Simulating Collusion: Challenging Conventional Estimation Methods*

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Abstract

The literature on estimating dark rates of collusion relies on hazard rate and capture-recapture models to estimate and explain the duration of collusive populations from detected offenses (sample). The detected cartels are non random samples of their populations. Hence, this study addresses the question of whether conventional methods derive consistent unbiased estimators for dark rates and variables explaining duration of collusion. We simulate collusive behavior of industries with different number of firms based on three classical models of collusion (Stigler [1964], Harrington and Chang [2009] and Bos et al. [2018]), additionally varying five variables of antitrust enforcement. The simulation provides a ground-truth data set of undetected and detected cartels; a population and its sample. Applying hazard rate and capture recapture estimation on the sample leads to a good prediction of the true probability of getting caught for the sample. However, it fails to predict the probability of being caught for the population; the estimates do not come close to the true probability of being caught.

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1 Introduction

Cartels are agreements between competitors operating on the same market to fix prices, quantities, and reduce competition to raise profits ultimately. The harm caused by illegal cartels is hard to estimate. An OECD survey conducted between 1996 and 2000 (OECD [2002]) shows "the amount of commerce affected by just 16 large cartel cases [...] exceeding USD 55 billion worldwide." Numerous studies identify detected cartels to be profitable, increasing allocative inefficiencies of markets (Connor [2003], Connor and Lande [2005], Connor and Bolotova [2006], Bolotova et al. [2008], Bolotova [2009], Connor [2013], Connor [2017] and Connor and Werner [2018]). Günster et al. [2017] extend the welfare analysis by showing that collusive industries also reduce innovation while strategically increasing market concentration - thus, ultimately reducing competition. De Loecker and Eeckhout [2018] show that mark-ups are rising attributable to increasing market concentration. Buccirossi et al. [2013] identify factor productivity being significantly higher in economies having solid antitrust enforcement. These findings hint at significant dark rates of anti-competitive activity. Current profit estimates based on detected cartels might just represent the tip of the iceberg of collusive activity (Bryant and Eckard [1991], Levenstein and Suslow [2006], Combe et al. [2008], Levenstein and Suslow [2011], and Ormosi [2014]).

The literature on estimating dark rates of collusion starts with hazard rate models to estimate population size and explain duration of cartel activity from detected offenses (sample). Hazard rates build on strong assumptions, e.g. no path dependency, random sampling, constant detection probability (Davies and Ormosi [2012]). Empirical research agrees that samples of collusive industries are non-random, showing two main selection biases. First, certain firms and industries are by structure prone to collude (Martin [1988], Motta [2004], Carree et al. [2010] and Russo et al. [2010]). Second, competition authorities (CA) selectively search markets either structurally prone to collude or having a detection track record (Carree et al. [2010], Connor [2010] and Veljanovski [2011]). Hence, the questions arise, *does the sample of detected cases represent its population and how do conventional methods perform in terms of prediction if not?*

We address this question by simulating collusive behavior in industries with different number of firms based on three classical models of collusion varying five variables characterizing the industry, enforcement and detection (Stigler [1964], Harrington and Chang [2009] and Bos et al. [2018]). The simulation provides a ground-truth data set of undetected and CA detected cartels; a population and its sample in various market and enforcement regimes. Applying harzard rate and capture recapture estimation on the samples leads to a good prediction of the true probability of getting caught for the sample. However, it fails to predict the probability of being caught for all cartels; the estimate does not come close to the true probability.

Our study starts with an overview of the literature empirically and theoretically analyzing collusion. In Section 3, we simulate several collusion models (Stigler [1964], Harrington and Chang [2009] and Bos et al. [2018]) and vary industry and enforcement characteristics to derive industries where firms form and dissolve cartels, being occasionally detected. Section 4 shows the summary statistics of the simulated populations and samples. Thereafter, we use the realizations to show that standard hazard rate models and capture recapture do not derive consistent and unbiased estimates of the population characteristics and explanation for duration (Section 5). Finally, we conclude.

2 Literature Review

As the main characteristic determining collusion is the number of firms in an industry, we allow for incomplete cartels and leniency programs, the theoretical literature review focuses on these areas. Martin [1988] and Motta [2004] provide a complete overview of all relevant firm and industry characteristics leading to collusion.[1] Early research on theoretical collusion starts with the seminal works of Chamberlin [1942] and Stigler [1964] on a firm's incentive to either enter or leave an already existing cartel. Friedman [1971] describes collusion as a (super)game, where firms collude non-cooperatively over an infinite sequence if their discount factor for future profits (patience) is sufficiently high. Selten [1973] also takes a game-theoretic approach by endogenizing cartel size as the number of firms. Salant et al. [1983] examine the effect of exogenous market structure on firm profits in a Cournot setting, showing that incomplete cartels may exist if they reach a joint market share of 80%. D'Aspremont et al. [1983] introduce stability conditions for incomplete cartels facing a competitive fringe. Donsimoni et al. [1986] show the possible existence of a stable (in)complete cartel to be dependent on the number of firms. A method for finding the equilibrium number of firms forming a cartel over time is proposed by Bloch [1996]. Diamantoudi [2005] extends this concept, allowing for a firm's foresight about the consequences of deviation.

The positive influence of cartel size on cartel stability is analyzed by Escrihuela-Villar [2008], while Escrihuela-Villar and Guillén [2014] demonstrate the advantage of price (Bertrand) versus quantity (Cournot) competition under the assumption of product differentiation. Assuming heterogeneous products, Deneckere and Davidson [1985] show incomplete cartels to be profitable. Donsimoni [1985] and Cramton and Palfrey [1990] analyze incomplete cartels with heterogeneous production costs. Shaffer [1995] and Martin [2002] show profits for incomplete cartels in reducing quantity, with the non-participating firms acting according to their best interests. Bos and Harrington [2010] model incomplete cartels with firms having heterogeneous capacity constraints. Incentive compatibility constraints (ICCs) dependent on expected fines for convicted collusion are modeled by Bos et al. [2018]. Stigler [1964], Harrington and Chang [2009] and Bos et al. [2018] are the base for our three models, where we additionally vary industry and enforcement variables.

Motta and Polo [2003], Chen and Harrington [2007] and Harrington [2008] model the effect of leniency programs on collusion and offer advice for an optimized implementation. While former models imply that either all firms or no firms apply for leniency, Harrington [2013] posits one single firm preemptively applying for leniency. Harrington and Chang [2015] extend this model by introducing leniency together with capacity constraints for the CA and endogenize intensity of non-leniency enforcement (Motta and Polo [2003] and Spagnolo [2006]). Empirical studies include Duso et al. [2014], Sovinsky and Helland [2019] and Heim et al. [2020]. These studies use the introduction of a leniency program as an exogenous shock to test whether horizontal cooperation (joint ventures and controlling shareholders) declines. In contrast, Miller [2009] shows increased cartel detection after the introduction of the leniency program in 1993, but no sign of an increase in deterrence in a sample of convicted cartels. Hellwig and Hüschelrath [2018] use the EU leniency program to estimate the characteristics of leniency applicants for a sample of detected cases. The observed anomaly of firms applying for leniency long after a cartel has collapsed is investigated by Zhou and Gärtner [2012], using hazard and probit models. While

[1] Based on Martin [1988] and Motta [2004], the main characteristics relevant for collusion are: market concentration, industry size, entry barriers, controlling ownership, regularity and frequency of orders, buyer power, demand elasticity, evolution of demand: demand shocks i.i.d. or correlated over time, product homogeneity, symmetry among firms, multi-market contacts, inventories and excess capacities, price transparency and exchange of information, pricing rules and contracts, product differentiation, cost differences, signaling, pricing rules.

the Diff-in-Diff studies do not quantify a reduction in collusive activity, the studies relying solely on samples of detected cartels base the analysis on information from convictions, ignoring selection biases.

