gaps that make a focal platform A indifferent between winning or losing the market, when the number of consumers is $N = 1$ and $N = n$, respectively. Hence:

$$\left(-\overline{Q}_A(1) + \beta \cdot 1 - \delta(EV_B^B - EV_B^A)\right) \cdot 1 + \delta EV_A^A = 0 + \delta EV_A^B, \tag{6}$$

$$\left(-\overline{Q}_A(n) + n\beta - \frac{\delta}{n}\left(EV_B^B - EV_B^A\right)\right) n + \delta EV_A^A = 0 + \delta EV_A^B. \tag{7}$$

Likewise, the threshold values of $\overline{Q}_B(1)$ and $\overline{Q}_B(n)$ are the quality gaps that make a focal platform B indifferent between winning or losing the market, when the number of consumers is $N = 1$ and $N = n$, respectively:

$$\left(\overline{Q}_B(1) + \beta \cdot 1 - \delta(EV_A^A - EV_A^B)\right) \cdot 1 + \delta EV_B^B = 0 + \delta EV_B^A, \tag{8}$$

$$\left(\overline{Q}_B(n) + \beta n - \frac{\delta}{n}(EV_A^A - EV_A^B)\right) n + \delta EV_B^B = 0 + \delta EV_B^A. \tag{9}$$

Let $V_i^{j*}(N)$ and $\overline{Q}_i^*(N)$ denote the equilibrium value functions and thresholds in the competitive, unregulated case. Solving conditions (2) - (9) yields the following result (the proof of all propositions are in the appendix):

Proposition 1. ***(Features of the unregulated equilibrium)*** *There exists a unique and symmetric Stochastic Markov Perfect Equilibrium. In this equilibrium:*

(*i*) *Value functions are positive and are higher for the focal platform than the non-focal one:* $V_i^{i*}(N) > V_i^{j*}(N) > 0$.

(*ii*) *The ability of a focal platform to maintain its focal position is independent of* δ*: when the market size in the current period is* $N \in \{1, n\}$*, a focal platform A (B) maintains its focal position if the quality gap is below (above)* $\overline{Q}_A^*(N) = \beta N$ $(\overline{Q}_B^*(N) = -\beta N)$.

(*iii*) *Evaluated at* $q = \overline{Q}_i^*(N)$*, a focal platform i* $(i = A, B)$ *charges a negative price.*

The first part of Proposition 1 shows that platforms have an incentive to fight for focality as winning focality increases their value function: $V_i^{i*}(N) > V_i^{j*}(N)$. At the same time, $V_i^{j*}(N) > 0$ implies that a non-focal platform has an incentive to wait for the "right moment" to fight for focality, i.e., wait for an innovative breakthrough such that its' quality realization is sufficiently higher than that of the focal platform.

The second part of Proposition 1 shows that in an unregulated market, the threshold values of q that enable a non-focal platform B to win is identical to the static threshold in Lemma 1, $q^{static} = \beta N$. Hence, as in the static game, there is inefficient incumbency: the incumbent platform can win due to its focal position even when its quality is inferior to that of the entrant. Dynamics do not mitigate the incumbency advantage. The intuition for this result is that, on one hand, dynamic considerations increase the incentive of the focal platform A to fight (and set a lower price) to maintain its focal position, because platform A would like to benefit from being focal in the next period. Yet, dynamics also increase the incentive of the non-focal platform B to compete for the focal position, again in order to benefit from being focal in future periods. Because the game is infinite and they have the same expected future quality, both platforms have the same incentive to gain focality in the next period and dynamics do not mitigate the problem of inefficient incumbency.

More precisely, the symmetry between the expected quality of the two platforms implies that their expected value functions in the future are equal, both from the focal position ($EV_A^A = EV_B^B$) and the non-focal position ($EV_A^B = EV_B^A$). This in turn means that the additional profits (losses) from winning (losing) the focal position are equal for both platforms (i.e. $\delta(EV_A^A - EV_A^B) = \delta(EV_B^B - EV_B^A)$), and, as those two terms cancel each other, the threshold quality gap follows immediately from each of the equations (6)-(9). As we show below, this result no longer holds under asymmetric regulation.[9]

The third part of Proposition 1 shows that in a dynamic game, inefficient incumbency requires a negative price from the incumbent. When the quality gap is such that the focal platform (either A or B) is indifferent between winning or losing focality, the price of the focal platform is negative. Intuitively, the focal platform's price depends on the realization of the quality gap and can be positive or negative. When the quality gap in a certain period is against the focal platform such that it needs to set a too low (negative) price, the platform becomes indifferent between winning or losing. This result implies that imposing regulation of non-negative price on the incumbent platform is binding on the solution to the equilibrium thresholds $\overline{Q}_i^*(N)$.

While dynamics do not change the threshold quality that enables a non-focal platform to take over the market, they do have an effect on the market outcome. As the following proposition shows, one-period expected welfare is unaffected by δ. Yet, profits (consumer surplus) decrease (increases) with δ. To show this, we normalize $q_{At} = q_0$ and $q_{Bt} = q_0 + q_t$, where q_t is the stochastic variable. Let W^{i*} $(i = A, B)$ denote the expected recursive social welfare when platform i is focal in an unregulated market. We have that W^{i*} is the solution to:

$$W^i = \rho\left(\int_{-\sigma}^{\overline{Q}_i^*(n)}((n\beta+q_0)n+\delta W^i)\frac{1}{2\sigma}dq + \int_{\overline{Q}_i^*(n)}^{\sigma}(n(n\beta+q_0+q)+\delta W^j)\frac{1}{2\sigma}dq\right) \tag{10}$$

$$+(1-\rho)\left(\int_{-\sigma}^{\overline{Q}_i^*(1)}((\beta+q_0)+\delta W^i)\frac{1}{2\sigma}dq + \int_{\overline{Q}_j^*(1)}^{\sigma}((\beta+q_0+q)+\delta W^j)\frac{1}{2\sigma}dq\right).$$

[9] Halaburda, Jullien and Yehezkel (2020) show that when platform B has a higher expected quality, dynamics reduce inefficient incumbency.

Notice that because of symmetry, $W^* \equiv W^{A*} = W^{B*}$.

Let $w^* = (1-\delta)W^*$ denote the per-period expected welfare in an unregulated market. The sum of the platforms' per-period expected profits is $\pi^* = (1-\delta)(EV_i^{i*} + EV_i^{j*})$. Notice that we sum the profits of both the focal and non-focal platform because in each period, each platform has a positive probability to win the market. Expected per-period consumer surplus is $cs^* = w^* - \pi^*$. We have:

Proposition 2. ***(Dynamics enhance consumers' surplus)*** *Per-period expected welfare in an unregulated market, w^*, is independent of δ. Yet, the expected sum of the platforms' per-period profit, π^*, is decreasing with δ, while the expected per-period consumer surplus, cs^*, is increasing in δ.*

The intuition for this result is the following. As higher δ means that future profits are of a higher value in the current period, an increase in δ drives both platforms to fight harder today for a focal position in future periods by lowering their prices. Thus, an increase in δ will lead to a decrease in the platforms' profits on one hand, and an increase in consumer surplus on the other, while the total welfare remains independent of δ, as the effects on profits and consumer surplus balance each other.

