A general model of endogenous timing in duopolistic competition

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Introduction

- An essential feature of duopolistic competition is whether firms take actions simultaneously or sequentially
- When firms compete in quantities, these models are commonly known as Cournot (1838) and Stackelberg (1934) competition, respectively
- Stackelberg solution concept for the standard duopolistic competition implies the arbitrary assignment of the roles of leader and follower for players that are, a priori, interchangeable.
- The issue of whether the competition should be modeled as simultaneous or sequential can be addressed from various points of view and has policy implications

Introduction

Point of view 1: preferences

- Dowrick (1986) studies the preferred role as a function of the slope of the reaction curves
- Amir and Grilo (1999) and Julien (2011) show conditions allowing followers to achieve higher profits than the leaders.
- Regarding price competition of firms with different capacities, Furth and Kovenock (1993) and Deneckere and Kovenock (1992) show that the firm with the lowest capacity strictly prefers being a follower.

Introduction

Point of view 2: efficiency

- Stackelberg equilibrium is more efficient than the Cournot one (see Amir and Grilo (1999), Robson (1990b), Anderson and Engers (1992) or Huck et al. (2001)).
- Haan and Maks (1996): entry-deterrence model to show that the prices are not necessarily lower when the postentry competition is Stackelberg instead of Cournot.
- Colombo and Labrecciosa (2019): from a differential game point of view, Cournot equilibria could be more efficient (total surplus) than Stackelberg

Introduction

Point of view 3: how such preferences translate into a market interaction where players assume the roles of leader or follower → endogenous timing

- Idea: an intrinsic difference (lower marginal cost, more production capacity, more quality) makes the game sequential and leads to the best role in the market

Intrinsic difference/Competition variable	Price competition	Quantity competition
Quality	Li (2014) Lambertini & Tampieri (2017)	Lambertini & Tampieri (2011) Jinji (2004)
Location	Lambertini (1997)	
Capacity		Lu & Poddar (2009)
Marginal Cost	Amir et al. (1999) van Damme & Hurkens (2004) Amir & Stepanova (2006)	Amir & Grilo (1999) van Damme & Hurkens (1999)
R&D		Amir et al. (2000) Tesoriere (2008)

Introduction

Point of view 3's strategy in a nutshell

- There is a "basic interaction" model: scenario for which they want to obtain the timing endogenously. These basic interactions are usually price or quantity competition plus the intrinsic difference
- Extend the basic interaction using an endogenizing game: the Game with Observable Delay (GOD) or the Game with Action Commitment (GAC) from Hamilton and Slutsky (1990) – thera are few others
- Each equilibrium of the extended game naturally induces a timing of movements in the basic interaction

This paper

- Sequential or simultaneous doesn’t depend on the intrinsic difference, depends only on:
  - Whether the game is supermodular or submodular
  - Whether a largest action of the rival decreases or increases the profits of the other firm
  - The endogenizing game (GOD or GAC)
  - Risk considerations (meaning, refining multiple equilibria by risk dominance)
- To know if a duopoly will be sequential or simultaneous, look at slopes, which endogenizing game better represents the industry, and then *believe* that risk dominance refines well multiple equilibria
- Example: A standard Stackelberg will emerge if firms compete in quantities (submodular), play a GAC, and risk dominance refines multiple equilibria. For GOD, no Stackelberg but Cournot.

Preliminaries – Endogenous timing models

- Game with Observable Delay (GOD) (Hamilton and Slutsky, 1990)
- Players can choose an action from the set {Ea,La}, where Ea stands for Early and La stands for Late
  - If players choose the same action, they obtain their Nash Equilibrium payoffs in the basic interaction.
  - If a player chooses E and the other choose L, the former obtains her leader equilibrium payoff and the latter her follower equilibrium payoff.