The empirical literature on estimating the number of unreported cartel cases starts with surveys on cartel activity. Beckstein and Gabel [1982] find a detection rate estimate of less than 50% based on questionnaires among American firms. Bryant and Eckard [1991] are the first to propose an econometric approach for estimating the annual probability of getting discovered for a sample of caught cartels. They use a statistical birth and death process with constant hazard rate to model illegal cartel formation and breakdown in the United States between 1961 and 1988 from a sample of successful Department of Justice (DoJ) investigations. They establish the rate of caught cartels to be around 13-17% for this sample of detected cartels. Also using hazard rates, Combe et al. [2008] derive a cartel detection rate of 12.9-13.3% for a sample of detected cartels in the European Union (EU) from 1969 to 2007. Levenstein and Suslow [2006] provide an overview of the entire literature of duration models of collusion. Levenstein and Suslow [2011] show determinants of cartel duration for detected cartels, estimating the hazard ratio depending on various firm/market characteristics for a sample of detected US and EU cartels. Ormosi [2014] proposes capture-recapture (CR) models to estimate the probability of getting caught. CR originates as a field method in life sciences, such as ecology, and is used to estimate population size, capture rates, and survival rates (Amstrup et al. [2010]). Ormosi [2014] finds 10-20% to be the probability of being caught for a sub-sample of convicted cartels in the EU over the period 1985-2009.

Harrington and Chang [2009] are the first to establish theoretically the conditions under which observed cartel duration equals actual duration. Taking observed duration to be representative of unobserved duration leads to significant over- or under-valuation of existing collusion. Detection might correlate with a variety of other factors, such as enforcement changes, the height of the overcharge, the duration itself, and strategic firm and agency behavior. We follow their reasoning to identify these characteristics for our population (sample) of (detected) cartels. Hyytinen et al. [2018] apply Harrington and Chang [2009] on a sample of legal cartels between 1951 and 1990 in Norway. In contrast to previous studies, one might perceive their sample as a good proxy for collusive activity in Norway. However, Hyytinen et al. [2018] cannot ensure all firms registering their cooperation, nor do they perfectly observe the state of registered cartels (alive or dead).[2]

All of the existing empirical studies rely on samples of caught (or registered) cases.[3] The detected cartels may differ significantly from the collusive population. It is unclear whether those estimated probabilities may be extrapolated to an unknown population (Bryant and Eckard [1991], Harrington and Chang [2009], Davies and Ormosi [2012] and Ormosi [2014]). For example, Asch and Seneca [1976] show that cartelized industries do not operate more profitably than non-cartelized industries, based on the idea that unsuccessful cartels are eventually caught. Another critical feature of the models used so far is that they rely on homogeneous Markov chains, i.e., formation, death, detection and deterrence rates are assumed to be constant and independent. However, those rates vary over time with the introduction of new laws and detection tools (Harrington and Chang [2009]). Further, the unit of analysis is the cartelized industry, not the firm. This excludes the possibility of

[2]The following studies analyze anti-trust offenses using simulations with a very different objective compared to our study. Paha [2011] simulates a model of collusive industries with endogenous cartel formation. Becker et al. [2018] combine a game-theoretic model with Monte Carlo (MC) simulations to measure the deterrent effect of cartel law enforcement. Davies and Ormosi [2013] and Davies et al. [2018] use simulations to estimate a quantification of harm deterred by antitrust policies, taking into account potential sample selection bias. Based on Harrington and Chang [2009], Katsoulacos et al. [2016] develop a framework for evaluating the effects of policy instruments on welfare, simulating different intervention parameters and allowing for repeat offenders. Gärtner [2014] uses MC simulations to show the existence of a preemptive push into leniency application.

[3]For an overview of all types of detected antitrust offenses in the US see Edwards [1950], Posner [1970], Gallo et al. [1985], Salop and White [1986], Gallo et al. [1986, 1994, 2000], Lin et al. [2000], Kovacic and Shapiro [2000], Ghosal and Gallo [2001], Posner [2001], Baker [2003] and Ghosal and Stennek [2007]. In analogy, see Carree et al. [2010] and Russo et al. [2010] for the EU.

incomplete cartels (Bos [2009]). Third, these methods do not account for firm- or market-based heterogeneity or, most importantly, time (Harrington and Chang [2009]). Moreover, individual observations are assumed to be independent (i.e., no repeat offenders), with the exception of Davies and Ormosi [2012] and Ormosi [2014]. Consequently, Bos and Harrington [2010] note that the existing studies only state comparative statics.

3 Simulating Collusion

Theoretical contributions focus usually on one or two factors when deriving the incentive to collude (i.e., incentive compatibility constraint). In our simulation, we relax several assumptions and vary model characteristics simultaneously. We derive collusive populations dependent on industry characteristics and CA detected samples. We simulate three classical models of collusion (Stigler [1964], Harrington and Chang [2009] and Bos et al. [2018]) varying industry size (number of firms (n_{firms})). Each firm's market share is $1/n_{firms}$.[4] There is no entry or exit in the industries. Additionally, we vary several variables of enforcement. From zero enforcement to varying probability of detection (ρ), fines (γ), leniency and its height (θ), and constant versus increasing detection probability for recidivism (*structured*). Table 1 shows all factors and models we vary.

Table 1: Simulation Parameters and Models

Parameter		Values	Model
Number of Firms	n_{firms}	$\{2, 3, .., 10\}$	Stigler [1964]
Detection Probability	ρ	{0.1,0.15,..,0.35}	Bos et al. [2018]
Fine (% of Profit)	γ	{0.7, 0.8, 0.9}	Bos and Schinkel [2006]
Leniency (% of Fine to Pay)	θ	{0,0.5,1}	Bos et al. [2018]
Constant (0) vs. Increasing (1) ρ	*structured*	{0,1}	Harrington and Chang [2009]

For each model, we calculate existing ICCs of collusion. We only model and simulate equilibria; firms collude or do not collude. The simulated data allow for a comparison to real data on firms and industries. The data generating process delivers detected as well as undetected cartels and their characteristics (i.e., start, end, duration, number of firms involved), and serves to evaluate estimation methods in Section 5. The data consists of number of cartels alive/dead, cartel/firm/industry characteristics, and duration of cartels like the empirical literature.

3.1 The Benchmark Model (Model I)

Consider an economy with k different industries, each with a certain number of firms ($n_{firms}(k)$). The firms in each industry compete in a repeated game, forming and leaving cartels depending on the ICCs. A firm colludes if its discount factor exceeds its ICCs and *vice versa*. A cartel forms and remains alive, if at least 80% of all firms collude (Salant et al. [1983]). If firms set the same price p, they face the same demand function $D_i = D(p)/n_{firms}$. Firms face no capacity constraints and produce homogeneous goods with constant marginal costs c. Demand is given by $D(p) = a - bp$ (with $a, b > 0$). It is a twice-differentiable, continuous, and strictly decreasing function. In each period t, firms set prices (p) simultaneously and non-cooperatively (Stigler [1964] and Friedman [1971]). Let $\pi_i^c = \pi(p)/n_{firms}$ be the current collusive profit of firm i and V_i^c the present discounted collusive value of profits for firm i given other firms colluding. If all firms collude and set price

[4] This translates into Hirschmann-Herfindahl Index (HHI) of $HHI = (1/n_{firms})^2$ as a proxy for market concentration. Note that HHI drops rapidly as firms have equal size.

$p^c > c$, then collusive profit is $\pi_i^c = \pi(p^c)/n_{firms}$ for each firm i. A firm can deviate, slightly undercutting the price and so supply the whole market. Then $\pi_i^d = \pi(p^c - \epsilon)$ is the current profit of the deviating firm with all other firms colluding. For small price deviations of ϵ, π_i^d gets very close to $\pi(p^c)$. Let V_i^d be the present discounted value of profits for firm i after deviation. Then $V_i^d = 0$ is the classic trigger-grim strategy. Stigler [1964] explains that competitive behavior is the strongest form of punishment.

