Vertical Mergers: No Foreclosure, Yet Harm to Consumers

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Abstract

I analyse a vertically related industry where downstream firms have a common supplier. A vertical merger can lead to foreclosure. However, competition upstream might prevent the merged entity from raising its downstream rivals’ input costs. I show that pre-merger margins can reveal whether the supplier is constrained by competitors. Therefore, observable margins can reveal whether competition prevents the merged entity from obtaining a higher input price post-merger. Absent foreclosure, a vertical merger eliminates double marginalization but weakens downstream (horizontal) competition. Because of weakened downstream competition, a vertical merger can benefit all firms – including non-integrated ones – and harm consumers.

JEL classification: L41; L42; L44; L12.

Keywords: Vertical Mergers; Foreclosure; Antitrust; Margins; Horizontal Competition.

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I Introduction

Vertical merger policy is hotly debated. In 2020 the DOJ & FTC released new vertical merger guidelines, but withdrew them in 2021 (due to disagreement within the FTC).\(^1\) In contrast, in the literature there exists broad agreement on a number of points. First, vertical mergers can lead to EDM (the elimination of double marginalisation).\(^2\) EDM benefits consumers via lower prices.\(^3\) Second, vertical mergers can lead the merged entity to harm its downstream rivals through RRC (raising rivals’ input costs).\(^4\) RRC harms consumers via higher prices. It is also understood that the merged entity can have an incentive to – instead – lower its rivals’ costs.\(^5\) Intuitively: when the EDM effect is very large, the merged entity is more willing to accommodate lower prices of rivals, and lowers input prices. Third, vertical mergers can weaken downstream (horizontal) competition.\(^6\) After the merger, the merged entity supplies a downstream rival and therefore benefits from its sales. Consequently, the merged entity competes less aggressively downstream. I call this the “stakeholder effect” because of the close link to the minority shareholding literature.\(^7\) The stakeholder effect harms consumers. Finally, in the literature the overall merger effect can mostly go either way – it is seldom clear.

Much of the literature on vertical mergers focusses on RRC.\(^8\) Similarly, for Antitrust authorities foreclosure is often the main concern when reviewing vertical mergers. Foreclosure can come in different forms. (1) The merged entity might set and obtain a higher price from downstream rivals. I call this “direct RRC”. Alternatively, the merged entity might stop to supply downstream rivals. This can lead downstream rivals to either (2) incur higher costs by sourcing inputs through other means, or (3) it can lead them to exit the market altogether.

\(^2\)See Spengler (1950) for an early exposition, and Waterson (1980).
\(^3\)Salop (2021) states that “Pass on of EDM should not be taken as automatic”, and that EDM is not necessarily merger-specific.
\(^6\)On similarities to horizontal mergers, see Riordan (1998), Chen (2001), Baker et al. (2019), Moresi and Salop (2021). In contrast, seminal work by Schelling (1960) shows instances in which a firm finds it profitable to delegate maximisation problems to elicit less aggressive competition. Related work on delegation within a firm and strategic incentives includes Fershtman and Judd (1987).
\(^7\)This effect has no agreed-upon name in the literature. Chen (2001) first identified this effect. He called it the “collusive effect”. Moresi and Schwartz (2017) call it the “input supply effect”. Moresi and Schwartz (2021) call it the “Chen effect”.
\(^8\)Other theories of harm lay out how a vertical merger can facilitate downstream collusion (Biancini and Ettinger, 2017) or upstream collusion (Nocke and White, 2007, 2010). Forward integration into an industry with high elasticity of demand can allow an upstream monopolist to limit arbitrage and better price discriminate across industries (see Perry (1978, 1980) on the classic ALCOA case).
Whereas much has been written on the *incentive* of the vertically integrated entity to raise rivals’ costs, less has been written on the merged entity’s *ability* to do so. The presence of upstream competitors may constrain the merged entity in the price it sets to non-integrated downstream rivals. Without the ability to engage in direct RRC, incentives to do so are inconsequential. My two main contributions focus on the inability of the merged entity to raise rivals’ costs.

First, I develop a test which can be used to answer the question: will a proposed merged entity be able to obtain a higher price from rival downstream firms? In other words, the question I address is: does upstream competition prevent the merged entity from obtaining a higher input price post-merger (i.e. from engaging in direct RRC)? The test is practical and transparent. It uses observable data on margins to predict merger effects. The prediction in existing models that the merged entity can engage in RRC can be consistent or inconsistent with observable data. The test serves to identify models which are consistent with observable facts of a particular industry or case under analysis.

Second, I show that absent foreclosure vertical mergers can harm consumers and benefit all firms – not only the merged entity but also its rivals. Precisely when a merged entity wants to but is unable to raise its rivals’ costs, downstream rivals benefit from weakened downstream competition, while input prices do not increase. Thus, in such cases a vertical merger can benefit all firms, while all consumer prices can increase.

The baseline model I consider is standard. In a vertically related industry, two downstream firms produce a differentiated consumer good. Each downstream firm needs one unit of an input per unit of output. There are one or more upstream producers of that input, each has a constant unit cost of production. In the first stage, each upstream producer submits a unit price (a bid) to each downstream firm, at which price it offers to supply any quantity. Producers are permitted to discriminate across downstream firms (i.e. to engage in third-degree price discrimination). In the second stage, each downstream firm sets its consumer price, and consumer demand is determined. Downstream firms then order inputs.9

To construct a novel merger test, I start by developing a result on the relation of upstream versus downstream margins. In Kadner-Graziano (2022) I show that, for any log concave demand function, an upstream monopolist producer earns a higher unit dollar margin than the downstream firm it supplies.10 Because for log concave demand functions the pass-through rate is lower than 100% (Amir, Maret, and Troege, 2004), the elasticity of demand is lower upstream than downstream, and therefore – by the intuition for the Lerner markup (Lerner, 1934) – the unit dollar margin upstream must be bigger than downstream.11

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9In section VII, I allow for suppliers to be chosen before demand is determined.
10E.g. the dollars of profit Apple versus one of its suppliers earns per iPhone sold.
11Adachi and Ebina (2014b) obtain this result for the special case of successive monopolies, in a Cournot setting.
In the present paper, I show this relation of margins also holds with downstream competition – and with the associated equilibrium feedback effects. When a monopolist supplier increases the price it sets to a downstream firm, say firm A, that downstream firm increases its price. The increase in firm A’s consumer price diverts demand to competing downstream firms. In equilibrium, other downstream firms raise their prices as well (downstream prices are strategic complements). This in turn leads to, in equilibrium, an even greater price increase by firm A. Price increases by competitors dampen firm A’s loss of quantity. Two factors thus explain why a monopolist supplier earns a higher margin than a downstream firm it supplies. First, keeping prices at other downstream firms fixed, the pass-through rate is less than 100%. Second, after accounting for price increases by other downstream firms, the loss in quantity at downstream firm A and the upstream elasticity of demand is lower still.