Finally, consider the alternative scenario in which the losing platform leaves the market and a new entrant is always born in the next period (see the results in Appendix B). In this case, the Stochastic Markov Perfect Equilibrium is identical to the static form, repeated infinitely. In particular, both profits and welfare are independent of δ. The intuition is that when platforms exit the market upon defeat, future profits of the losing platform (whether it is the focal or the non-focal platform) are zero. In turn, both platforms may apply all future profits they attain from gaining the focal position, into subsidizing consumers in the current period, and, because in expectancy, the profits from gaining the focal position are equal for both platforms, the threshold is identical to the static game: $\overline{Q}_A(N) = \beta \cdot N$ and $\overline{Q}_B(N) = -\beta \cdot N$, $(N = \{1, n\})$, and equilibrium profits are independent of δ.

6 Symmetric regulation

To address the problem of inefficient incumbency, consider symmetric regulation: competition authorities forbid both platforms from charging negative prices. When both platforms can only charge positive prices, the game is equivalent to a static game, in which the platform that can attract consumers with a positive price (equivalently, positive current-period profits) wins, given that the competing platform charges a price equal to 0. As we showed in the static benchmark in Section 3, a focal platform A (B) maintains its focal position when the market size is $N = \{1, n\}$ if $q < N\beta$ ($q > -N\beta$). Because by assumption $\sigma > N\beta$, in the Stochastic Markov Perfect Equilibrium $\overline{Q}_A(N) = N\beta$ and $\overline{Q}_B(N) = -N\beta$. The following corollary summarizes this result.

Corollary 1. ***(Symmetric regulation has no effect on market efficiency)*** *Suppose that the two platforms are prohibited from charging negative prices. Then, in equilibrium, $\overline{Q}_A(N) = N\beta$ and $\overline{Q}_B(N) = -N\beta$, ($N = \{1, n\}$). Hence, market efficiency (the quality of the wining platform) is identical to that in the unregulated market and the only effect of price regulation is higher prices.*

Intuitively, symmetric regulation cannot correct the asymmetric advantage that the focal platform has, and hence maintains the competitive advantage of the focal platform. The market exhibits the same level of inefficient incumbency as under an unregulated market. While symmetric regulation does not affect total social welfare, it decreases consumer surplus as it is harmful to consumers who need to pay higher prices.

The inability of symmetric regulation to correct the market inefficiency calls for asymmetric regulation, which we discuss in the next section.

7 Asymmetric regulation: banning predatory pricing by the incumbent

In this section we consider asymmetric regulation, when competition authorities prohibit only the incumbent platform from dominating the market with a predatory price. The main conclusion of this section is that when the market remains constant over time, asymmetric regulation reduces the problem of inefficient incumbency and is therefore welfare enhancing. Yet, when market size varies between periods, asymmetric regulation may create the problem of inefficient entry: an entrant of low quality wins the market. As a result, asymmetric regulation can reduce social welfare.

Suppose now that competition laws prohibit the incumbent from dominating the market with a price below marginal costs (zero in our model). The incumbent is condemned for anti-competitive behavior only when it wins the market with a negative price, while its price is not subjected to antitrust scrutiny when it loses the market anyway. [10] The entrant platform has a grace period in which it can charge an introductory price in order to gain a foothold in the market. In the context of this infinite horizon game, when an entrant becomes an incumbent after winning the previous period, the former entrant faces the restriction of a price above cost, while the former incumbent can now charge an introductory price.

To solve for the Stochastic Markov Perfect Equilibrium given a ban on predatory pricing, we add the restriction that $p_i^i(N) \geq 0$ in any equilibrium in which the incumbent wins the market. From Proposition 1, this restriction is binding at least at $q = \overline{Q}_i(N)$. Therefore, $\overline{Q}_i(N)$ are defined by the restriction that $p_i^i(N) = 0$ instead of the conditions that defined $\overline{Q}_i(N)$ in the unregulated market (equations (6) - (9)). Because $p_A^A(N)$ is decreasing in q while $p_B^B(N)$ is increasing in q, we have that when $\overline{Q}_i(N)$ are defined by $p_i^i(N) = 0$, it follows that $p_A^A(N) > 0$ for all $q < \overline{Q}_A(N)$ and $p_B^B(N) > 0$ for all $q > \overline{Q}_B(N)$. Replacing conditions (6) - (9) with the conditions:

$$p_A^A(1) = \beta - \overline{Q}_A(1) - \delta(EV_B^B - EV_B^A) = 0, \tag{11}$$

$$p_A^A(n) = n\beta - \overline{Q}_A(n) - \frac{\delta}{n}(EV_B^B - EV_B^A) = 0, \tag{12}$$

$$p_B^B(1) = \beta + \overline{Q}_B(1) - \delta(EV_A^A - EV_A^B) = 0, \tag{13}$$

$$p_B^B(n) = n\beta + \overline{Q}_B(n) - \frac{\delta}{n}(EV_A^A - EV_A^B) = 0, \tag{14}$$

[10] When the incumbent is prohibited from charging a negative price in an equilibrium in which the incumbent loses the market, there is no Stochastic Markov Perfect Equilibrium with pure strategies. Intuitively, in this case a platform does not have an incentive to win the market because when it does, it cannot profitably hold on to it. We report this result in Appendix C.

and rearranging yields the asymmetric regulation value functions and thresholds, denoted by $V_i^{j**}(N)$ and $\overline{Q}_A^{**}(N)$.

Proposition 3. ***(The asymmetric regulation equilibrium)*** *Consider asymmetric regulation in which a focal platform cannot charge a negative price. Then, there is a unique Stochastic Markov Perfect Equilibrium with:*

$$\overline{Q}_A^{**}(1) = \beta - \delta\beta\left(1 + \rho(n^2 - 1)\right), \quad \overline{Q}_A^{**}(n) = \beta n - \frac{\delta\beta}{n}\left(1 + \rho(n^2 - 1)\right), \tag{15}$$

$$\overline{Q}_B^{**}(1) = -\beta + \delta\beta\left(1 + \rho(n^2 - 1)\right), \quad \overline{Q}_B^{**}(n) = -\beta n + \frac{\delta\beta}{n}\left(1 + \rho(n^2 - 1)\right). \tag{16}$$

In equilibrium, the value functions are positive at least when n is not too high and $\sigma > 2\beta$. Moreover, it is always the case that $V_i^{i**}(N) > V_i^{j**}(N)$.