Firms are more prone to collude if their patience is high. Patience is modeled as a discount factor (δ) of future profits. The discount factor is identical for each firm in the industry, being $\delta = 1/(1+r)$ with r being the real interest rate. Each firm's discount factor changes over time following a random walk of r. Harrington [1989] explains how discount factors may vary between firms due to imperfect capital market information or principal-agent problems.[5,6] Based on three different economic models (Stigler [1964], Harrington and Chang [2009] and Bos et al. [2018]), profits vary with relaxing assumptions and variables, affecting the industry-wide ICCs (see Table 1). We solve for the correspondent ICC (δ), thereby defining willingness of a firm to join a cartel given its discount factor. The incentive to join or leave a cartel is summarized by the following incentive compatibility constraint:

$$\pi_i^c + \delta V_i^c \geq \pi_i^d + \delta V_i^d, (i = 1, ..., n_{firms}) \tag{1}$$

Model I is solely based on Stigler [1964], who models the ICCs of firms endogenizing the numbers of firms in the industry. The recursively defined present discounted value of collusive profits by firm i is $V_i^c = \pi_i^c + \delta V_i^c$. Substituting profits in (1) leads to $(1 + \delta + \delta^2 + ...)\pi(p^c)/n_{firms} \geq \pi(p^c)$. As $\sum_{t=0}^{\infty} \delta^t = 1/(1-\delta)$, when solving for the industry discount factor, the ICC based on the number of firms in a market is

$$\delta \geq 1 - 1/n_{firms} \tag{2}$$

3.2 Allowing for Enforcement and Leniency (Model IIa and IIb)

Bos et al. [2018] model the effect of competition policy on formation and dissolution of illegal cartels. In essence, this simulation expands on the previous by allowing for CA detection and leniency notification by firms involved in a cartel. With a probability $\rho \in [0, 1]$, the cartel is discovered, penalized and dissolves.[7] When discovered, the firms may pay a fine (F). The fine is modeled as a linear function of realized collusive profit: $F = \gamma\pi(p)/n_{firms}$ (Bos and Schinkel [2006]).[8] Model IIa and IIb allow for detection and fines.

In Model IIb, we additionally introduce leniency in the middle of the time horizon.[9] Let $\theta \in [0, 1]$ be the

[5] The real interest varies stochastically over time, depending on such things as fiscal and monetary policy, access to capital markets, debt structure (Fisher [1930]). Modeling other aspects stochastically might be of interest, too. For example one might vary δ itself, demand (Rotemberg and Saloner [1986]) and cost shocks, product differentiation or any other variable mentioned by Martin [1988] and Motta [2004].

[6] For the random walk let $X_1 = 0$ with $p(X_{t+1} = X_t + 1) = 0.5$ and $p(X_{t+1} = X_t - 1) = 0.5$. Then $r_{t+1} = f(X_t) = (2 * X - 1)/100$ and $\delta_t = g(r_t) = atan(X * 2)/\pi + 0.5$. Note that any function based on a cumulative distribution function is equally valid for this exercise.

[7] In one single time step, the simulated probability of detection is $\rho * 20/1000$.

[8] See Bos and Schinkel [2006] and Allain et al. [2011] for an overview of fine calculation.

[9] Leniency programs for cartels were introduced for the first time in 1978 and 1996 in the US and EU, respectively. They provide immunity from sanctions to firms who notify the CA of the cartel before investigations start. See for the US: DoJ Leniency Program of 4 October 1978; DoJ Corporate Leniency Policy of 10 August 1993; DoJ Leniency Policy for Individuals of 10 August 1994. See for the EU: Notice on immunity from fines and reduction of fines in cartel cases (Leniency Notice) of 18 July 1996; Commission Notice on immunity from fines and reduction of fines in cartel cases of 19 February 2002; Commission Notice on Immunity from fines and reduction of fines in cartel cases (New Leniency Notice) of 8 December 2006.

fraction of fine for the first firm applying for leniency, then 1 indicates that there is no option to apply for leniency while 0 corresponds to full immunity from fines for first applicant. Let $V^n = 0$ and $V^c = \pi(p) + \delta(1-\rho)V^c + \delta\rho V^n - \rho F(p)$ be competitive and collusive profit, respectively, then present discounted value of profits, if firms collude, depends on detection, fines, and leniency. After detection, the cartel dissolves and prices decrease to the competitive level. Profit from deviation and present discounted value of profits after deviation are $\pi^d(p) + \delta V^n$ (grim punishment).[10]

Introducing enforcement changes firms' ICC for entering and leaving a cartel. Intuitively, collusive profit including the probability of detection and being fined must exceed deviation profit without the risk of fine, else firms will never enter a cartel. Hence, ICCs for entry become

$$\pi_i(p) + \delta(1-\rho)V_i^c + \delta\rho V_i^n - \rho F(p) \geq \pi_i^d(p) + \delta V_i^n, \tag{3}$$

Substituting profits and fine in (3) leads to $\pi(p)/n_{firms} + (1-\rho)\sum_{t=1}^{\infty}\delta^t\pi(p)/n_{firms} - \rho\gamma\pi(p)/n_{firms} \geq \pi(p)$. Fine reduction by leniency (θ) is not relevant for the ICC to enter, as leniency application is only possible for firms participating in a cartel. Note that $\theta = 1$ corresponds to the case when there is no leniency program (Model IIa). Simplifying and solving for the discount factor (δ) leads to the following ICC thresholds to enter a cartel, allowing for detection, fines, and leniency:

$$\delta \geq (n_{firms} + \rho\gamma - 1)/(n_{firms} + \rho\gamma - \rho) \tag{4}$$

Next to the incentive to enter a cartel, we derive the incentive to leave a cartel. Profit from collusion must be smaller than profit from deviation. Both sides include the probability of being detected and fined, as any firm participating in a cartel faces the risk of detection, sanctioning, and leniency applications even when exiting. Incorporating enforcement, the ICCs for exit become

$$\pi_i(p) + \delta(1-\rho)V_i^c + \delta\rho V_i^n - \rho F(p) \geq \pi_i^d(p) + \delta V_i^n - \theta\rho F(p). \tag{5}$$

Substituting profits and fine in (5) gives $\pi(p)/n_{firms} + (1-\rho)\sum_{t=1}^{\infty}\delta^t\pi(p)/n_{firms} - \rho\gamma\pi(p)/n_{firms} \geq \pi(p) - \theta\rho\gamma\pi(p)/n_{firms}$. Solving for δ leads to the following ICC to leaving a cartel, allowing for detection probability, fines, and leniency:

$$\delta \geq (n_{firms} + \rho\gamma - 1 - \theta\rho\gamma)/(n_{firms} + \rho\gamma - \rho - \theta\rho\gamma) \tag{6}$$

3.3 Allowing for Strategic Enforcement and Leniency (Model IIIa and IIIb)

In Model II, the probability of detection stays constant over time. There is no path dependency in the sample. This aims at producing a random sample. At least, the chance of being detected is not *structured*. In Models IIIa and IIIb, the probability of detection changes over time. The empirical literature hints at recidivism being a structural phenomenon in samples of collusion. Harrington and Chang [2009] endogenize formation and dissolution of cartels under changing (increasing) detection probabilities. Hence, we address selective behavior by a CA, leading to a *selective sample*. We let the probability of getting caught increase with every time a

[10] Note that this simulation does not allow for Type II errors. A firm will not be caught when not being active in a cartel. By analogy, a firm may not self-report under a leniency program if it is not currently active in a cartel.

cartel is caught. The detection probability becomes $\rho_{new} = \rho + \rho/2^{nTC}$, with nTC being the number of times getting caught. From a strategic perspective, a firm's sales generated by the cartel do not change, however the cost of collusion increases in getting caught. By construction, the *selective* population and sample of cartels decrease over time with the total number of firms being constant in our industries. Harrington [2018] suggests divestiture on colluding firms as a remedy, especially for repeat offenders. As with Model IIb, Model IIIb allows for leniency after 500 time periods. In Model IIIa, $\theta = 1$ and firms do not receive any immunity from fines.

4 Descriptive Statistics - Realizations of Simulations

Table 1 shows the variation of the variables we alter. For each of 972 possible different combinations of parameters, we run a simulation over a time horizon of $t = 1'000$. Each simulation is repeated 300 times with individual seed. This gives us a total of 291'600 simulated industries, each containing two to nine firms. Model IIa and IIIa contain 48'600 industries each and there is no possibility to apply for leniency over the whole time. Model IIb and IIIb incorporate 97'200 each. Here, we introduce the possibility of leniency application in the middle of the time horizon. Leniency applicants pay only a share of $\theta \in \{0, 0.5\}$ of a fine when detected. Finally, detection probability stays constant over time ($structured = 0$) in Model IIa and IIb, while it increases for repeat offender ($structured = 1$) in Model IIIa and IIIb. Fines (γ) start at 70% of cartel profits and rise up to 90%.