In addition, I show the relation of upstream to downstream margins also holds when an upstream producer supplies competing downstream firms. Intuitively, if a producer supplies several downstream firms, its elasticity of total demand is lower still: if the supplier increases the input price to downstream firm A, it profits from extra sales diverted to any other downstream firm it supplies. This third factor adds to the two factors described in the preceding paragraph.

The margin of a monopolist supplier is bigger than that of each downstream firm it supplies. A monopolist supplier is unconstrained by competition. The test I develop states: if the supplier, on its sales to a downstream firm, earns a smaller margin than that downstream firm, then the supplier is constrained by upstream competition pre-merger and the merged entity cannot obtain a higher price from that downstream firm post-merger. Because if the merged entity were able to obtain a higher price post-merger, then the supplier would have already charged a higher price pre-merger. In this way, margins can reveal the existence of constraints sufficient to rule out that the merged entity can engage in direct RRC.

The test is useful precisely because it is robust to a series of extensions which portray the complexity and intertwined nature of real supply chains. The relation of margins also holds with more than two downstream firms. It also holds when downstream firms use not only one but any number of inputs. It also holds when different downstream firms use different sets of inputs, and when they are supplied by the same or by different upstream producers.

To utilise margins to reveal ability to engage in RRC is new. Inderst and Valletti (2011) analyse pre-merger margins of the merging parties, and argue these are insufficient to predict post-merger incentives. Instead, I compare margins to determine post-merger ability. Other papers also compare upstream to downstream margins, but not in a merger environment (Bresnahan and Reiss, 1985 and Adachi and Ebina, 2014a). They use variable proportions: see their production cost function on p.822. Adachi and Ebina (2014a) analyse a setting with symmetric Cournot competition both upstream and
Absent foreclosure, the consumer welfare effect of the merger is determined by the trade-off between benefits from EDM and harm from the stakeholder effect (weakened downstream competition). The former effect is small when, pre-merger, the supplier earns a small margin on sales to the downstream firm it integrates with. The latter effect is large when, pre-merger and among other factors, the supplier earns a large margin on the non-integrated downstream firm. Overall, the merger can harm consumers.

Consumer harm is clearest in a diagonal merger, one where the downstream firm integrates with a producer from whom it does not source inputs, but who supplies a downstream rival. In such a merger, there is no EDM effect. Absent foreclosure, the only merger effect is the stakeholder effect. Consequently, all firms benefit, and all consumer prices rise. The merger is unambiguously detrimental to consumers of all downstream firms. This highlights that – contrary to Chen (2001) – my model requires neither efficiency gains nor non-standard costs, neither for merger profitability nor for consumer harm.

For example, suppose Samsung acquires a supplier of Apple, without previously having had any relationship with that supplier. Apple should be worried about an increase in input prices if that supplier has pricing power. But margin data indicates each Apple supplier is constrained, such that Samsung could not raise prices post-merger, and hence the acquisition would only render Samsung less aggressive on the smartphone market. Consequently both smartphone makers would benefit, whereas consumers would lose out.

The typical foreclosure concern raised in both theory and practice pits the merged entity against its downstream rivals, see Chen (2001), Rey and Tirole (2007) among many others. Opposite to this view, in the present paper I outline a theory of harm where the merged entity is unable to engage in RRC and, as the example illustrates, the merger can benefit not only the merged entity, but also its downstream rivals. In clear contrast to the existing literature, I show that some vertical mergers benefit a non-integrated downstream firm more than the merged entity itself, i.e. the increase in profit is greater for the non-integrated downstream firm. (Intuitively, this occurs when the principle merger effect is to reduce the merged entity’s downstream aggressiveness.)

A vertical merger with a constrained supplier is akin to a horizontal merger: it has two countervailing effects that resemble those of horizontal mergers – some form of efficiency gain (here the EDM) versus a loss of horizontal competition (here the stakeholder downstream. They find Cournot competition induces a ratio of margins which depends on the number of upstream and downstream firms. This differs to my results, where suppliers set prices.

14 On diagonal mergers, see Zenger (2020).
15 Chen (2001) analyses the incentive of the non-integrated downstream firm in choosing a supplier. He assumes there to be switching costs, due to relationship-specific investments. Such costs are non-standard in the literature.
16 See Kadner-Graziano (2022).
17 Samsung could not raise prices, except if Samsung itself was the constraint on that supplier.
Thus a vertical merger can constitute yet another form of market concentration. Crucially, this has implications for empirical research: estimates of concentration should not only encompass common ownership and cross-ownership, but also ownership of suppliers to competitors.19

The remainder of the paper is structured as follows. In section II, I present the baseline model and obtain the comparative result of upstream versus downstream margins. In section III, I analyse vertical mergers and develop a new test. I also discuss welfare effects. In section IV, I briefly consider diagonal mergers. Section V contains the Antitrust discussion. Section VI relates to extensions and to robustness. In section VII, I discuss the effects of changing the timing of the game. In section VIII, I conclude. Proofs are deferred to the appendix.

II A relation of upstream to downstream margins

A Baseline model

I model a vertically related industry. There are two downstream firms. Each produces a consumer good. The two consumer goods are differentiated. Each downstream firm needs one unit of an essential input per unit of output (each has a Leontief production function). Downstream firm \(i \in \{A, B\}\) can produce the input in-house at constant unit marginal cost \(c_i \geq 0\).

There are three upstream producers of the input. The second and third-most efficient producers have, respectively, constant unit marginal costs of production \(c^{[2]}\) and \(c^{[3]}\), with \(c^{[3]} \geq c^{[2]} > 0\). Without loss of generality, the most efficient has zero production costs.

There are two stages. In the first stage all upstream producers simultaneously submit bids to the two downstream firms. To each downstream firm, each upstream producer submits a unit price at which it offers to supply any quantity demanded by that downstream firm. Each upstream producer can price discriminate across downstream firms.