There is a Stochastic Markov Perfect Equilibrium with positive value functions when the spread of stochastic qualities is sufficiently wide. Intuitively, value functions are positive when the non-focal platform has a sufficiently high probability to win the market, which is the case when the spread of potential quality gap is high. In what follows, suppose that n is not too high and $\sigma > 2\beta$.

To evaluate the effect of asymmetric regulation on efficiency, recall that in the absence of regulation there is inefficient incumbency when a focal platform A (B) wins even though the entrant platform B (A) is more efficient: $q > 0$ ($q < 0$). This in turn holds when the market size is N and $\overline{Q}_A(N) > 0$ ($\overline{Q}_B(N) < 0$). A second type of inefficiency, that does not emerge in an unregulated market or in the symmetric regulation market, is *inefficient entry*. This inefficiency occurs when an entrant of inferior quality can nevertheless enter and dominate the market. Intuitively, inefficient entry can emerge under asymmetric regulation because only the incumbent is restricted from charging a negative price. In the context of this model, there is inefficient entry when the non-focal platform B (A) wins even though the incumbent platform A (B) is more efficient: $q < 0$ ($q > 0$). This in turn holds when the market size is N and $\overline{Q}_A(N) < 0$ ($\overline{Q}_B(N) > 0$). The following two corollaries show how asymmetric regulation affects the possibilities of inefficient incumbency and inefficient entry. We start with the case in which there are only stochastic qualities, while the market size is always $N = 1$. Substituting $n = 1$ into (15) and (16), we obtain the following result.

Corollary 2. ***(Asymmetric regulation reduces inefficient incumbency when the market size is constant over time)*** *Consider an asymmetric regulated market in which a focal platform cannot charge a negative price and suppose that the market is characterized only by stochastic qualities, i.e., $n = 1$. Then, in comparison with an unregulated market, asymmetric regulation reduces the problem of inefficient incumbency and does not create a problem of inefficient entry. That is:*

(i) $\beta > \overline{Q}_A^{**}(1) > 0$ *and* $-\beta < \overline{Q}_B^{**}(1) < 0$*;*

(ii) Inefficient incumbency decreases with δ, i.e., $\overline{Q}_A^{**}(1)$ ($\overline{Q}_B^{**}(1)$) is decreasing (increasing) with δ. This effect is stronger as network externalities increase:

$$\frac{\partial^2 \overline{Q}_A^{**}(1)}{\partial\delta\partial\beta} < 0 \quad \textit{and} \quad \frac{\partial^2 \overline{Q}_B^{**}(1)}{\partial\delta\partial\beta} > 0;$$

(iii) As $\delta \to 1$, the market converges to the efficient entry outcome, $\overline{Q}_A^{**}(1) = \overline{Q}_B^{**}(1) = 0$.

Figure 2 illustrates the results of Corollary 2. As the figure shows, asymmetric regulation decreases the region in which there is inefficient incumbency (i.e., the region in which $\overline{Q}_A^{**}(1) > q > \overline{Q}_B^{**}(1)$) and this decrease becomes stronger as platforms become more forward-looking. Moreover, as $\delta \to 1$, the two regions of inefficient incumbency vanish. In contrast to Corollary 3 below on a stochastic market size, when the market is not expected to change over time, although the non-focal platform can increase its future profits by winning the current period and secure the focal position, a fixed size market over time ensures that the competitive advantage that a non-focal platform gains from the asymmetrical restrictions will not be strong enough to facilitate its entry into the market with a lower quality than the incumbent.

As for the effect of network externalities, β, notice that β increases the problem of inefficient incumbency regardless of whether there is regulation or not. That is, $\overline{Q}_A^{**}(N)$ ($\overline{Q}_B^{**}(N)$) is increasing (decreasing) in β. This is because as network externalities become more important to consumers, focality provides a stronger competitive advantage to the incumbent platform. Yet, as platforms become more forward-looking, the positive effect of asymmetric

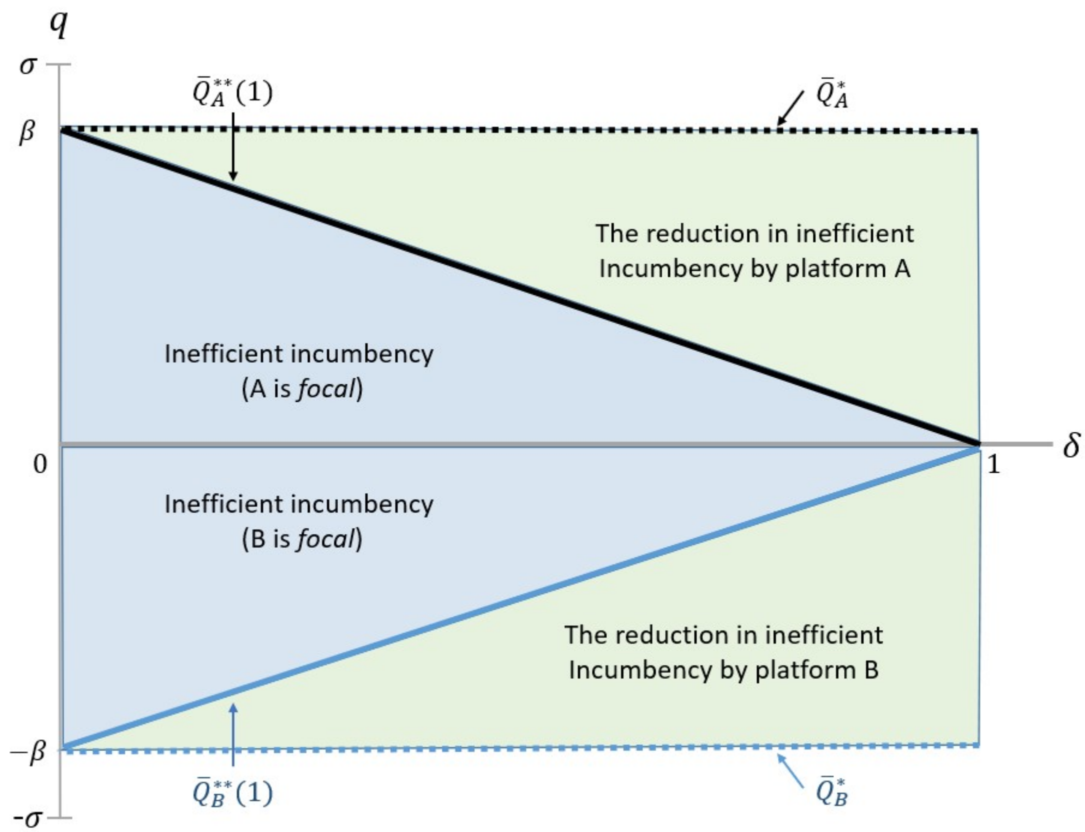

Figure 2: The regions of inefficient incumbency of both focal platforms, fixed size market

regulation in reducing inefficient incumbency becomes stronger as network externalities increases. Intuitively, as network effects increase, the ability of the incumbent to exploit its focal position increase, which makes asymmetric regulation a stronger tool in reducing this incumbent's ability.