Table 2: Model I: Cartels and Market Size (n_{firms})

Market Size (n_{firms})	ICC Threshold	Avg. Number of Cartels	Avg. Duration
2	0.50	299.83	999
3	0.67	299.81	999
4	0.75	299.71	967
5	0.80	295.48	534
6	0.83	158.58	31
7	0.86	4.96	9
8	0.88	0.21	9

By construction, the number of firms drives the simulated data (Stigler [1964] and Selten [1973]). The fewer the number of firms in an industry, the higher the likelihood of collusion. If the market size is very small with three firms ($n_{firms} = 3$), the discount factor, starting at 0.85, will hardly ever drop below the critical ICC level of 0.67. Consequently, firms enter the cartel and do not leave, resulting in industry-wide stable cartels with a long duration. The theoretical and empirical literature supports this finding. Industries with few players show significantly longer cartel duration (Bryant and Eckard [1991], Levenstein and Suslow [2006], Combe et al. [2008]), Carree et al. [2010], Russo et al. [2010], Levenstein and Suslow [2011] and Carree et al. [2012]). For simulations based on Model I (Stigler [1964]), we do not see any cartels appearing in industries with more than eight firms (Table 2, see also Selten [1973]). For the remaining analysis, we focus on the data provided by Models II and III. As our analysis requires detected cartels (sample) to estimate duration models with the aim of estimating dark rates and explaining duration.

Figure 1 and Figure 2 show one example of the same industry consisting of five firms. Each firm's discount factor follows the same random walk in all panels for ease of comparison. The lower dark blue line in all panels indicates no enforcement (Model I), with identical thresholds for entry and exit at a discount factor of 0.8.

Colored realization of discount factors shows the five firms' patience (δ). Whenever their discount factor lies above the critical ICC threshold, they enter the collusive phase and *vice versa*. Intuitively, all ICCs incorporating enforcement are substantially larger than the baseline model. Enforcement increases the costs of collusion while profits are identical in all regimes.

Figure 1: Model II ($n_{firms} = 5$) with Constant Detection ($\rho = 15\%$) and Leniency (b)

In Figure 1, the left and right panels indicate enforcement with constant detection probabilities (Bos et al. [2018]) without (Model IIa) and with leniency (Model IIb), respectively. In Panel a) of Figure 1, the straight lines above the benchmark line (dark blue) indicate entry (blue) and exit (red) Iinfor a detection probability of 15% ($\rho = 0.15$). Bryant and Eckard [1991], Combe et al. [2008] and Ormosi [2014] find similar detection probabilities in their estimations. Fines are a percentage of profit ($\gamma = 0.9$). In panel a), there is no fine reduction from leniency ($\theta = 1$). We amend these variables to different values to generate a rich population and sample for our estimations. The ICC thresholds for entry and exit differ. Entry happens at a higher discount factor than exit. For entering a cartel, a firm's patience has to exceed the entry threshold. Once the firm is part of the cartel, it faces the risk of a fine even if it decides to leave. In Panel a), the industry is most of the time in an incomplete cartel with 80% market share as one firm (dark green) is not sufficiently patient to participate. In Model IIa and b, firms are more prone to collude, if industries have only few firms, detection probability is low and fines are small. Increasing the number of firms, detection probability, and fines rises the ICC thresholds; firms collude less frequently and drop out of collusion more often.

With the introduction of leniency after 500 time periods (Panel b)), firms who deviate from the cartel and apply for leniency can significantly reduce sanctions by 50% ($\theta = 0.5$), increasing deviation profits and making exit more attractive (i.e., higher ICC). Panel b) shows that the ICC for exit increases in time $t = 500$, making deviation an interesting option. Around time $t = 550$, a second firm becomes too impatient (dark green line drops below the orange ICC exit). The cartel dissolves as at least 80% of the firms in an industry must remain for cartels to last (Salant et al. [1983]). Due to the increased ICC for exit, cartels last longer if there is no possibility for firms to apply for leniency. Consequently, the introduction of leniency leads to shorter duration in Model IIb. Note that for full leniency reduction, ICC for exit equals ICC for entry (blue line).

Figure 2: Model III ($n_{firms} = 5$) with Increasing Detection ($\rho = 15\%$) and Leniency (b)

In Figure 2, we show the impact of strategic or selective enforcement. Whenever the CA identifies a cartel in an industry, the probability of being caught does not remain at 15% but increases by $\rho/2^{nTC}$, with nTC being the number of times getting caught. All other variables are identical to Figure 1 ($\gamma = 0.9$, $n_{firms} = 5$, $\theta = 1$ in a) and $\theta = 0.5$ in b)) As in Figure 1, all ICCs lie above the benchmark (Model I). Note that the ICC starting point is identical as $\rho = 0.15$ and all other variables are the same. Contrasting Figure 1, this probability increases with every detection, happening twice within the time horizon of 1'000. Therefore, Model III leads to shorter and fewer cartels as detection happens more frequently and every detection increases the cost to collude. Introducing leniency (Panel b) has the same effect as before; it increases the ICC to exit after half of the time horizon, making deviation from the cartel more lucrative.

Figure 3 shows the sum of simulated cartels for all three models on the population and sample level. The blue line represents the sum of all active cartels for every point in time. The red line represents the number of detected cartels out of all operating cartels. The difference between the blue and red line indicates *undetected* cartels. In all three models, the variables are identical whenever possible ($\rho = 0.15, \gamma = 0.9$) while we have 300 industries with varying number of firms (n_{firms}). In Model I, ICCs depend only on the number of firms in the industry (Panel a)). Hence, ICCs for entry and exit are lower (Figure 1 and Figure 2). Therefore, the number of cartels is higher around 450 cartels in Model I compared to Models II and III with around 350 and 250 cartels on average, respectively. However, we do see a slight decrease of cartels over time. As there is no detection, the sample is zero indicated with a red line at the bottom.