In the second stage the two downstream firms simultaneously set their respective uniform consumer price, \(P_A\) and \(P_B\). The product market clears and quantities \(Q_A\) and \(Q_B\) are determined. Each downstream firm then orders inputs externally from an upstream producer, or produces inputs internally.20 Downstream firm \(i \in \{A, B\}\) pays unit price \(v_i\) and earns profit \(\Pi_i = (P_i - v_i)Q_i\). In equilibrium, the most efficient upstream producer earns profit \(\Pi_S = \sum_i v_i Q_i\).

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19 On empirical studies of common ownership, see e.g. Azar, Schmalz, and Tecu (2018). Azar, Raina, and Schmalz (2019) analyse, in addition, cross-ownership.

20 If two suppliers offer the same price to downstream firm \(i\), the upstream firm with the lowest cost is chosen.
Downstream firm $i$’s demand strictly decreases and is weakly log concave in consumer price $P_i$: $Q_i' < 0$ and $(\ln Q_i)'' \leq 0$. Downstream products are imperfect substitutes, with $\frac{\partial Q_i}{\partial P_i} > 0$ for $i \neq j$. Moreover, I make the following three assumptions. Assumptions 1 and 2 are standard (see e.g. Miklós-Thal and Shaffer, 2021).

**Assumption 1** (Gross substitutes). At equilibrium prices $P_i^*$ and $P_j^*$, $|Q_i'| > \frac{\partial Q_i}{\partial P_i}$.

Assumption 1 states: at the equilibrium, if all consumer prices rise by the same small amount, then demand for each consumer product decreases.\(^{21}\)

**Assumption 2** (Concave profit). The Hessian of $\Pi_S(v_i, v_j)$ is negative definite.

Assumption 2 ensures that, for any $v_j$, the supplier’s profit is strictly concave in $v_i$.\(^{22}\) Assumption 3 is not necessary, it is made for conciseness of the algebraic exposition.\(^{23}\)

**Assumption 3** (Additively separable). $\frac{\partial Q_i}{\partial P_j} = 0$ for $i \neq j$.

Finally, while I assume complete information in the baseline model, this assumption is not necessary for my results (as discussed in the robustness section VI).

## B Equilibrium

The game is solved by backwards induction. Once consumer demand is determined, each downstream firm sources inputs from the cheapest source: either it produces internally

\(^{21}\)This assumption ensures sensible comparative statics (in particular it ensures that a downstream firm raises its consumer price in response to an increase in its unit input cost).

\(^{22}\)Assumption 2 is not necessary for the results in section II, but is necessary for results in section III. It ensures that a constrained supplier prices at the constraint rather than below.

\(^{23}\)Assumption 3 is not necessary in the sense that it could be replaced by a weaker – though less transparent – assumption. Assumption 3 removes a second-order effect. It is satisfied for any demand function where the price of a competing downstream firm enters as an additively separable term. E.g. as in Shubik-Levitan linear demand systems. However – unlike in Shubik-Levitan linear demand systems – I allow for $Q''_i \neq 0$. 

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**Figure 1: Pre-merger setting**

![Diagram](image-url)
at \( c_i \), or orders inputs from the supplier who offered the lowest unit price. In stage 2, each downstream firm \( i \in \{A,B\} \) sets its price \( P_i \) to maximise its profit. At the optimal consumer price \( P_i^* \),

\[
\frac{\partial \Pi_i}{\partial P_i} = Q_i(P_i, P_j) + (P_i - v_i)Q_i'(P_i, P_j) = 0
\]  

(1)

Prices are strategic complements:

\[
\frac{\partial^2 \Pi_i}{\partial P_i \partial P_j} = \frac{\partial Q_i}{\partial P_j} > 0
\]

In stage 1, upstream producers set their offer prices to each downstream firm. The most efficient upstream producer sets (bids) unit price \( v_i \) to downstream firm \( i \). Let \( v_i^{*u} \) denote the monopoly price: the price the most efficient producer would set if it were unconstrained by any outside option of the downstream firm. Then, the most efficient producer sets the profit-maximising price:

\[
v_i^* \equiv \arg \max_{v_i} \Pi_S = \min \{v_i^{*u}, c_i\}
\]

(2)

If the supplier cannot set its monopoly price, it is constrained – either at \( c_i^2 \) (the unit cost of the second-most efficient potential supplier) or at \( c_i \) (beyond which a downstream firm would produce the component in-house). One may rewrite the above as

\[
v_i^* = \min \{v_i^{*u}, \tilde{c}\}
\]

where \( \tilde{c} = \min \{c_i, c_i^2\} \). I keep the two different costs as separate because the distinction matters for some merger results.

**Definition 1.** Supplier \( i \) is **unconstrained** if \( v_i^* = v_i^{*u} \).

**Definition 2.** Supplier \( i \) is **constrained** if \( v_i^* \neq v_i^{*u} \).

The most efficient producer has profit

\[
\Pi_S = v_i Q_i(P_i^*(v_i, v_j), P_j^*(v_j, v_i)) + v_j Q_j(P_j^*(v_j, v_i), P_i^*(v_i, v_j))
\]

(3)

At the optimal unconstrained (monopoly) price \( v_i^{*u} \),

\[
\frac{\partial \Pi_S}{\partial v_i} = Q_i + v_i \left( Q_i' \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i} \right) + v_j \left( Q_j' \frac{dP_j^*}{dv_i} + \frac{\partial Q_j}{\partial P_i} \frac{dP_i^*}{dv_i} \right) = 0
\]

(4)

24The second order condition yields \( \frac{\partial^2 \Pi_i}{\partial P_i^2} \bigg|_{P_i=P_j} = \frac{2Q_i^2 - Q_i''}{Q_i} < 0 \). Weak log concavity of demand is equivalent to \( Q_i^2 \geq Q_i'Q_i'' \), the second order condition thus shows \( P_i^* \) yields the downstream firm’s unique profit-maximising price.

25By Assumption 3 \( \frac{\partial Q_i'}{\partial P_j} = 0 \) for \( i \neq j \).