1 \ 2	Ea	La
Ea	π_i^N, π_j^N	π_i^L, π_j^F
La	π_i^F, π_j^L	π_i^N, π_j^N

Preliminaries – Endogenous timing models

Game with Action Commitment (GAC) (Hamilton and Slutsky, 1990)

- Players can either commit to an action or wait: the action space in the pre-play stage for each player is $A \cup \{W\}$, where A is the action space in the basic interaction and W represents the option of waiting.
  - If both players commit to an action, they play simultaneously in the basic interaction and must use those chosen actions (commitment) – *note that this action may not be the exogenous Nash value*
  - If one commits to an action and the other waits, they play sequentially in the basic interaction. The leader plays the action chosen, the follower plays her best response after learning the leader's action.
  - If both players wait, they play simultaneously in the basic interaction

Preliminaries – risk dominance

1 \ 2	A	B
A	80 , 80	70 , 0
B	0 , 70	100 , 100

- Can we say more about which of the two equilibria is more likely to arise?
- (B,B) = (100,100) is payoff dominant – better for both of them
- But, if firm 1 is unsure about what firm 2 will play, how can she ensure ‘not to loose much’? How can she manage risk?

Preliminaries – risk dominance

	1/2	1/2
1 \ 2	A	B
A	80 , 80	70 , 0
B	0 , 70	100 , 100

- The best response of firm 1 to a half-and-half mixed strategy is to play A, because it has a larger expected payoff
- Risk from lack of coordination is minimized

Preliminaries – risk dominance

Risk dominant Nash equilibrium

1 \ 2	A	B
A	80 , 80	70 , 0
B	0 , 70	75 , 100

Non-payoff dominant Nash equilibrium

- It may happen that, with multiple equilibria, there is no payoff dominance, but there is risk dominance
- If the action space is continuous: Bicentric prior and tracing procedure -- generalization of the process of assuming mixing of possible equilibria, and best responding.

Model and results

- Basic interaction: $\pi_i(x_1, x_2) \quad i = 1, 2.$

- Definition 1: Competition is supermodular if $\frac{\partial^2 \pi_i}{\partial x_i \, \partial x_j} > 0.$
  - Best response functions will be increasing: (x_1, x_2) are strategic complements

- Definition 1: Competition is submodular if $\frac{\partial^2 \pi_i}{\partial x_i \, \partial x_j} < 0.$
  - Best response functions will be decreasing: (x_1, x_2) are strategic substitutes

Model and results (GOD)

- Consider first supermodularity and $\partial\pi_i/\partial x_j > 0$ (e.g. differentiated Betrand)

$\pi_1(x_1^L, x_2^F) > \pi_1(x_1^N, x_2^N)$ because player 1 could mimic Nash eq. but can do better

$\pi_1(x_1^N, x_2^N) > \pi_1(x_1^L, x_2^N)$ because x_1^L is not best response to x_2^N

- Then $x_2^F > x_2^N$ because $\partial\pi_1/\partial x_j > 0$
- And, $x_1^L > x_1^N$ because best responses are increasing (supermodularity)

- Since the same applies to player 2, we have that x_i^L and x_i^F are both above x_i^N $\forall i = 1,2$

Model and results (GOD)

- Next, only one of three things may happen:

  $x_1^L > x_1^F$ and $x_2^L > x_2^F$, or only one of the two holds

- Lemma: if $x_2^L > x_2^F$, then player 1 has a second mover advantage, that is $\pi_1^N < \pi_1^L < \pi_1^F$.

  Proof: $\pi_1(x_1^F, x_2^L) > \pi_1(x_1^L, x_2^L)$ because x_1^L is not best response to x_2^L

  $\pi_1(x_1^L, x_2^L) > \pi_1(x_1^L, x_2^F)$ because because $\partial\pi_i/\partial x_j > 0$

  implying $\pi_1^N < \pi_1^L < \pi_1^F$.

- Therefore, at least one player has a second mover advantage

Solving the endogenous timing

- $\pi_1^N < \pi_1^L < \pi_1^F$ (ad)
- $\pi_2^F < \pi_2^N < \pi_2^L$

1 \ 2	Ea	La
Ea	π_i^N, π_j^N	π_i^L, π_j^F
La	π_i^F, π_j^L	π_i^N, π_j^N

- $\pi_1^N < \pi_1^L < \pi_1^F$ (ad)
- $\pi_2^N < \pi_2^F < \pi_2^L$

1 \ 2	Ea	La
Ea	π_i^N, π_j^N	π_i^L, π_j^F
La	π_i^F, π_j^L	π_i^N, π_j^N

- This last case also happens when have both with second mover advantage

Model and results (GOD)

- Consider next supermodularity and $\partial\pi_i/\partial x_j < 0$ (e.g. firms compete on own product advertising for market shares) – the analysis is very similar and the equilibria are also sequential

- Theorem 1: when the game is supermodular and the endogenizing game is GOD, the resulting game will be sequential