The above results, however, do not follow to the case where there are both stochastic qualities and market sizes.

Corollary 3. ***(Asymmetric regulation creates inefficient entry when market size is stochastic)*** *Consider an asymmetric regulated market in which the focal platform cannot charge a negative price and suppose that the market is characterized by both stochastic qualities and market sizes, i.e., $n > 1$. Then,*

(*i*) *In periods of low market size ($N = 1$), there is inefficient incumbency when platforms are short-sighted and inefficient entry if platforms are forward-looking. That is: $\overline{Q}_A^{**}(1) > 0$ and $\overline{Q}_B^{**}(1) < 0$ when $\delta \in [0, 1/(1 + \rho(n^2 - 1))]$ and $\overline{Q}_A^{**}(1) < 0$ and*

$\overline{Q}_B^{**}(1) > 0$ *when* $\delta \in [1/(1+\rho(n^2-1)), 1]$. *Moreover,* $\overline{Q}_A^{**}(1)$ *(*$\overline{Q}_B^{**}(1)$*) is decreasing (increasing) in* n.

(*ii*) *In periods of high market size (*$N = n$*), there is only inefficient incumbency:* $\overline{Q}_A^{**}(n) > 0$ *and* $\overline{Q}_B^{**}(n) < 0$ *for* $\forall \delta \in [0, 1)$. *Moreover,* $\overline{Q}_A^{**}(n)$ *(*$\overline{Q}_B^{**}(n)$*) is increasing (decreasing) in* n.

(*iii*) *The problem of inefficient incumbency is higher in periods of high demand than in periods of low demand:* $\overline{Q}_A^{**}(n) > \overline{Q}_A^{**}(1)$ *and* $\overline{Q}_B^{**}(n) < \overline{Q}_B^{**}(1)$.

Figure 3 illustrates the results of Corollary 3. Consider the case of a low market size: $N = 1$

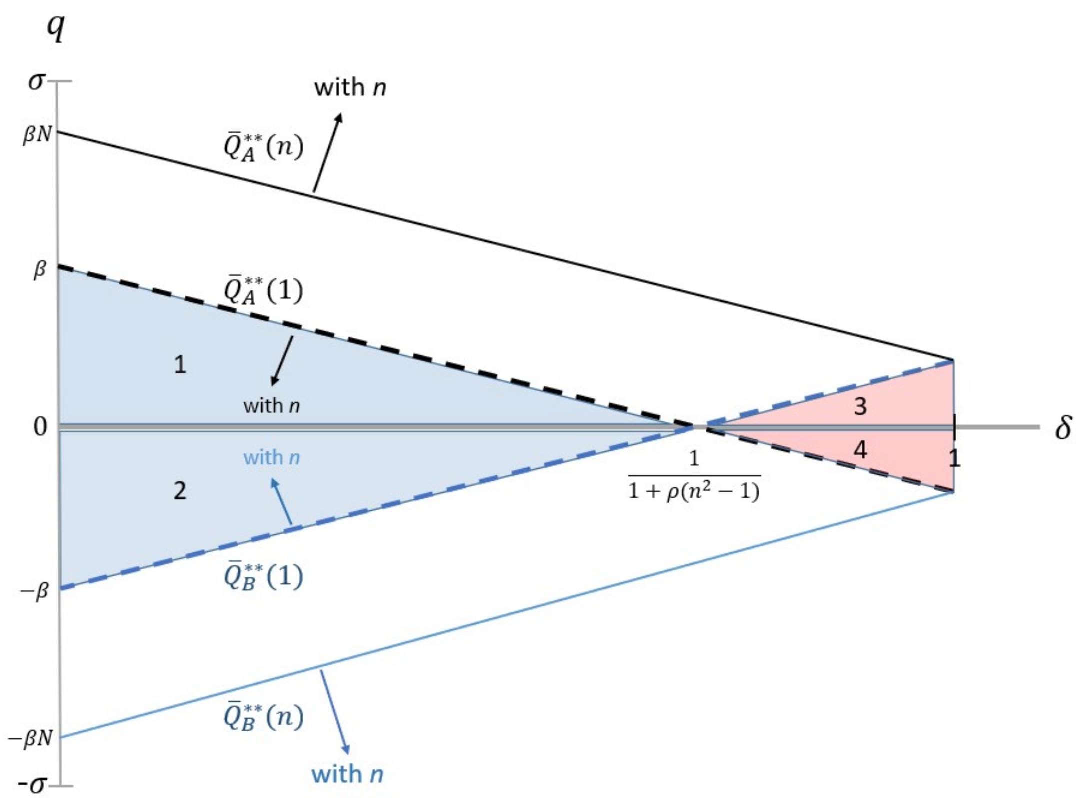

Figure 3: The regions of inefficient incumbency and inefficient entry, regulated market with stochastic market size

(part *(i)* of Corollary 3). The figure shows that when δ is small, there is a region of inefficient incumbency (region 1 (2) for a focal A (B)) with: $\overline{Q}_A^{**}(1) > 0 > \overline{Q}_B^{**}(1)$. As δ increases, this region shrinks and vanishes at $\delta = 1/(1+\rho(n^2-1))$. Then, a further increase in δ results in a region with the problem of inefficient entry (region 3 (4) for a non-focal B (A)) with

$\overline{Q}_B^{**}(1) > 0 > \overline{Q}_A^{**}(1)$. This region increases as δ increases. The intuition for this result is that when the market size is small, platforms expect that in future periods market size is likely to be higher than in the current period. This creates a strong incentive for the non-focal platform to set a negative price because the platform incurs losses from this negative price for only a small market size, and expects to gain high revenues in future periods that can compensate for these short-run losses. Because the focal platform cannot charge a negative price, the non-focal platform gains a competitive advantage, which can be strong enough to enable it to win the market with a lower quality than the focal platform. Part (i) of Corollary 3 also shows that as n increases, the region of inefficient incumbency decreases while the region of inefficient entry increases. Intuitively, the greater the expected market size in a high realization, the stronger the competitive advantage that an asymmetric regulation provides the entrant in periods of low market size realization.

When the market size is large, part (ii) of Corollary 3 shows that there is only the problem of inefficient incumbency, which decreases as δ increases. Yet, the region of inefficient incumbency is wider in periods of high market size than in periods of low market size. The intuition for this result is that the non-focal platform finds it too costly to charge a negative price when current market size is high, which provides a competitive advantage to the focal platform.