26By Assumption 2 the second order condition is satisfied.
C Constrained or unconstrained – what margins reveal

I aim to identify when a particular supplier is constrained. To detect a constrained supplier I compare the downstream margin with the upstream margin of an unconstrained supplier. Rearranging (1), the unit dollar margin of downstream firm $i$, $m_i^u$, is

$$P_i^* - v_i = -\frac{Q_i(P_i^*, P_j)}{Q_i'(P_i^*, P_j)}$$  \hfill (5)

Rearranging (4), the supplier’s unit dollar margin on sales to downstream firm $i$ when it is unconstrained, $m_{Si}^u$, is

$$v_i^{*u} = -\frac{Q_i}{Q_i' + \frac{\partial Q_i}{\partial P_i} \frac{\partial P_i}{\partial v_i}} - \frac{Q_j^*}{Q_j' + \frac{\partial Q_j}{\partial P_j} \frac{\partial P_j}{\partial v_i}} \quad \hfill (6)

Observation 1. The supplier is constrained in the price it sets to downstream firm $i$ if and only if $m_{Si}^u < m_{Si}^{*u} = -\frac{Q_i}{\frac{\partial Q_i}{\partial v_i}} - \frac{Q_j^*}{\frac{\partial Q_j}{\partial v_i}}$.

The equivalence in Observation 1 is of little practical use in identifying whether a supplier is constrained, because the derivatives in (6) are difficult to estimate. To develop a practical and transparent test which can be used to identify whether a supplier is constrained, I proceed by comparing upstream to downstream margins.

To compare (5) and (6) I calculate the total derivatives $\frac{dP_i^*}{dv_i}$ and $\frac{dP_j^*}{dv_i}$. I rewrite the first-order condition (1) as

$$Q_i\left(P_i^*(v_i, v_j), P_j^*(v_j, v_i)\right) + \left(P_i^*(v_i, v_j) - v_i\right)Q_i'\left(P_i^*(v_i, v_j), P_j^*(v_j, v_i)\right) = 0$$  \hfill (7)

and totally differentiate with respect to $v_i$ and $v_j$ respectively. Solving yields\footnote{Workings are provided in the proof of Theorem 1. Both denominators are positive, because $Q_i'^2 > Q_i Q_j'$ (due to log concavity) and because $|Q_i'| > \frac{\partial Q_i}{\partial P_i}$ (by Assumption 1). As a specific example on the equilibrium pass-through rate: if downstream firms each face the same demand function $Q_i = \alpha - P_i + \frac{1}{2}P_j$, with $\Pi_i = (P_i - v_i)Q_i$, then $\frac{dP_i^*}{dv_i} = \frac{2}{2x^2 - \frac{1}{2}x^{\frac{1}{2}}} = \frac{2}{2 - \frac{1}{4}} = \frac{4}{3} < 1$.}

$$\frac{dP_i^*}{dv_i} = \frac{2 - \frac{1}{2}Q_i''}{\left(2 - \frac{1}{2}Q_i''\right)\left(2 - \frac{1}{2}Q_j''\right)} > 0$$  \hfill (8)

$$\frac{dP_j^*}{dv_i} = \frac{2 - \frac{1}{2}Q_j''}{\left(2 - \frac{1}{2}Q_i''\right)\left(2 - \frac{1}{2}Q_j''\right)} > 0

Plugging in (8) into (5) and (6) yields the following key result. The supplier’s unit dollar margin on sales to downstream firm $i$, when it is unconstrained, weakly exceeds downstream firm $i$’s unit dollar margin.
Theorem 1. (Relation of margins) The margin of an unconstrained supplier, on sales to downstream firm i, weakly exceeds downstream firm i’s margin: \( m^u_{Si} \geq m^*_i \).

The relation of upstream to downstream margins as shown in Theorem 1 is simple. However, the intuition and effects which underlie the result are intricate. In stage 1, the supplier sets its profit-maximising unit input price to each downstream firm by accounting for stage 2 downstream equilibrium effects.

When the supplier increases the input price to downstream firm A, downstream firm A passes-through less than 100% of the increase to its consumer price – keeping prices of other downstream firms constant (price effect 1). (This follows from the log concavity of demand.) In equilibrium, because downstream prices are strategic complements, downstream firm B increases its price (price effect 2) and — now accounting for the change in price of other downstream firms — downstream firm A increases its consumer price further (price effect 3). Overall, price effects 2 and 3 necessarily dampen — rather than exacerbate — the loss of demand at firm A caused by price effect 1 (because the equilibrium price increase of firm A only occurs as a reaction to the greater demand it enjoys thanks to prices increases at other firms).

Thus, the elasticity of demand upstream is lower than downstream because – holding prices of other downstream firms fixed – an increase in the input price to downstream firm A is dampened (“absorbed”) by that downstream firm (it passes through less than 100%) and because – accounting for equilibrium changes in downstream prices – the loss of quantity at downstream firm A is dampened. By the Lerner intuition, a lower elasticity of demand upstream implies a higher (unconstrained) margin upstream than downstream.

An additional factor contributes to the margin upstream being larger than downstream. When a supplier increases the input price to one downstream firm, it profits from extra sales at the other downstream firm it supplies. Therefore, the supplier’s elasticity of “total” demand (of units sold to both downstream firms) is lower still.

The price effects 1, 2, and 3 are shown algebraically below. Keeping prices of other downstream firms constant, the unconstrained upstream margin exceeds the downstream margin because pass-through is less than 100% (price effect 1):

\[
- \frac{Q_i}{Q_i' \frac{dP_i}{d\sigma}} \geq - \frac{Q_i}{Q'_i}
\]

Accounting for downstream price reactions of other firms (price effect 2) and feedback effects (price effect 3) further increases the unconstrained upstream margin relative to the downstream margin:

\[
- \frac{Q_i}{Q_i' \frac{dP_i}{d\sigma} + \partial Q_i \frac{dP_i}{d\sigma} } > - \frac{Q_i}{Q'_i \frac{dP_i}{d\sigma}}
\]

To the extent that the supplier supplies other downstream firms, its unconstrained margin
is higher still relative to the downstream firm’s margin:

\[-v_j \frac{dQ_j}{dv_i} \geq 0\]

where

\[
\frac{dQ_i}{dv_i} = Q'_i \frac{dP^*_i}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP^*_j}{dv_i} < 0
\]

\[
\frac{dQ_j}{dv_i} = Q'_j \frac{dP^*_j}{dv_i} + \frac{\partial Q_j}{\partial P_i} \frac{dP^*_i}{dv_i} \geq 0
\]

An increase in \(v_i\) leads to a decrease in quantity sold by downstream firm \(i\), and an increase in quantity sold by downstream firm \(j\).\(^{28}\)

The price effects 1, 2 and 3 are illustrated in the figure below. An increase in \(v_A\) shifts out downstream firm A’s best response function. Holding \(P^*_B\) fixed, \(P^*_A\) rises by less than the increase in \(v_A\). In equilibrium, both downstream prices increase.