Model and results (GOD)

- Consider next submodularity and $\partial\pi_i/\partial x_j < 0$ (e.g. classic Cournot)

$\pi_1(x_1^L, x_2^F) > \pi_1(x_1^N, x_2^N)$ because player 1 could mimic Nash eq. but can do better

$\pi_1(x_1^N, x_2^N) > \pi_1(x_1^L, x_2^N)$ because x_1^L is not best response to x_2^N

- Then $x_2^F < x_2^N$ because $\partial\pi_1/\partial x_2 < 0$
- And, $x_1^L > x_1^N$ because best responses are decreasing (submodularity)

- Since the same applies to player 2, we have that $x_i^F < x_i^N < x_i^L \quad \forall i = 1,2$

Model and results (GOD)

- Lemma: if $x_2^L > x_2^N$, then player 1 has a first mover advantage, that is $\pi_1^F < \pi_1^N < \pi_1^L$.

Proof: $\pi_1(x_1^L, x_2^F) > \pi_1(x_1^N, x_2^N)$ because player 1 could mimic Nash eq. but can do better

$> \pi_1(x_1^F, x_2^N)$ because x_1^F is not best response to x_2^N

$> \pi_1(x_1^F, x_2^L)$ because because $\partial\pi_1/\partial x_j < 0$

- Therefore, both players have first mover advantage and $\pi_i^F < \pi_i^N < \pi_i^L$

Solving the endogenous timing

- $\pi_1^F < \pi_1^N < \pi_1^L$
- $\pi_2^F < \pi_2^N < \pi_2^L$

	Ea	La
Ea	π_i^N, π_j^N	π_i^L, π_j^F
La	π_i^F, π_j^L	π_i^N, π_j^N

- Therefore, the endogenous timing is (Ea, Ea): a sequential game does not arise → No Stackelberg for GOD
- Consider next submodularity and $\partial\pi_i/\partial x_j > 0$ (e.g. firms may invest in product differentiation, prices are given – *we think*) – the analysis is very similar the equilibria is still simultaneous

Model and results (GAC)

- Submodularity's $\pi_1^F < \pi_1^N < \pi_1^L$ does not depend on the endogenizing game.
- If we solve for equilibria using GAC instead of GOD, we obtain that $(x_1^L, W_2\ ), (x_1^N, x_2^N)$ and $(W_1\ , x_2^L)$ are all possible

- There is no payoff dominant strategy
- Risk dominance analysis: only the sequential equilibria survives → Stackelberg appears if the endogeneizing game is GAC and risk dominance refines
- Intuition: both firms playing the Stackelberg quantity is too risky

- Supermodularity and GAC: three equilibria, only the sequential survive risk refinement

Conclusions

	Supermodular	Submodular
GOD	Sequential	Simultaneous
GAC	Sequential (risk)	Sequential (risk)

- We have mix-and-match theorems. E.g.:
  - If $\partial\pi_1/\partial x_2 > 0$ and $\partial\pi_2/\partial x_1 < 0$, the timing is simultaneous if the game is supermodular and is sequential if the game is submodular for GOD
- What seems very relevant, then, is which industries are better characterized by GOD and which ones are better characterized by GAC

Conclusions – not in the paper

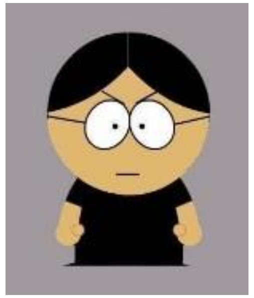
- Any intrinsic difference will help to identify which firm will be the leader when the game is sequential
- We believe this can also be traced to primitives
- Consider an investment that makes a firm tough: $d\pi_1/dI_2 < 0$. For example, investing in decreasing the marginal cost makes a Cournot competitor tougher.
- If such investment took place before, then we expect to fill up…

Conclusions – not in the paper

	Supermodular		Submodular	
GOD	Tough	Soft	Tough	Soft
	Sequential Firm with largest investment leader (may require risk)	Sequential To Be done	Simultaneous	Simultaneous
GAC	Tough	Soft	Tough	Soft
	Sequential Firm with largest investment leader (may require risk)	Sequential To Be done	Sequential Firm with largest investment leader (may require risk)	Sequential To Be done

Thanks!

Questions and (nice) comments are more than welcome



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