Combining the results, it follows that asymmetric regulation enhances efficiency when platforms are short-sighted. For low values of δ, a further increase in δ increases the efficiency of imposing asymmetric regulation. Yet, starting from $\delta = 1/\left(1 + \rho(n^2 - 1)\right)$ onward, a further increase in δ has conflicting effects on the efficiency of asymmetric regulation. On one hand, regulation has the positive effect of reducing the region of inefficient incumbency when the market size is high. At the same time, regulation increases the region of inefficient entry when the market size is low. In such a case, when competition authorities consider a stricter policy on prohibiting incumbent platforms from charging predatory prices, they need to prioritize between the potential problems of inefficient incumbency and inefficient entry. The next section provides a welfare analysis of asymmetric regulation.

8 The effect of asymmetric regulation on consumers, platforms and welfare

We now turn our attention to social welfare. We focus on asymmetric prohibition in which only the focal platform is prohibited from charging negative prices, and ask whether social welfare is higher under regulatory restrictions. The main conclusion of this section is that because asymmetric regulation decreases the problem of inefficient incumbency when market size is constant over time, asymmetric regulation increases social welfare. Yet, because asymmetric regulation may create the problem of inefficient entry when market size is stochastic, asymmetric regulation enhances welfare when δ is low, but can be harmful to welfare for high values of δ.

Under a regulated market, W^i is:

$$W^i = \rho\left(\int_{-\sigma}^{\overline{Q}_i(n)}((n\beta+q_0)n+\delta W^i)\frac{1}{2\sigma}dq+\int_{\overline{Q}_i(n)}^{\sigma}(n(n\beta+q_0+q)+\delta W^j)\frac{1}{2\sigma}dq\right) \quad (17)$$

$$+(1-\rho)\left(\int_{-\sigma}^{\overline{Q}_i(1)}((\beta+q_0)+\delta W^i)\frac{1}{2\sigma}dq+\int_{\overline{Q}_j(1)}^{\sigma}((\beta+q_0+q)+\delta W^j)\frac{1}{2\sigma}dq\right),$$

and W^{i**} is the solution to (17) evaluated at $\overline{Q}_i^{**}(N)$. Notice that because of symmetry, $W^{**} \equiv W^{A**} = W^{B**}$.

Let $w^{**} = (1-\delta)W^{**}$ denote the per-period expected welfare in an asymmetric regulated market. Recall that w^* is the per-period expected welfare at the unregulated market. Our next proposition details the results from comparing w^* with w^{**}.

Proposition 4. ***(The effect of asymmetric regulation on per-period social welfare)*** *Consider an asymmetric regulated market in which the focal platform cannot charge a negative price. When n is close to 1, asymmetric regulation increases welfare for all $\delta \in (0,1]$. Yet, when n and δ are high, asymmetric regulation decreases welfare. In particular:*

(i) *Evaluated at $\delta = 0$, $w^{**} = w^*$ and w^{**} is increasing in δ;*

(ii) *w^{**} is concave in δ;*

(iii) *There is a threshold of* δ, $\delta^* \equiv \frac{2n(1+(n-1)\rho)}{(n+\rho-n\rho)(1(n^2-1)\rho)}$, *such that if* $\delta^* < 1$ *(which holds if* n *is high and for intermediate values of* ρ*),* $w^{**} > w^*$ *when* $\delta \in [0, \delta^*]$ *and* $w^{**} < w^*$ *otherwise.*

Figure 4 illustrates the results of Proposition 4. Panel (a) shows the case where n is small

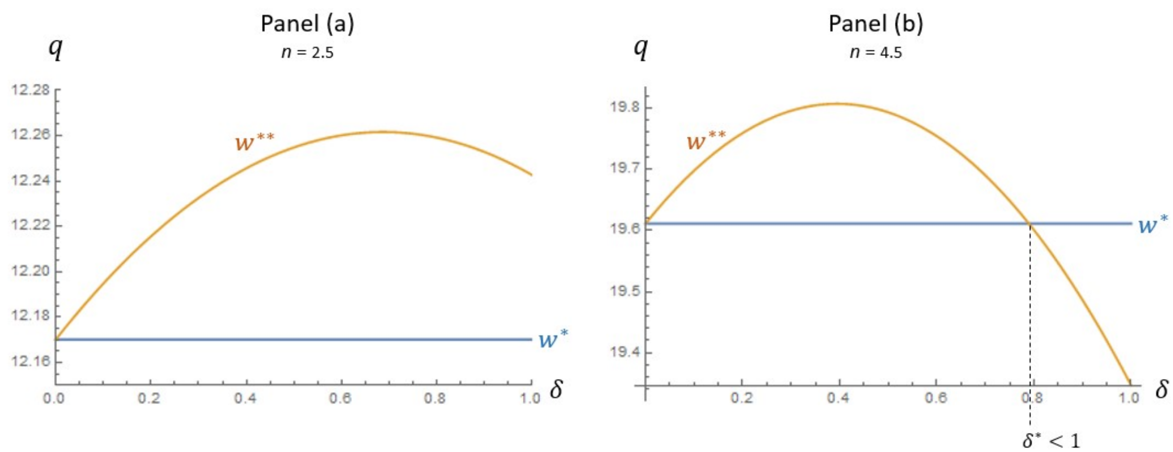

Figure 4: Total welfare under unregulated and regulated markets as a function of δ ($\rho = 0.3$, $\beta = 1$ and $\sigma = 7$)

such that $w^{**} > w^*$ for all values of $\delta > 0$. Panel (b) shows the case where $\delta^* < 1$, such that $w^{**} > w^*$ when $\delta \in [0, \delta^*]$ and $w^{**} < w^*$ otherwise.[11]

The intuition for this result is the following. When market size is constant over time (n is close to 1, or ρ is close to either 0 or 1), asymmetric regulation always decreases inefficient incumbency without creating the problem of inefficient entry. Therefore, under these parameters asymmetric regulation is welfare enhancing for all δ, as shown in panel (a). Suppose now that market size changes over time (n is higher than 1 and ρ is intermediate). For low δ, asymmetric regulation reduces inefficient incumbency under both states of market size and again enhances welfare. For high δ, asymmetric regulation reduces inefficient incumbency at the high realization of market size but creates the problem of inefficient entry at the low realization of market size. As δ increases, the second effect becomes stronger and asymmetric regulation reduces welfare.

[11]In both cases, we have verified that the value functions are positive and that the equilibrium includes an internal values of $\overline{Q}_i^*(N)$ and $\overline{Q}_i^{**}(N)$.

Next, we turn to profits and consumer surplus. As in the unregulated case, we define expected per-period profits as $\pi^{**} = (1-\delta)(EV_i^{i**} + EV_i^{j**})$ and the expected per-period consumer surplus is $cs^{**} = w^{**} - \pi^{**}$. We have:

Proposition 5. ***(The effect of asymmetric regulation on profits and consumers)*** *Platforms' expected joint per-period profit is lower under asymmetric regulation than in an unregulated market: $\pi^{**} < \pi^{*}$, while expected per-period consumer surplus is higher under asymmetric regulation: $cs^{**} > cs^{*}$.*

The intuition for this result is the following. Out of the two competing platforms in each period, the incumbent is able to monetize the consumers' network effects through a high price, due to its focal position. The entrant can only monetize its quality advantage, as it is required to overcome the bias in consumers' beliefs. As a result, other things being equal, consumers are better off when the non-focal platform wins the market. Now, in the asymmetric regulation market, switches in focality are more frequent than in an unregulated market, resulting in a higher expected consumer surplus and lower expected profits.