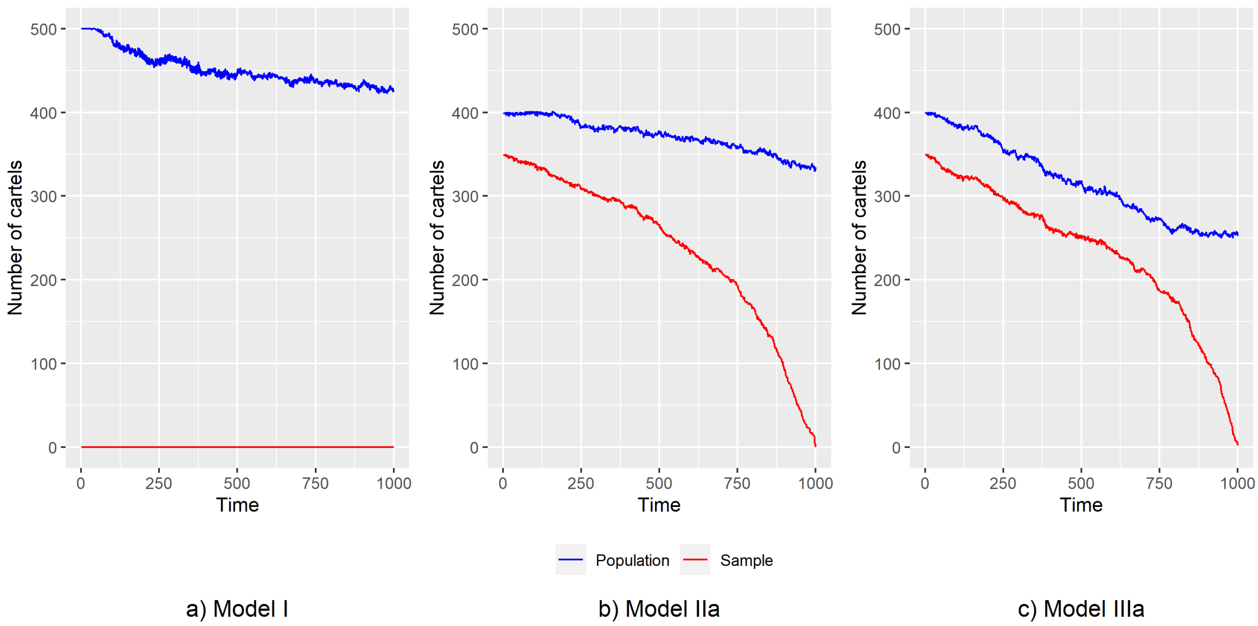

Figure 3: Number of Simulated Cartels over Time

In Model II and III, ICCs additionally depend on leniency (θ being 0, 0.5, and 1). Panel b) shows population and sample for a constant detection probability ($\rho = 0.15$) in Model II. The sample in Panel b) incorporates collusion being CA detected and leniency application based. With a constant number of firms in the industry, the convicted sample declines. Also, we add all cartels active at a point in time. Towards the end of the sampling horizon, some of the active cartels have simply not be found. Consequently, the number of active cartels in the population remains fairly constant why the sample of detected cartels declines significantly in Panels b) and c) due to the selection process. Empirical work in the area shows the same decline at the onset and end of the sampling horizon (Bryant and Eckard [1991], Levenstein and Suslow [2006] and Combe et al. [2008]). Panel c) shows population and sample for increasing detection probabilities (starting with $\rho = 0.15$), increasing ICC for entry and exit for each individual after every capture. Consequently, the number of cartels is lower than in Models I and II. As a higher percentage of cartels gets detected and re-detected the sample gets closer to its population compared to Model II. For the Hazard rate estimation, we expect a larger discrepancy in the estimated parameters whenever the difference between population and sample is large.

To identify discrepancies between population and sample, Table 3 and Table 4 show descriptive statistics summarizing the population and its sample summarizing models (Model IIa, IIb, IIIa and IIIb). We neglect Model I as it does not incorporate detection (no sample). We only describe the population and sample of cartels as depicted in the previous Figure 3. We do not report information on phases when industries do not show collusion as this information is also not available in reality and does not enter the econometric specifications. The percentage per model constitutes to population or sample may vary as some models lead to more or less (detected) industries being in collusion. Even in Model II, where we have a constant and equal probability of being caught, we see that selection takes place.

Table 3 describes the population of all industries being in collusion. The simulation is based on a total of 300 industries for every possible combination of parameters in Table 1. Each run over 1'000 periods. On average, there are about three to four firms in a cartel (Stigler [1964]). Most often collusion takes place in industries with three firms. Note that the minimum and maximum number of companies in a collusive industry is two and seven, respectively (Selten [1973]). The imposed fine being a percentage of profit (γ) ranges from 0.7-0.9, translating to 70 to 90%. The medium fine imposed is 80% of generated profits by construction. Next to detection involving sanctions, we allow for regimes with, with some, and without leniency (θ). Hence, the minimum of $\theta = 0$ (no immunity from fines) while its maximum is $\theta = 1$ (full immunity from fines). There is also the possibility to get some immunity from fines ($\theta = 0.5$). If the model allows for leniency, θ changes after 500 time periods from 1 to either 0.5 or 0. Leniency only affects ICC for exit, so we include summary statistics about cartels ending after time period 500, which applies to 57% of all cartels.

Table 3: Summary Statistics: Population of Cartels

	Mean	Median	SD	Min	Max	Skew	N
n_{firms}	3.49	3.00	1.27	2.00	7.00	0.35	710612
Fines (γ)	0.80	0.80	0.08	0.70	0.90	0.02	710612
% Fines Leniency (θ)	0.67	1.00	0.43	0.00	1.00	-0.70	710612
Start Leniency ($t > 500$)	0.57	1.00	0.49	0.00	1.00	-0.30	710612
structured	0.47	0.00	0.50	0.00	1.00	0.11	710612
unstructured	0.53	1.00	0.50	0.00	1.00	-0.11	710612
Detection (ρ)	0.22	0.20	0.08	0.10	0.35	0.10	710612
Duration	125.26	62.00	168.71	1.00	1000.00	2.41	710612
Cartel Start	433.51	439.00	327.52	1.00	1000.00	0.08	710612
Cartel End	557.77	576.00	313.74	1.00	1000.00	-0.14	710612
Detected Cartel	0.61	1.00	0.49	0.00	1.00	-0.45	710612
Model IIa	0.16	0.00	0.36	0.00	1.00	1.89	710612
Model IIb	0.37	0.00	0.48	0.00	1.00	0.54	710612
Model IIIa	0.14	0.00	0.35	0.00	1.00	2.03	710612
Model IIIb	0.33	0.00	0.47	0.00	1.00	0.72	710612

The probability of detection (ρ) affecting the ICC ranges between 10 and 35% with a median of 20%. Hence, the rising probability of the *structured* sample does not drive up the probability of being caught a lot. It is transformed to $\rho * 20/1000$ probability for an individual cartel to be detected by a CA in one single time step. This results in 61% of all cartels finally being detected. In Table 4, this variable is 1 by definition. All cartels in the sample are detected. Model IIa,b and IIIa,b each account for about half of the cartels, parting the population equally in structured and unstructured cartels. On average, a cartel in the population lasts 125 time periods. But duration is highly skewed, with a median of 62 and few cartels being alive over the whole time horizon.

Table 4 describes the sample of all industries being caught when colluding, consisting of 61% of the population. On average, there are about two to three firms in a cartel. The median of n_{firms} is equal to the population, but the mean is only 2.97 instead of 3.49. Cartels with a small number of firms have less incentive to break because of lower ICC for entry and exit. They mostly dissolve when detected. This leads to a larger share of industries with few firms in the sample. Additionally, this leads to longer duration, explaining that mean duration in the sample is 16% longer than in the population.

A similar reasoning holds for the percentage of fines to be paid by leniency applicants (θ). It increases ICC for exit and thereby reduces duration. The average percentage of fines to be paid by leniency applicants is higher in the sample (78%) than in the population (67%). Detection probability (ρ) plays an ambiguous role. On the one hand, a higher ρ increases ICC for entry and exit, leading to fewer and shorter cartels. On the other

Table 4: Summary Statistics: Sample of Detected Cartels

	Mean	Median	SD	Min	Max	Skew	N
n_{firms}	2.97	3.00	0.99	2.00	7.00	0.69	434009
Fines (γ)	0.80	0.80	0.08	0.70	0.90	0.02	434009
% Fines Leniency (θ)	0.78	1.00	0.36	0.00	1.00	-1.33	434009
Start Leniency ($t > 500$)	0.45	0.00	0.50	0.00	1.00	0.18	434009
structured	0.49	0.00	0.50	0.00	1.00	0.04	434009
unstructured	0.51	1.00	0.50	0.00	1.00	-0.04	434009
Detection (ρ)	0.23	0.25	0.08	0.10	0.35	-0.08	434009
Duration	146.03	92.00	157.86	1.00	1000.00	1.97	434009
Cartel Start	326.59	277.00	294.38	1.00	1000.00	0.46	434009
Cartel End	471.62	455.00	290.18	1.00	1000.00	0.12	434009
Detected	1.00	1.00	0.00	1.00	1.00	-	434009
Model IIa	0.17	0.00	0.38	0.00	1.00	1.74	434009
Model IIb	0.34	0.00	0.47	0.00	1.00	0.69	434009
Model IIIa	0.17	0.00	0.38	0.00	1.00	1.73	434009
Model IIIb	0.32	0.00	0.47	0.00	1.00	0.78	434009

hand, a higher ρ increases the share of detected cartels. In our simulation, mean of detection probability in population and sample only differ by 1 percentage point. Fines (γ) are equally distributed as in the population with no visible difference.