**Figure 2: An increase in \(v_A\) – illustrative example**

Using Theorem 1, I now provide a sufficient condition to identify a supplier who is constrained. In contrast to the equivalence in Observation 1, the sufficient condition below is transparent and practical.

**Corollary 1** (Constrained supplier). If \(m^*_M < m^*_i\), then the supplier is constrained in the price it sets to downstream firm \(i\).

\(^{28}\)Workings are provided in the proof of Theorem 1.
For each unit of input sold, the supplier earns a margin $m_{Si}^*$, while the downstream firm earns a margin $m_i^*$ per unit of consumer product sold. If the supplier earns a smaller margin than downstream firm $i$, then the supplier is constrained in its pricing to that downstream firm. The supplier would like to set a higher price, but cannot because the downstream firm has some outside option it would switch to in case the supplier’s offer were higher. Thus, if $m_{Si}^* < m_i^*$ the supplier is unable to extract a higher price.

Even if the supplier earns a large margin (even if it has a large competitive advantage over other potential suppliers or over in-house production by the downstream firm), the supplier may nonetheless be constrained. Whether it is constrained can be identified by comparing unit dollar (not percentage) margins, as shown in Corollary 1.

### III Vertical merger

In this section I focus on ability. I use the relation of upstream to downstream margins to develop a practical and transparent test. The test serves to address the following question: yes or no, will a vertical merger enable the merged entity to extract a higher input price from the rival downstream firm?

I distinguish between two cases of foreclosure. The merged entity is able to engage in **direct RRC** if it can set and obtain a higher price from the non-integrated downstream firm post-merger, relative to the price it obtained pre-merger. Here the higher input price is paid to the merged entity, which thus benefits directly from its rival’s higher cost. The merged entity is able to engage in **indirect RRC** if – by stopping to supply the non-integrated downstream firm – the latter’s input cost increases (because, for example, it pays a higher price to another supplier or because it switches to more costly in-house production). Here the higher input price is not paid to the merged entity, which benefits only indirectly from its rival’s higher cost through weakened downstream competition. If stopping to supply causes a sufficiently large cost increase, the rival exits the market.29 (There are no terms used uniformly across the academic and legal Antitrust literatures to distinguish between direct versus indirect RRC.)30

### A Post-merger equilibrium

Let the supplier merge with a downstream firm, say downstream firm A. If the merged entity continues to supply the non-integrated firm, the joint entity $M$ maximises profit

$$\Pi_M = P_A Q_A(P_A, P_B) + v_B Q_B(P_B, P_A)$$

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29 This is sometimes referred to as “complete exclusion” (Rey and Tirole, 2007).

30 The first case is sometimes referred to as “foreclosure” or “RRC”, or also as “partial foreclosure” (Moresi and Schwartz, 2021). The second case is also sometimes referred to as “foreclosure” or “RRC”, or also as “full foreclosure” (Baker et al., 2019), or “complete foreclosure” (Rogerson, 2019). However, “full foreclosure” sometimes refers to the case where the downstream rival exits the market.
The profit function of the non-integrated downstream firm B remains unchanged:

$$\Pi_B = (P_B - v_B)Q_B(P_B, P_A)$$

The game is again solved by backwards induction. The FOC of the non-integrated firm remains unchanged, it is still given by (1). The first order condition for the merged entity’s downstream price differs from (1). At the optimal consumer price $P_A^{*M}$, the post-merger first order condition yields:

$$\frac{\partial \Pi_M}{\partial P_A} = Q_A(P_A, P_B) + P_A Q'_A(P_A, P_B) + v_B \frac{\partial Q_B}{\partial P_A} = 0$$

Compared to the pre-merger first order condition (1), there are two countervailing effects. First, the EDM effect: whilst downstream firm A had unit input cost $v_A$, the merged entity has unit cost 0. EDM decreases the consumer price charged by the merged entity, $P_A$. Second, the merged entity earns $v_B$ per unit sold by the non-integrated firm, whereas pre-merger downstream firm A did not profit from sales of its competitor. Because, post-merger, the merged entity profits from sales of its downstream competitor, the merged entity competes less aggressively downstream than downstream firm A did pre-merger. I call this the stakeholder effect, to highlight the link to the minority shareholdings (common ownership) literature.\textsuperscript{31} The stakeholder effect increases the consumer price $P_A$. Algebraically, the term $v_B \frac{\partial Q_B}{\partial P_A} > 0$ stands for this incentive to behave less aggressively downstream. Ceteris paribus, the higher $\frac{\partial Q_B}{\partial P_A}$, the greater the reduction in downstream competition. Chen (2001), who first described weakened downstream competition, calls this the “collusive effect”, Moresi and Schwartz (2017) (who do not consider a merger) the “input supply” effect, and Moresi and Schwartz (2021) the “Chen effect”.

In stage 1, the merged entity sets the optimal input price to the non-integrated downstream firm

$$v_B^{*M} \equiv \arg\max_{v_B} \Pi_M = \min\{v_B^{*uM}, c^{[2]}, c_B\}$$

where $v_B^{*uM}$ is the input price the merged entity sets if it is unconstrained (a monopolist supplier). The optimal unconstrained unit price solves

$$\frac{\partial \Pi_M}{\partial v_B} = Q_B + v_B \left(Q'_B \frac{dP^*_B}{dv_B} + \frac{\partial Q_B}{\partial P_A} \frac{dP^*_A}{dv_B}\right) + \frac{dP^*_A}{dv_B} Q_A + P_A \left(Q'_A \frac{dP^*_A}{dv_B} + \frac{\partial Q_A}{\partial P_B} \frac{dP^*_B}{dv_B}\right) = 0$$

The FOC of the merged entity differs from (4), the FOC of the supplier pre-merger, in two ways. First, the merged entity now earns $P_A$ rather than $v_A$ from each unit of the consumer product A.\textsuperscript{32} Second, an increase in $v_B$ allows the merged entity to increase

\textsuperscript{31}The merged entity does not earn a share of profits, its earnings are instead directly proportional to the quantity sold by the downstream rival and customer.

\textsuperscript{32}Ceteris paribus the lower the diversion rate $\frac{\partial Q_A}{\partial P_B}$, the lower the incentive of the merged entity to engage in RRC – as in the vGUPPI of Moresi and Salop (2013).
its own downstream price $P_A$, from which it benefits for any given quantity level $Q_A$. Algebraically, this refers to the term $\frac{dP_A}{dQ_A} Q_A$.