The results of Propositions 4 and 5 indicate that at least for low values of δ, asymmetric regulation is both welfare enhancing and increases consumer surplus. Yet, for high values of δ, asymmetric regulation may decrease total welfare when its negative effect on the platforms' profits outweighs the positive effect such regulation has on consumers.

9 Conclusion

This paper examines the competitive effects of the prohibition of predatory pricing in markets with network externalities. We consider an infinitely repeated game between two competing platforms. The platform that dominated the market in the previous period becomes the focal, incumbent platform in the current period, in that consumers expect other consumers to join it. Hence, consumers' beliefs serve as a barrier to entry, when they are biased towards the incumbent platform. We study the market outcomes of price restricting policies on either both the incumbent and the entrant, or asymmetric regulation on only the incumbent.

Our paper contributes to the understanding of predatory pricing in platform competition in three ways. First, we model and highlight the role of predatory pricing in the context of platform competition, and evaluate the effects of focality on the motivation to adopt predatory pricing and its effect on welfare. We study both the short and long-run effects of predatory pricing by considering an infinitely repeated game, in which there is competition in every period.

Second, our results have policy implications for the regulation of platforms. We show that a symmetrical price restriction (on both platforms) always decreases consumer surplus as it softens price competition, yet has no effect on market efficiency and welfare. In contrast, we show that an asymmetrical price restriction, where only the focal platform is prohibited from charging predatory pricing, improves the ability of an entrant to compete with the focal incumbent, and therefore abate the problem of focality, even though they do not entirely eliminate it. This result holds as long as the size of the market remains constant between periods.

Yet, in our third and main contribution, we show that when the size of the market varies over time, asymmetrical price restrictions may lead to inefficiencies and a decrease in social welfare, by facilitating the entry of a lower quality entrant. More precisely, we identify the role of an unstable market size, in creating a potential tradeoff in prohibiting predatory pricing: on one hand, an incumbent focal platform of low quality can use predatory pricing to dominate the market, when facing an entrant of superior quality, and on the other, restricting the incumbent from charging predatory prices may have the welfare reducing effect of enabling a lower-quality entrant to take over the market. Our results show that prohibiting predatory pricing may result in the latter case when the market size varies over time and when platforms are forward-looking.

Appendix A

Below are the proofs of Propositions 1 - 5.

Proof of Proposition 1:

Solving conditions (2) - (9) and letting $EV_i^{j*} = (1-\rho)V_i^{j*}(1) + \rho V_i^{j*}(N)$, yields 12 equations with 12 variables: 8 value functions, $V_i^{j*}(N)$, and 4 thresholds, $\overline{Q}_i^*(N)$, for $i = \{A, B\}$ and $N = \{1, n\}$. Solving them yields the unique symmetric outcome with:

$$V_i^{j*}(1) = \frac{(1-\delta\rho)(\sigma-\beta)^2}{4\sigma(1-\delta)} + \frac{\delta n\rho(\sigma-n\beta)^2}{4\sigma(1-\delta)}, \tag{18}$$

$$V_i^{j*}(n) = \frac{\delta(1-\rho)(\sigma-\beta)^2}{4\sigma(1-\delta)} + \frac{n(1-\delta+\delta\rho)(\sigma-n\beta)^2}{4\sigma(1-\delta)}, \tag{19}$$

$$V_i^{i*}(1) = V_i^{j*}(1) + \beta, \tag{20}$$

$$V_i^{i*}(n) = V_i^{j*}(n) + n^2\beta, \tag{21}$$

and $\overline{Q}_A^*(N) = \beta N$, $\overline{Q}_B^*(N) = -\beta N$. It is straightforward to see that because $0 < \delta < 1$, $0 < \rho < 1$ and $\sigma > n\beta$, all value functions are positive. Moreover, it is straightforward to see that $V_i^{i*}(N) > V_i^{j*}(N)$.

Next, we turn to show that evaluated at $q = \overline{Q}_i^*(N)$, $p_i^{i*}(N) < 0$, where $p_i^{i*}(N)$ is the price of a focal platform i when the market size is N. Notice that evaluated at $q = \overline{Q}_i^*(1)$, $p_i^{i*}(1)$ solves: $p_i^{i*}(1) + \delta EV_i^{i*} = \delta EV_i^{j*}$. Hence, $p_i^{i*}(1) = -\delta(EV_i^{i*} - EV_i^{j*}) = -\beta(1-\rho+n^2\rho) < 0$, where the second equality follows from substituting $V_i^{i*}(1) = V_i^{j*}(1) + \beta$ and $V_i^{i*}(n) = V_i^{j*}(n) + n^2\beta$. Likewise, evaluated at $q = \overline{Q}_i^*(n)$, $p_i^{i*}(n)$ solves: $np_i^{i*}(1) + \delta EV_i^{i*} = \delta EV_i^{j*}$. Hence, $p_i^{i*}(1) = -\frac{\delta}{n}(EV_i^{i*} - EV_i^{j*}) < 0$.

Proof of Proposition 2:

The solution to (10) is:

$$w^* = \frac{1}{4}\left[\left(4\beta(1+\rho(n^2-1)\right) - \frac{\beta^2\left(1+\rho(n^3-1)\right)}{\sigma} + \left(1+(n-1)\rho\right)\left(4q_0+\sigma\right)\right], \tag{22}$$

which is independent of δ. Next, expected per-period profits is:

$$\pi^* = (1-\delta)\left(\rho(V_i^{i*}(n) + V_i^{j*}(n)) + (1-\rho)(V_i^{i*}(1) + V_i^{j*}(1))\right) \tag{23}$$

$$= \frac{1}{2}\left(\frac{\beta^2(1+\rho(n^3-1))}{\sigma} + \sigma(1+\rho(n-1))\right) - \delta\beta\left(1+\rho(n^2-1)\right),$$

which is decreasing with δ because $\beta\left(1+\rho(n^2-1)\right) > 0$. Finally, expected one-period consumer surplus is:

$$cs^* = w^* - \pi^* = \tag{24}$$

$$\beta(1+\delta)\left(1+\rho(n^2-1)\right) + \frac{1}{4}\left(1+\rho(n-1)\right)(4q_0-\sigma) - \frac{3\beta^2\left(1+(n^3-1)\rho\right)}{4\sigma},$$

which is increasing with δ because $\beta\left(1+\rho(n^2-1)\right) > 0$.