5 Results - Challenging Classical Estimation Methods

To identify potential problems using HR and CR models on selective samples, we run the econometric models on the population and its sample. We start with hazard rate estimation following closely on Bryant and Eckard [1991] to derive the probabilities of survival, death, and thereby the probability of detection. Thereafter, we aim to explain cartel duration perceived to be the main success factor of collusion (Levenstein and Suslow [2006]). Finally, we apply CR models to the population and the sample. By construction, the probability of being caught is predetermined.

5.1 Challenging Hazard Rate Estimation

Hazard rate analysis has been used to estimate the probability of getting caught for detected cartels and to explain cartel duration.[11] Hazard rate estimation is based on crucial assumptions of independence over time, random sampling, constant detection probabilities, etc. If one wants to run hazard rate estimations on the sample of caught cartels to make inferences about the whole population, the sample must be representative of its population. In Models IIa and IIb, birth and death of a cartel are considered to be independent events in the hazard analysis. Models IIIa and IIIb are adapted to our assumption of selection bias in enforcement strategies with increasing detection probabilities. We show in our simulations, that mean duration differs for cartels in the sample versus the whole population. The summary statistics on the population and its sample hint already at potentially problematic sampling biases.

[11] The most basic model is

$$\lambda(t) = \lim_{dt \to 0} \frac{Pr(t \leq T < t + dt | T \geq t)}{dt} = \frac{f(t)}{S(t)} = -\frac{S'(t)}{S(t)}$$

where t represents time and S is the hazard function. The hazard function can also be represented as a cumulative hazard function $\Lambda(t) = -\log S(t)$.

Table 5 shows estimated mean arrival time and mean duration following Bryant and Eckard [1991][12]. In Model I without enforcement, cartels are never detected and only dissolve on their own. Consequently, mean duration of cartels is longer than in Models II and III, also for their population duration. Models IIa and IIb with *unstructured* enforcement show average duration in the group of undetected cases to be about 33% shorter compared to detected cases (sample). Models IIIa and IIb allow for strategic enforcement with increasing detection probability for repeat offenders. The mean duration in the group of undetected cases is only around half of the mean duration in the sample.

Table 5: Mean Duration and Probability of Death Estimation

	Mean Arr. T. τ^{-1}	Births τ	Mean Dur. λ^{-1}	Deaths λ	Alive τ/λ	Obs. N
Model I						
All Cases (Population)	0.140	7.12	190.76	0.005	1359	7122
Model IIa,b						
Detected Cases (Sample)	0.005	220.92	160.83	0.006	35532	220923
Undetected Cases (Population - Sample)	0.007	153.36	109.45	0.009	16786	153357
All Cases (Population)	0.003	374.28	139.78	0.007	52317	374280
Model IIIa,b						
Detected Cases (Sample)	0.005	213.09	130.68	0.008	27845	213086
Undetected Cases (Population - Sample)	0.008	123.25	71.79	0.014	8847	123246
All Cases (Population)	0.003	336.33	109.10	0.009	36693	336332

$\tau^{-1} = timespan/N$
$\lambda^{-1} = \sum duration/N$

On the one hand, there are combinations of variables such as few firms in an industry, small fines, and low detection probability, where cartels do not end, except when detected. This leads to a longer duration in the population. Detection shortens the lifespan of a cartel, leading to shorter average duration in the detected sample. On the other hand, there are combinations of variables such as many firms in an industry, large fines, and high detection probability, where cartels dissolve and form again frequently. This leads to a shorter duration in the population. Cartels with longer duration are more likely to get detected, translating into cartels in the detected sample having longer duration. Consequently, the detected sample may either over- or underestimate the duration of the whole population (Harrington and Wei [2017]).

What does the sample tell us about its population? To cite Bryant and Eckard [1991] "*If the life of a caught conspiracy is typically* no longer *than that of an uncaught conspiracy, then duration estimates apply (also as lower bounds), and our probability estimate is an upper bound on the probability of uncaught conspiracy failure (i.e., demise by natural causes).*" In our simulation, this assumption that the life of a caught cartel is on average no longer than that of an uncaught cartel only holds in the group of industries with small number of firms, simulated in Model II with constant detection probability. For industries with large number of firms, and for all industries simulated in Model III with increasing detection probability, the average length of cartel life is between 1/10 and 3 times longer for detected cases than for undetected cases. Consequently, the mean duration of detected cartels either overestimates or underestimates the mean duration of all cartels depending on parameters and model. In analogy, the probability of death in a certain time period for detected cartels is

[12] Adjusting notation Bryant and Eckard [1991]: τ^{-1} Mean interarrival time in timesteps; τ Probability of cartel "birth" on a given timestep; λ^{-1} Mean cartel duration in timesteps; λ Probability of cartel "death" on a given timestep

only a good estimate for some parameters and models. Most often, it over- or underestimates the probability of death for all cases. Of course, the probability of being caught in a certain time unit for detected cartels is always an upper bound for the whole population of cartels. Undetected cartels are not caught by definition.

As we find that mean duration, arrival time, as well as cartels active differ significantly between sample and population, we are interested if hazard regressions lead to consistent and unbiased estimators when run on population and sample. Following Levenstein and Suslow [2011], we estimate the hazard rate coefficient depending on various firm and industry characteristics for the population of all cartels in Table 6 and for its sample of detected cartels in Table 7. The hazard rate coefficient shows the increase of risk concerning a change in one variable, holding all other variables constant. We define risk as the death of the cartel, assuming cartels dissolve after detection. In the sample of caught cartels, time of detection and death are equal. In the group of undetected cartels, some cartels experience the event of death because firms drop out whenever the firm-specific discount factor falls below the ICC threshold. Cartels may still be active at the end of the observed time period of $t = 1'000$. The regression models (Model A to F) are identical whether run on the population or its sample. Models A to C are comparable to empirical studies using real cartel data as they encompass information known to researchers trying to explain the duration of real cartels.

Table 6: Hazard Rate Coefficients Population

	Model A	Model B	Model C	Model D	Model E	Model F
n_{firms}	0.275	0.275	0.267	0.276	0.367	0.366
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fines (γ)		-0.096	-0.090	-0.089	-0.086	-0.088
		(0.015)	(0.015)	(0.015)	(0.015)	(0.015)
% Fines Len. (θ)			-0.647	-0.651	-0.671	
			(0.004)	(0.004)	(0.004)	
Start Leniency ($t > 500$)			-0.629	-0.606	-0.495	-0.127
			0.004	0.004	0.004	0.003
structured				0.277	0.396	
				(0.003)	(0.003)	
Detection (ρ)					3.439	3.389
					(0.017)	(0.017)
Model IIa						-0.201
						(0.004)
Model IIIa						0.194
						(0.004)
Model IIIb						0.390
						(0.003)
Constant	-4.149	-4.072	-3.972	-4.154	-4.756	-5.328
	(0.006)	(0.013)	(0.014)	(0.014)	(0.016)	(0.015)
N	710'612	710'612	710'612	710'612	710'612	710'612
Model Fit - Log	-1324915	-1324895	-1307798	-1301836	-1281591	-1291173

Table 6 shows the impact of all variables we vary on the duration of the cartels in the population. All regressors except the intercept are significantly different from zero at all conventional significance levels (i.e., $\alpha > 0.001$). We do not indicate coefficients being statistically different from zero as our hypothesis is whether they are different from the true estimate. The main determinant of cartel duration is the number of firms. If the number of firms in an industry increases by 1 unit, the risk coefficient for a cartel to dissolve increases by 0.275 to 0.37 for the population of all cartels. In general, coefficients hardly change during all specifications for the population-based estimations (Models A to F). The impact of the variation of fines as a percentage of profits (γ) on the risk of cartel death is small. Hazard risk coefficients are 0.09 for all cartels and 0.13 for detected cartels, regarding the increase of γ by one unit. We vary γ between 0.7 and 0.9. The decrease in risk for higher fines seems not intuitive, as higher fines increase ICC. But, if leniency is not possible ($\theta = 1$), ICC for exit do not depend on fines (6). In this case, an increase in γ only increases ICC for entry. Less firms enter collusion, but duration of those who enter stays the same. Consequently, average duration can increase and risk of dissolution decreases.