B  The test: ability to engage in direct RRC

I now develop a new, practical, and transparent test for vertical mergers. As shown in Theorem 1 and Corollary 1, pre-merger data (facts on margins) can reveal whether a supplier is constrained. If the supplier is constrained in the price it sets to the non-integrated downstream firm pre-merger, then the merged entity cannot engage in direct RRC post-merger.

Proposition 1 (Test). If $m^{*}_{SB} < m^{*}_{B}$, then the merged entity is unable to engage in direct RRC post-merger.

For a given proposed merger, this test can be used to assess whether pre-merger data indicates that the merging supplier is able to extract a higher input price post-merger from the non-integrated downstream firm. Knowing that the supplier’s margin is smaller than that of the non-integrated downstream firm suffices to know that the merged entity is unable – it does not have the ability – to extract a higher price from the non-integrated downstream firm. If it were able to do so post-merger, it would have raised the price pre-merger already. In other words, and for the model considered herein, if the supplier’s margin is smaller than that of the third party downstream firm, then it is inconsistent with facts of the given case that the merged entity can engage in RRC.

Remark. Even if the supplier is unconstrained pre-merger, a constraint could lie just above its pre-merger price ($c^{[2]}$ could lie just above $v^{eM}_{B}$), such that its ability to engage in RRC may be limited.

C  Ability to engage in indirect RRC

If pre-merger the supplier is constrained at $c_{B}$, then the merged entity is unable to engage in indirect RRC. (The non-integrated downstream firm would switch to in-house production if the merged entity raised the input price.) Therefore, in this case, the merged entity finds it profitable to continue to supply the non-integrated downstream firm.

Instead, if pre-merger the supplier is constrained by the second-most efficient upstream producer, then the merged entity may or may not be able to engage in indirect RRC. If $c^{[3]} = c^{[2]}$, the merger cannot lead to indirect RRC, because the second-most efficient producer is constrained by the third-most efficient producer and cannot extract a higher price than $v_{B} = c^{[2]}$. Again, in this case the merged entity finds it profitable to continue to supply the non-integrated downstream firm.

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33The merged entity has an incentive to engage in direct RRC if and only if $v^{eM}_{B} > v^{eM}_{B}$. It has an incentive to raise the input price beyond the pre-merger unconstrained price if and only if $v^{eM}_{B} > v^{eM}_{B}$. 

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Observation 2. If pre-merger the supplier is constrained at $c_B$ and/or if $c^{[3]} = c^{[2]}$, then the merged entity is unable to engage in indirect RRC, and finds it strictly profitable to continue to supply.

Remark. The condition $c^{[3]} = c^{[2]}$ can be interpreted as the existence of a competitive fringe.

Only if $c^{[2]} < c_B$ and $c^{[2]} < c^{[3]}$ does the merged entity have the ability to engage in indirect RRC. But even then it may not have the incentive to do so. Intuitively, indirect RRC is not profitable if it only leads to a small input cost increase for the non-integrated downstream firm, but to a large loss of input sales revenues for the merged entity. Or, alternatively, if the diversion of sales from the non-integrated firm to the merged entity is small. “Vertical arithmetic” is an existing tool practitioners use to assess the incentive of such foreclosure (see Zenger, 2020 for more on this).

D Consumer welfare

In a merger with a constrained supplier, the merger may or may not result in foreclosure. Nevertheless – even without foreclosure (absent direct and indirect RRC) – the merger effect can be negative, i.e. consumer welfare can decrease.

Proposition 2 (Consumer Welfare). If the merged entity continues to supply, and if it wants to engage in RRC but $m^*_{SB} < m^*_B$, then a vertical merger does not result in foreclosure. Moreover,

- consumer welfare increases when benefits from the EDM effect outweigh harm from the stakeholder effect. But
- consumer welfare decreases when harm from the stakeholder effect outweighs benefits the EDM effect.

EDM shifts the best response function of the merged firm’s downstream entity. It exerts a downward-pricing effect on the merged entity’s consumer price. Because prices are strategic complements, EDM also causes a decrease in the price of the non-integrated downstream firm.

However, there is a countervailing effect: the stakeholder effect. Because the merged entity benefits from sales of its downstream customer and rival, the merged entity’s best response function shifts and it behaves less aggressively downstream. Because prices are strategic complements, the stakeholder effect also causes an increase in the price of the non-integrated downstream firm.

Intuitively,

- if the EDM effect is large (if $v_A$ is large) and if the stakeholder effect is small (if $v_B \frac{\partial Q_B}{\partial P_A}$ is small), then the main merger effect is to bring about efficiencies. Consumer prices can decrease and consumers can benefit from the merger. Conversely,
• if the EDM effect is small and the stakeholder effect large, the stakeholder effect can be the dominating effect. Consumer prices can increase and consumers can be harmed by the merger.

E Merger profitability

A vertical merger is always profitable to the merged entity. The merged entity becomes closer to a multi-product firm which internalises the beneficial effect of an increase in its consumer price $P_A$ on demand for the other consumer product. The stakeholder effect leads the merged entity to raise its downstream price and lose quantity. Therefore, the merged entity can find it profitable to lose market share, both quantity-based and sales-based market share.$^{34}$

More surprisingly, a vertical merger can profit the non-integrated downstream firm more than the merging parties. I provide the intuition for this result for a case where $c_A = 0$. Without EDM (i.e. when $c_A = 0$) and without foreclosure (i.e. when $m^*_B < m^*_B$), the only merger effect is to render the merged entity’s less aggressive downstream. The shift in the merged entity’s best response function leads to further increases in $P_B$ and $P_A$ in equilibrium. But overall, the first order effect is an increase in $P_A$ with a reduction in $Q_A$, whereas firm B not only increases $P_B$ but also enjoys an increase in its quantity sold $Q_B$.

Proposition 3. There are vertical mergers after which the profit of the non-integrated downstream firm rises by more than the profit of the merging parties.

IV Diagonal merger

A diagonal merger is a special form of a vertical merger. In a diagonal merger, a downstream firm integrates with a competitor’s upstream supplier, without previously having much or any direct business interaction with that supplier.$^{35}$ Here, I take a diagonal merger to be one where a downstream firm has no previous relationship with the supplier it integrates with. The results hitherto allow for a swift analysis of diagonal mergers.

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$^{34}$For an example where the merged entity loses quantity-based and sales-based market share, see the proof of Proposition 3 and the subsequent Remark.