Proof of Proposition 3:

Solving conditions (2) - (5) and (11) - (14) yields:

$$V_i^{j**}(1) = V_i^{j*}(1) - \frac{\left(1+\rho(n^2-1)\right)^2}{4n\sigma(1-\delta)}\beta^2\delta^2(n(1-\delta\rho)+\delta\rho), \tag{25}$$

$$V_i^{j**}(n) = V_i^{j*}(n) - \frac{\left(1+\rho(n^2-1)\right)^2}{4n\sigma(1-\delta)}\beta^2\delta^2(1+\delta(n-1)(1-\rho)), \tag{26}$$

$$V_i^{i**}(1) = V_i^{j**}(1) + \beta, \tag{27}$$

$$V_i^{i**}(n) = V_i^{j**}(n) + n^2\beta, \tag{28}$$

and (15) - (16). Notice that it is always the case that $V_i^{i**}(N) > V_i^{j**}(N)$. Yet, because the second terms in $V_i^{j**}(1)$ and $V_i^{j**}(n)$ are negative, we need to verify that $V_i^{j**}(1)$ and $V_i^{j**}(n)$ are positive. To this end, evaluating $V_i^{j**}(1)$ and $V_i^{j**}(n)$ when there is no stochastic market size, $n=1$, yields

$$V_i^{j**}(1)\big|_{n=1} = V_i^{j**}(n)\big|_{n=1} = \frac{(\sigma-\beta(1+\delta))(\sigma-\beta(1-\delta))}{4\sigma(1-\delta)}, \tag{29}$$

which is positive if $\sigma > \beta(1+\delta)$, which in turn is positive for all values of δ if $\sigma > 2\beta$. Because $V_i^{j**}(1)$ and $V_i^{j**}(n)$ are continuous in n, we have that $V_i^{j**}(1)$ and $V_i^{j**}(n)$ are positive as long as $\sigma > 2\beta$ and n is not too high.

Proof or Proposition 4:

The solution to (17) evaluated at $\overline{Q}_i^{**}(N)$, is:

$$w^{**} = w^* + \delta \left[\frac{\beta^2(1+(n-1)\rho)(1+(n^2-1)\rho)}{2\sigma} \right] \tag{30}$$

$$-\delta^2 \left[\frac{\beta^2(n(1-\rho)+\rho)(1+(n^2-1)\rho)^2}{4n\sigma} \right].$$

Evaluating w^{**} at $\delta = 0$, the second and third terms vanish, hence $w^{**} = w^*$. Because the two terms in the squared brackets of w^{**} are positive and unaffected by δ,

$$\left.\frac{\partial w^{**}}{\partial \delta}\right|_{\delta=0} = \frac{\beta^2(1+(n-1)\rho)(1+(n^2-1)\rho)}{2\sigma} > 0, \tag{31}$$

which proves part (i). The welfare,w^{**}, is concave in δ because the two terms in the squared brackets of w^{**} are positive and unaffected by δ , which proves part (ii). Finally, there are two solutions to $w^* = w^{**}$ at $\delta = 0$ and $\delta = \frac{2n(1+(n-1)\rho)}{(n+\rho-n\rho)(1+(n^2-1)\rho)} \equiv \delta^*$, where $\delta^* < 1$ when $n > 3.55$ and $\frac{1+n-n^2}{2(1-n^2)} - \frac{\sqrt{1-4n+6n^3-4n^5+n^6}}{2(n-1)^2(1+n)} < \rho < \frac{1+n-n^2}{2(1-n^2)} + \frac{\sqrt{1-4n+6n^3-4n^5+n^6}}{2(n-1)^2(1+n)}$.

Proof of Proposition 5:

From equations (25) - (28), it follows that $V_i^{j**}(N) < V_i^{j*}(N)$ and $V_i^{i**}(N) < V_i^{i*}(N)$, for $N = \{1, n\}$. We therefore have that $\pi^{**} < \pi^*$. Next, $cs^{**} = w^{**} - \pi^{**}$ or:

$$cs^{**} = cs^* + \frac{\delta\beta^2}{4n\sigma}\left(1+(n^2-1)\rho\right)\left(2(n+n\rho(n-1)) + \delta n(1-\rho)+\rho)(1+(n^2-1)\rho\right), \tag{32}$$

where the second term is positive for all $\delta > 0$ because $n > 0$ and $0 < \rho < 1$.

Appendix B: The losing platform leaves the market

This appendix considers the case in which the losing platform in each period leaves the market and a new entrant is always born in the next. The main conclusion of this appendix is that the Stochastic Markov Perfect Equilibrium is identical to the static form, repeated infinitely. In particular, both profits and welfare are independent of δ. The intuition is that when platforms exit the market upon defeat, future profits of the losing platform (whether it is the focal or the non-focal) are zero. In turn, both platforms may apply all future profits they attain from gaining the focal position, into subsidizing consumers in the current period, and, because in expectancy, the profits from gaining the focal position are equal for both platforms, the threshold is identical to the static game: $\overline{Q}_A(N) = \beta \cdot N$ and $\overline{Q}_B(N) = -\beta \cdot N$, $(N = \{1, n\})$, and equilibrium profits are independent of δ.

Consider the solution to Section 6 of an unregulated market. Suppose that we now add the restriction that when a platform loses in a certain period, it leaves the market indefinitely. This assumption changes the derivations of $V_A^A(1)$ (as described before in equation (2)). Now, when a focal platform A loses the current period, future profits are zero. Hence:

$$V_A^A(1) = \int_{-\sigma}^{\overline{Q}_A(1)} \left[\beta \cdot 1 - q - \delta EV_B^B + \delta EV_A^A\right] \frac{1}{2\sigma} dq. \tag{33}$$

Because symmetry implies that $EV_A^A = EV_B^B$, this equation can be re-written as:

$$V_A^A(1) = \int_{-\sigma}^{\overline{Q}_A(1)} [\beta - q] \frac{1}{2\sigma} dq. \tag{34}$$

Notice that fixing $\overline{Q}_A(1)$, the value function $V_A^A(1)$ is independent of δ, and is equal to the expected profits of platform A in the static benchmark.

Next we move to $V_A^A(n)$ (as described before in equation (3)). Following a similar intuition, we have that when a focal platform A loses the current period, future profits are zero. We have:

$$V_A^A(n) = \int_{-\sigma}^{\overline{Q}_A(n)} \left[\left(n\beta - q - \frac{\delta}{n} EV_B^B\right) n + \delta EV_A^A\right] \frac{1}{2\sigma} dq \tag{35}$$

Because $EV_A^A = EV_B^B$, this equation can be re-written as:

$$V_A^A(n) = \int_{-\sigma}^{\overline{Q}_A(n)} \left[\left(n\beta - q\right) n\right] \frac{1}{2\sigma} dq.$$

As before, notice that fixing $\overline{Q}_A(n)$, $V_A^A(n)$ is independent of δ.