Table 7: Hazard Rate Coefficients Sample

	Model A	Model B	Model C	Model D	Model E	Model F
n_{firms}	-0.028	-0.028	-0.045	-0.037	0.036	0.036
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Fines (γ)		-0.135	-0.124	-0.126	-0.132	-0.132
		(0.019)	(0.019)	(0.019)	(0.019)	(0.019)
% Fines Len. (θ)			0.031	0.026	0.003	
			(0.006)	(0.006)	(0.006)	
Start Leniency ($t > 500$)			-0.382	-0.373	-0.304	0.306
			(0.004)	(0.004)	(0.004)	(0.003)
structured				0.170	0.269	
				(0.003)	(0.003)	
Detection (ρ)					3.088	3.087
					(0.020)	(0.020)
Model IIa						-0.011
						(0.005)
Model IIIa						0.276
						(0.005)
Model IIIb						0.260
						(0.004)
Constant	-4.540	-4.433	-4.370	-4.485	-5.596	-5.589
	(0.008)	(0.017)	(0.018)	(0.018)	(0.019)	(0.019)
N	434’009	434’009	434’009	434’009	434’009	434’009
Model Fit - Log	-710195	-710169	-702003	-700465	-688617	-688608

The introduction of leniency leads to a higher risk to dissolve. If full fines have to be paid ($\theta = 1$) instead of full immunity ($\theta = 0$), firms are reluctant to drop out and the risk of death decreases by 0.67. On the contrary, the introduction of leniency leads to a lower risk to dissolve in the sample of detected simulated cartels. The absence of leniency increases the risk to dissolve by 0.003. This finding supports the previously found sample selection bias; longer-lasting cartels get detected more frequently. Therefore, the duration of detected cartels is longer and the risk of dissolution is lower. In the second half of the time horizon, the risk of dissolving decreases for cartels in the population by 0.5, but for cartels in the sample only by 0.3, holding all other variables constant. On the contrary, increasing detection probability (*structured*) increases the risk of dissolving for all cartels by 0.4 in the population. However, for the detected cartel sample it only shows a coefficient of 0.27. A change in parameter ρ (detection probability), which is set at the beginning of the simulation, increases the risk by 3.44 for cartels in the population, and by 3.09 for cartels in the sample.

The most surprising finding is a change in the sign of the estimate on n_{firms} in the regression of the sample. In Table 7, Models A to D show a negative coefficient for the number of cartel members. The larger the cartel, the lower the probability of death and detection. As smaller cartels last longer, their detection and death risk is larger. When controlling for the introduction of leniency and the probability of being caught the coefficient turns positive. This clearly indicates that using HR estimations to explain cartel duration is highly dependent on variables included in the specification and hardly ever do coefficients found in the population estimations come close to the ones in the model focusing on the sample. To facilitate the comparison of the same models estimated on the sample and the population, Table 8 compares the confidence intervals of the estimated hazard rate coefficients for population versus sample for Model E.

Table 8: Model E: Hazard Rate 95% Confidence Intervals

	Population		Sample	
Model E	CFI - Beginn	CFI - End	CFI - Beginn	CFI - End
n_{firms}	0.365	0.370	0.033	0.039
Fines (γ)	-0.116	-0.056	-0.169	-0.096
Leniency (θ)	-0.679	-0.662	-0.007	0.014
Start Leniency ($t > 500$)	-0.503	-0.488	-0.312	-0.296
structured	0.391	0.401	0.263	0.276
Detection prob. (ρ)	3.406	3.473	3.048	3.127
Constant	-4.787	-4.725	-5.634	-5.558

Table 8 shows that with the exception of the fine being proportional to cartel profits (γ), the confidence intervals of the estimations do not overlap between estimations of the population and sample. The estimated values of the sample do not deliver consistent and unbiased estimates of the population even when including information like the probability of being caught and how it may *structurally* change over time. They overestimate the effect of the parameters describing leniency (θ and Start Leniency ($t > 500$)). They underestimate the effect of the parameters n_{firms}, *structured*, and ρ. Relying on hazard rate estimation for the evaluation of enforcement strategies and explaining factors leading to long-lasting collusion might lead to wrong conclusions and thereby, wrong policy recommendations.

5.2 Challenging Capture Recapture Estimation

Ormosi [2014] describes the use of CR models in estimating cartel detection rates. CR is originally used to estimate capture, survival rates and the size of animal populations (see Amstrup et al. [2010] and McCrea and Morgan [2015]). In a pre-specified area, part of an animal population is captured, marked, and released over time. Afterward, another portion is captured in exactly the same area and the number of marked individuals within the sample is counted. Since the number of marked individuals within the second sample should be proportional to the number of marked individuals in the whole population, an estimate of the total population size can be obtained by dividing the number of marked individuals by the proportion of marked individuals in the second sample. This process is then repeated a number of times, or even on an ongoing basis. The most simple representation is Amstrup et al. [2010]:

$$N = \frac{mc}{r}$$

where N represents the population size (of cartels as before), m the number of captured and marked animals, c the total number of captures during the second visit and r the number of recaptures on the second visit. The analysis relies on a likelihood specification with limited capability for modeling heterogeneity across industries. Citing difficulties with too many parameters needed to allow capture probabilities to depend on covariates, Ormosi [2014] suggests grouping industries into a small number of types. He finds that "*cartel detection stays between 10% and 20% most of the time between 1985 and 2009,*" for a sample of convicted EU cartels over the period 1985-2009. Similar to mean duration and hazard rate estimation, his result "*relies on colluding firms* *that have been detected at least once.*" The CR method used by Ormosi [2014] assumes instantaneous annual sampling of the CA. It allows for heterogeneity among firms in the form of trap response. The probability of survival or detection may change after a capture. Among all fitted models in Ormosi [2014], the most likely is $\phi(./.)p(t)$, with ϕ the rate of survival and p the probability of detection. In this model, survival rate is constant over time except being different in the year following detection. The probability of detection is time-dependent and varies every year. Other forms of heterogeneity like different firm or industry characteristics are discussed in Ormosi [2014], but not estimated.

Analogous to Ormosi [2014], we annualize our simulated dataset, reducing the 1000 time periods to 84. We apply CR estimation on the same variation of models, resulting in the similar order of best-fitted models as in Ormosi [2014].[13] We run CR estimations for the samples generated by Models II and III for few (2-3) and many (4-7) firm industries colluding to allow for some parameterization.

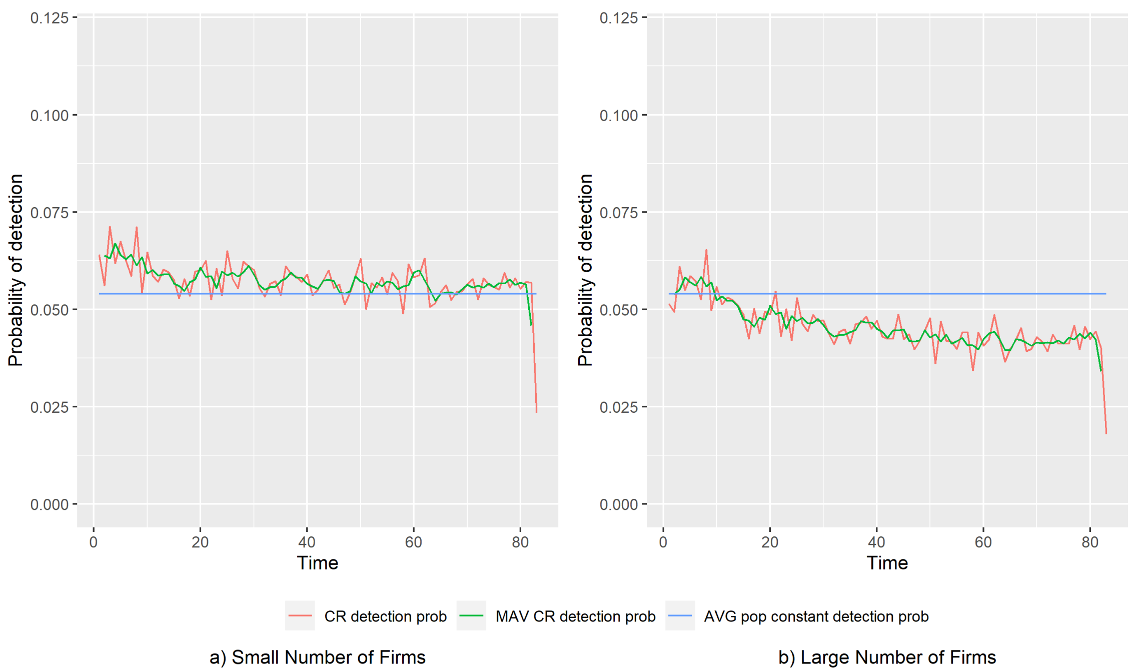

Figure 4: Model II: CR Estimated Detection Probability
(Panel a) Few Firms (77'930 Obs.) (Panel b) Many Firms (106'671 Obs.)