Consider a diagonal merger, where downstream firm A integrates with the supplier of its rival, downstream firm B. (Downstream firm A produces inputs in-house.) Such a merger results in only two effects. The merger entails no EDM effect, simply because downstream firm A does not source the input from the upstream producer it integrates with (see e.g. Zenger, 2020). Recent literature has shown that, in a vertical merger, the merged entity may have an incentive to lower its rival’s cost due to the EDM effect. Because a diagonal merger entails no EDM, in a diagonal merger the merged entity always has an incentive to engage in RRC. Whether the diagonal merger leads to RRC effects depends on the merged entity’s ability. Finally, a diagonal merger always weakens downstream competition – it always leads to the stakeholder effect.

The relation of margins shown in section II also holds for the setting in Figure 3. Hence the test (Proposition 1) also applies to diagonal mergers. If $m^*_{SB} < m^*_{B}$, then the merged entity is unable to engage in direct RRC post-merger. The same intuition as in the previous section applies: if the input supplier is constrained pre-merger, it cannot increase price post-merger. Even absent foreclosure, a diagonal merger is profitable.

Results on indirect RRC in a diagonal merger are the same as in a vertical merger. However, the effects on the downstream rival and on consumers are different, due to the absence of EDM.

**Proposition 4 (Welfare).** If the merged entity continues to supply and if $m^*_{SB} < m^*_{B}$, then a diagonal merger always

- benefits the downstream rival, and
- harms consumers.

In vertical mergers, the downstream rival might lose profits – when it is hurt more by EDM than it gains from the stakeholder effect. In contrast, in a diagonal merger there is no EDM, thus absent foreclosure the only effect is the stakeholder effect, in which case the merger unambiguously benefits the downstream rival. With or without direct RRC, a diagonal merger strictly harms consumers of all downstream firms.
V Antitrust discussion

A The problem

Vertical mergers can benefit consumers, but they can also raise a whole swath of concerns.\textsuperscript{36} Further complicating merger reviews, a vertical merger may have different effects on different consumer groups: while customers of the merged entity might benefit from the merger through a lower price (thanks to EDM), instead customers of non-integrated downstream firms might be harmed (due to RRC).\textsuperscript{37} Merger reviews by Antitrust authorities often involve a trade-off between benefits from EDM and harm from RRC (Riordan and Salop, 1994), whose equilibrium sizes are intertwined (Karlinger et al., 2020). But which effect overrides? Existing theory does not provide a general answer to this question. (Riordan (1998) for example shows harmful effects can dominate,\textsuperscript{38} whereas e.g. Loertscher and Reisinger (2014) show the EDM effect can exceed foreclosure effects.\textsuperscript{39})

Foreclosure is a key concern raised in US and EU vertical merger guidelines.\textsuperscript{40} Much is known on incentives of the merged entity to engage in RRC, but few papers in the literature focus on the ability of the merged entity to engage in such an anticompetitive strategy. The present paper contributes a practical test to the existing toolbox of Antitrust authorities, one which deals with the ability to foreclose.

\textsuperscript{36}For more concerns, see Hart and Tirole (1990), Baker et al. (2019). Instead, e.g. Perry (1989) mentions the avoidance of transaction costs as a determinant of vertical integration.
\textsuperscript{37}Some customers might also switch from one firm to another.
\textsuperscript{38}He argues his model provides a basis to presume that “backward integration by a dominant firm has an anticompetitive effect even when significant efficiencies are taken into account”. He models capacity-constrained firms, with one dominant downstream firm and a competitive fringe. This model does not feature differentiated product price competition downstream.
\textsuperscript{39}They model a downstream market with quantity competition.
\textsuperscript{40}US Antitrust guidelines state:

“\textquote{A vertical merger may diminish competition by allowing the merged firm to profitably use its control of the related product to weaken or remove the competitive constraint from one or more of its actual or potential rivals in the relevant market. For example, a merger may increase the vertically integrated firm’s incentive or ability to raise its rivals’ costs by increasing the price or lowering the quality of the related product. The merged firm could also refuse to supply rivals with the related products altogether (\textquote{foreclosure}).}” U.S. Department of Justice & The Federal Trade Commission (2020), p.4.

Similarly, the European Commission merger guidelines state

“\textquote{Input foreclosure arises where, post-merger, the new entity would be likely to restrict access to the products or services that it would have otherwise supplied absent the merger, thereby raising its downstream rivals’ costs by making it harder for them to obtain supplies of the input under similar prices and conditions as absent the merger.} \textquote{\textquote{\textquote{}}}}” European Commission (2014), p.215.
B  The test

In the setting considered in this paper, I show that the margin of a monopolist supplier must exceed that of any downstream firm it supplies. Therefore, if the observed unit dollar margin of a supplier is smaller than that of the downstream firm, it must be that the supplier cannot set its optimal monopoly (or “unconstrained”) price. Thus it must be constrained.\textsuperscript{41} It could be constrained for a number of reasons. For example by a competing supplier, by the threat of in-house production of the downstream firm, or (as laid out in section VI) because past a certain price the downstream may simply not use the input at all.

The test developed herein states: if pre-merger the supplier has a lower unit dollar margin than the non-integrated downstream firm, then the supplier is constrained and cannot engage in RRC post-merger.\textsuperscript{42} The test yields a binary conclusion: it shows whether or not RRC effects can be excluded altogether. Even if the supplier who merges provides an essential input to downstream firms, and is dominant in the sense of having a large margin and cost advantage over any rivals, the test can reveal the existence of constraints sufficient to rule out that the merged entity can engage in RRC.

The test only requires margin data for the supplier and for the non-integrated downstream firm. It does not require any additional data to what is required for existing tools used to analyse foreclosure incentives. Vertical arithmetic (VA) and the vGUPPI also need data on margins.

Intuitively, my result on the relation of upstream to downstream margins is related to the Lerner markup. The latter relates a firm’s unit percentage margin to the elasticity of demand it faces. In contrast, my result relates the unit dollar margins of an upstream supplier to a downstream firm. Intuitively, I can relate absolute rather than percentage margins because the quantities sold by the upstream and downstream firms are the same (or more generally with fixed proportions, they are a fixed ratio of each other). For the ability to engage in RRC, percentage margins do not convey the information that dollar margins do.

C  How to use the test in conjunction with existing tools

Several methods have been developed to estimate harm from vertical foreclosure, notably VA to assess the incentive of the merged entity to engage in full foreclosure (see Zenger,\textsuperscript{41} Moresi and Schwartz (2017) intuitively explain that a firm which supplies a downstream customer and competitor would want that competitor to act more (rather than less) aggressively downstream. In their model, the supplier is a monopolist upstream. Their intuitive finding may not hold however when the upstream supplier is constrained.