Next we move to $V_A^B(1)$ (as described before in equation (4)). If a non-focal platform A

does not win the market, it leaves forever and earns 0. Hence:

$$V_A^B(1) = \int_{-\sigma}^{\overline{Q}_B(1)} \left[-q - \beta - \delta EV_B^B + \delta EV_A^A\right] \frac{1}{2\sigma} dq. \tag{36}$$

Recalling again that $EV_A^A = EV_B^B$, this equation can be re-written as:

$$V_A^B(1) = \int_{-\sigma}^{\overline{Q}_B(1)} \left[-q - \beta\right] \frac{1}{2\sigma} dq, \tag{37}$$

which again is independent of δ.

Next we move to $V_A^B(n)$ (as described before in equation (5)). Recalling again that $EV_A^A = EV_B^B$ we have:

$$V_A^B(n) = \int_{-\sigma}^{\overline{Q}_B(n)} \left[(-q - \beta n)\, n\right] \frac{1}{2\sigma} dq. \tag{38}$$

Recalling that the two platforms are ex-ante identical in terms of their expected quality, we apply symmetry and obtain: $V_B^B(N) = V_A^A(N)$ and $V_B^A(N) = V_A^B(N)$, for $N \in \{1, n\}$.

Next, we move to the derivation of $\overline{Q}_A(N)$ and $\overline{Q}_B(N)$ (as described before in equations (6-9)) under the restriction. We start with $\overline{Q}_A(1)$ and $\overline{Q}_A(n)$. The quality gap that makes a focal platform A indifferent between winning and losing the market solves:

$$\left(-\overline{Q}_A(1) + \beta \cdot 1 - \delta EV_B^B\right) \cdot 1 + \delta EV_A^A = 0,$$

Because $EV_A^A = EV_B^B$, this equation can be re-written as

$$-\overline{Q}_A(1) + \beta \cdot 1 = 0. \tag{39}$$

Which is identical to the threshold value in the static game: $\overline{Q}_A(1) = \beta$. Likewise, $\overline{Q}_A(n)$ is the solution to:

$$\left(-\overline{Q}_A(n) + n\beta - \frac{\delta}{n} EV_B^B\right) n + \delta EV_A^A = 0.$$

Because $EV_A^A = EV_B^B$, this equation can be re-written as

$$\left(-\overline{Q}_A(n) + n\beta\right) n = 0, \tag{40}$$

Hence, $\overline{Q}_A(n) = n\beta$, as in the static game. Turning to the threshold values of $\overline{Q}_B(1)$ and $\overline{Q}_B(n)$ when the number of consumers is $N = 1$ and $N = n$, respectively:

$$\overline{Q}_B(1) + \beta = 0, \tag{41}$$

$$\left(\overline{Q}_B(n) + \beta n\right) n = 0. \tag{42}$$

We summarize the results above in the following corollary.

Corollary 4. ***(Losing platform leaves the market)*** *Consider an unregulated market and, in each period, the losing platform leaves the market indefinitely. Then, the Stochastic Markov Perfect Equilibrium is identical to the static equilibrium.*

Appendix C: The focal platform is prohibited from charging a negative price in an equilibrium in which the focal platform loses the market

This appendix considers asymmetric regulation: competition authorities prohibit the incumbent from charging a negative price when the incumbent either wins or loses the market. We can provide an analytical solution given that $n \to 1$, and we find that when δ is close to 0, there is no pure strategy Stochastic Markov Perfect Equilibrium. The intuition for this result is that when the incumbent cannot compete with the entrant with a negative price and the incumbent loses the market, a platform cannot profitably maintain a focal position in the market. More precisely, we find that when δ, and n are sufficiently small, the solution involves $V_i^{i**}(N) < 0$, which cannot occur in equilibrium.

To this end, consider the solution to Section 7 on asymmetric regulation. Suppose that we add the restriction that in an equilibrium in which a focal platform loses the market, it cannot charge a negative price. This assumption changes the derivations of $V_A^B(1)$ (as described before in equation (4)), because when a focal platform B loses the market, it charges $p_B^B(1) = 0$ instead of $p_B^B(1) = -\delta(EV_B^B - EV_B^A)$. Substituting $p_B^B(1) = 0$ instead of

$p_B^B(1) = -\delta(EV_B^B - EV_B^A)$ in equation (4) we have:

$$V_A^B(1) = \int_{-\sigma}^{\overline{Q}_B(1)} \left[(-q - \beta \cdot 1 - 0) \cdot 1 + \delta EV_A^A\right] \frac{1}{2\sigma} dq \tag{43}$$

$$+ \int_{\overline{Q}_B(1)}^{\sigma} \left[\delta EV_A^B\right] \frac{1}{2\sigma} dq.$$

Next we move to $V_A^B(n)$ (as described before in equation (5)). Following a similar intuition, we have that when a focal platform B loses the market, it charges $p_B^B(n) = 0$ instead of $p_B^B(1) = -\frac{\delta}{n}(EV_B^B - EV_B^A)$. Substituting $p_B^B(n) = 0$ instead of $p_B^B(1) = -\frac{\delta}{n}(EV_B^B - EV_B^A)$ in equation (5) we have:

$$V_A^B(n) = \int_{-\sigma}^{\overline{Q}_B(n)} \left[(-q - \beta n - 0)\, n + \delta EV_A^A\right] \frac{1}{2\sigma} dq \tag{44}$$

$$+ \int_{\overline{Q}_B(n)}^{\sigma} \left[\delta EV_A^B\right] \frac{1}{2\sigma} dq.$$

Solving for conditions (2), (3), (43) and (44), the constraints on a winning focal platform, (11) - (14), and letting $n \to 1$, we have:

$$V_A^A(1) = V_A^A(n) = \frac{1}{8(1-\delta)^2\delta^2\sigma^2}\Bigg[-(1-\delta)\sigma(-\beta^2\delta^2 + 2\beta(-2+\delta)\delta\sigma + (4 - \delta(4+5\delta))\sigma^2) \tag{45}$$

$$-\sqrt{(1-\delta)^2\sigma^2((\beta\delta + (-2+3\delta)\sigma)^2(\beta^2\delta^2 + 2\beta\delta(-2+3\delta)\sigma + (2+\delta)^2\sigma^2))}\Bigg].$$

Evaluating (3) at $\delta \to 0$, we have that the term in the squared brackets is $-4(\sigma^3 + \sigma^6) < 0$ while the term in the denominator approaches 0. We therefore have that $V_A^A(1) \to V_A^A(n) \to -\infty$ as $n \to 1$ and $\delta \to 0$ implying that there is no pure-strategy Stochastic Markov Equilibrium. Because $V_A^A(1)$ and $V_A^A(n)$ are continuous in n and δ, the same argument holds at least when n and δ are not too high.

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