In Model II, the constant detection probability is represented by the blue line in Figure 4. In the simulated population of cartels, the probability of getting detected is constant over time. The CR estimate for the probability of detection depends on the number of firms in an industry for the sample of detected cartels. The red and green lines provide the actual estimate and its moving average over five periods in Figure 4 and Figure 5. Panel a) and b) show the CR results for few (2-3) and many firms (4-6) being in the cartelized industries, respectively in both figures. The estimated probability of detection varies highly over the time horizon. For industries with few and many firms, it highly over- or underestimates the probability of being caught, respectively. The discrepancy between the actual simulated constant probability of being caught is significantly different from the CR estimated one.

Figure 5 shows the result for the sample based on Model III. It allows for increasing detection probability.

[13]Using the same CR software MARK (White [2022]), we first reproduce results for the EU in Ormosi [2014], using identical European Commission detected cartels. Thereafter, we apply the same estimation method on our simulated data sets; the population and its sample.

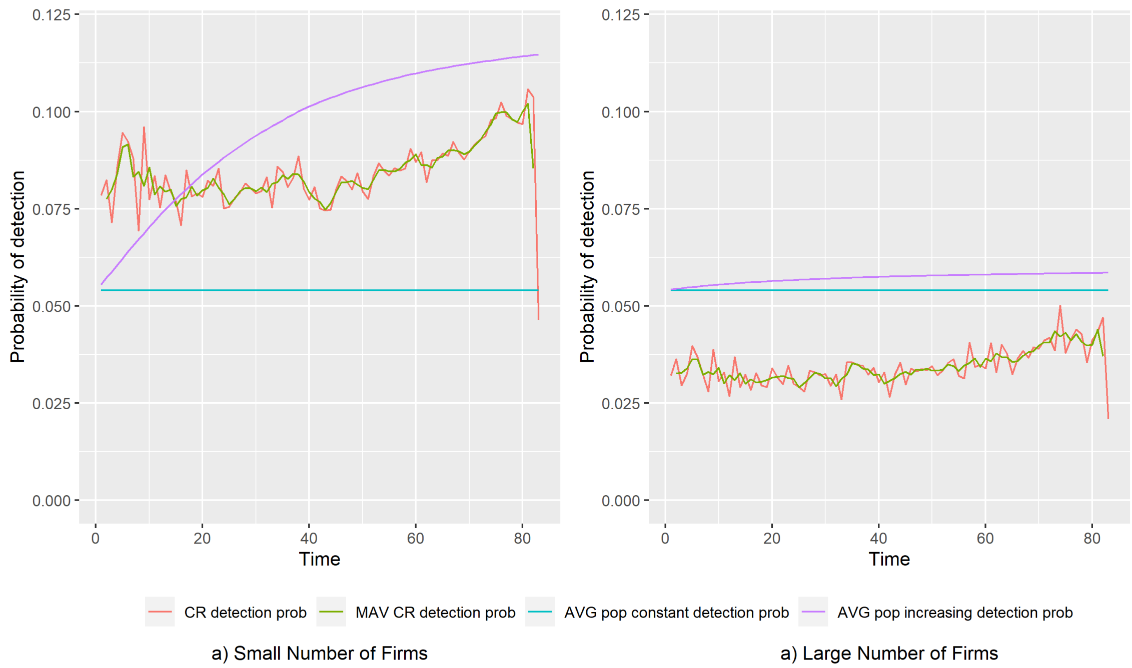

Figure 5: Model III (increasing ρ): CR Estimated Detection Probability over Time for Cartels with (a) Small Number of Firms (78'244 Obs.) (b) Large Number of Firms (106'686 Obs.)

For every cartel in the simulated population of cartels, the ex-ante probability of getting detected depends on time and increases in every detection. Previous results show that the population and detected sample are more alike compared to Model II. Panel a) of Figure 5 shows that the probability of detection (ρ) increases significantly over the simulation horizon (lila line). For the sake of transparency, we also state the starting probability of getting caught in light green. In industries with few firms, the CR estimated ex-post probability of detection is underestimated. For industries with many firms, the CR estimated ex-post probability of detection is underestimated.

Consequently, CR estimations, similar to Hazard Rate estimations, do not show good estimates for the probability of being caught. Since CR models are parametric estimations of zero parameters, one cannot account for various explanatory variables as we did in the duration models shown above. However, any result based on CR and HR models should be interpreted with caution especially when the difference between the population and its sample might be significant. Hyytinen et al. [2018], relying on populations of legal cartels in Scandinavia, may shed some light on similarities and differences between populations and their samples.

6 Contribution

So far the standard literature in microeconomics on collusion employs hazard rate and capture-recapture models to empirically evaluate antitrust enforcement. The duration and probability of re-captures is used to estimate the total cartel population size. We question the reliability of these estimation techniques. Necessary assumptions of the estimation models like no path dependencies and non-random sampling are violated. More specifically, there exist sample selection biases, eventually leading to biased and inconsistent hazard and capture-recapture rate estimations.

To evaluate the quality of an estimation technique, research in statistics offers two possibilities. First, one may either analytically derive the impact of sample selection (Heckman [1978] and Heckman [1979]). Harrington and Chang [2009] confirm the shortcomings of using duration of uncovered cartels as being representative of actual collusion. However, they do only vary one variable at a time. The second option is to simulate the data generating processes and run the conventional method to evaluate its predictive power. We simulate collusive behavior of industries with different number of firms based on three classical models of collusion (Stigler [1964], Harrington and Chang [2009] and Bos et al. [2018]), additionally varying five variables of antitrust enforcement (Table 1). The simulation provides a ground-truth data set of undetected and detected cartels; populations and their samples.

Already descriptive statistics hint at the existence of sample selection biases even for regimes where we do not induce them (Model II (*unstuctured*)). On the one hand, there are combinations of variables (few firms in an industry, small fines, and low detection probability), where cartels last very long and only end with detection. This leads to long duration in the population. On the other hand, there are combinations of variables (many firms in an industry, large fines, and high detection probability), where cartels dissolve and form again frequently. We observe shorter duration in the population. Consequently, the detected sample may either over- or underestimate the duration of the whole population. Using hazard estimations leads to biased and inconsistent results for the number of cartels being alive as well as their duration. When trying to explain cartel duration, coefficients derived for the sample and the population differ significantly. Hence, they do not result in reliable estimates. Using CR estimations relying on fewer assumptions than hazards and allowing for non-random samples does also not lead to an unbiased estimation of the probability of being caught.

Conclusively, we cannot recommend the use of hazard rate and capture-recapture estimation when sample selection is likely and the sample is not representative of its population (Heckman [1979]). This might also be applicable outside our area of interest whenever trying to deduct population sizes (e.g., animal population sizes, spread of viruses, size of illegal behavior). In analogy, using CR and HR models to identify the probability of dying or getting caught might lead to significant over- or underestimation. When explaining duration of any type of illegal offense, employing hazard rate models on the sample might lead to finding diverging coefficients when compared to the population. Given these three shortcomings, one should use findings cautiously when providing policy recommendations in anti-trust. Any existing law or legal change (like the introduction of the leniency program) might just induce a new or different sample selection bias, increasing the discrepancy between population and sample. If the sample of detected cases does represent its population, conventional methods perform poorly in terms of predicting and explaining the behavior at the population level. They should not be used to evaluate policies.

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