\textsuperscript{42}Section VII discusses examples of how to render the upstream price endogenous to the merger. These examples use, as Chen (2001), non-standard assumptions on costs. I therefore exclude such costs from my main model.
and the vGUPPI developed by Moresi and Salop (2013)\(^{43}\) used to gauge the degree to which the merged entity has the incentive to increase price. The test developed herein is complementary to these existing tools.

When Antitrust authorities are concerned about foreclosure in the form of RRC, the test provides a practical tool to help determine whether or not empirical facts indicate the merged entity would be unable to set and obtain a higher input price from a downstream rival. If the merged entity is deemed able to engage in RRC, then the vGUPPI can be used to gauge incentives regarding the magnitude of a possible post-merger price increase.\(^{44}\) (Though, as discussed, the merged entity might have an incentive to lower rivals’ costs.) However, even if the upstream supplier is unconstrained, it is possible that a constraint (e.g. the cost of the second-most efficient firm) lies just above the unconstrained price, such that the RRC effect might be limited.

VA may or may not indicate that the merged entity would find it profitable to stop to supply downstream rivals. In any case, the test developed herein is relevant to VA. It is relevant because it shines light on the alternative outcome to stopping to supply (in particular, on whether or not RRC is possible in the alternative outcome).

### D Welfare

Surprisingly perhaps, I present a theory of harm where the merged entity has no pricing power upstream, neither pre- nor post-merger. The loss of downstream competition is by itself sufficient to cause consumer harm.\(^{45}\)

If the merged entity has the incentive but not the ability to engage in RRC, nor the incentive to stop supplying, then the merger leads to only two effects: EDM and the stakeholder effect. The trade-off can go either way. Even without any RRC, a vertical merger can be profitable and strictly harm consumers.

An example setting where a vertical merger would harm consumer welfare is where

- \(v_A\), the pre-merger price charged to the integrating downstream firm, is small, such that the EDM effect is small, and where

- \(v_B\) is relatively large, such that the stakeholder effect is large.

In such a situation the vertical merger yields no RRC effect, barely any EDM, but a significant stakeholder effect. Intuitively, a limited size of the EDM effect stems from the same cause as the absence of RRC: pre-merger constraints on the supplier.

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\(^{43}\)Moresi and Salop (2013) develop the vGUPPI both with and without input substitution.

\(^{44}\)The vGUPPI can be used to estimate the unilateral incentive of the merged entity to raise the input price, and to estimate the extent to which a downstream firm unilaterally wants to raise its consumer price (Moresi and Salop, 2013).

\(^{45}\)In Canada, the Competition Bureau Canada uses total welfare (rather than consumer welfare) to assess mergers. This stems from a view that Antitrust should be focused on allocative efficiency, not redistribution, see Sanderson (1997) and Duhamel (2003).
VI Robustness

The results of sections II-V are robust to a series of extensions, which are laid out in the following subsections A and B. Furthermore, in subsection C I show the main findings hold even if the integrated downstream firm is a multi-product firm. Finally, in subsection D I consider different informational assumptions and different price formation mechanisms.

A More than two downstream firms

$n$ symmetric downstream firms, with Shubik demand

In sections II and III there were two asymmetric downstream firms. Now suppose there is a number $n \geq 2$ of symmetric downstream firms, where each downstream firm $i$ faces Shubik demand

$$Q_i(P) = \alpha - P_i - \phi(P_i - \frac{1}{n} \sum_{i=1}^{n} P_i)$$

with $\phi > 0$. The Shubik linear demand system fits Assumptions 1 - 2. In particular, it fits with the gross substitutes assumption. Each downstream firm has constant unit input price (cost) $v_i \geq 0$, with profit $\Pi_i = (P_i - v_i)Q_i(P)$.

The upstream firm which supplies the $n$ downstream firms with the essential input has profit

$$\Pi_S = \sum_{i=1}^{n} v_i Q_i(P^*(v))$$

Zimmerman and Carlson (2010) derive the formula for the equilibrium pass-through rate. They show that the equilibrium pass-through rate decreases with the number of firms $n$. Building on the results of Zimmerman and Carlson (2010) I obtain the following result regarding pass-through:

**Proposition 5.** With $n \geq 2$ and a Shubik linear demand system, the equilibrium pass-through rate is always (strictly) below 1, and the margin of an unconstrained supplier exceeds that of any downstream firm it supplies.

The result of Proposition 5 can be visualised by plotting the pass-through formula of Zimmerman and Carlson (2010) on the graph below:

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46This demand function is taken from Zimmerman and Carlson (2010, p.15).
Given the equilibrium pass-through rate is smaller than 1, the result on the relationship between the upstream and downstream margins still holds. Hence Theorem 1, Corollary 1 and the test (Proposition 1 also hold with $n$ downstream firms and Shubik demand: the unit dollar margin of an unconstrained supplier (strictly) exceeds the unit dollar margin of the downstream firm.

$n$ asymmetric downstream firms, with passive beliefs

Suppose $n \geq 2$ asymmetric firms compete downstream. Such that there is differentiated product competition. Moreover, let each downstream firm source not only one but any number of different components. In stage 2, based on input prices all downstream firms simultaneously set the price for their consumer good.

In Kadner-Graziano (2022) I deal with this level of generality. To do so, I use the “passive beliefs” assumption: following Moresi and Salop (2013) I assume downstream firms do not know the input prices of their competitors, that downstream firms form point beliefs about the input prices of their competitors, and that these beliefs are correct in equilibrium. The key technical and simplifying consequence of such beliefs is that, when a supplier sets its optimal input price $v_i$ to downstream firm $i$, a change in $v_i$ changes only $P_i$, whilst it does not change the consumer prices of other downstream firms (as they set their prices based on their point beliefs regarding $v_i$.) (In this sense, the supplier believes downstream firms other than firm $i$ to be “passive” with respect to a change in $v_i$.)

With passive beliefs, the key results of sections II and III continue to hold. In particular, Theorem 1, Corollary 1, and the test (Proposition 1) continue to hold. Intuitively, if a downstream firm faces a log concave consumer demand, then the downstream firm

\[ \frac{\partial P_i}{\partial v_i} \]

\[ 0 \]

\[ 5 \]

\[ 10 \]

\[ 10 \]

\[ 20 \]

\[ 0 \]

\[ 0.5 \]

\[ 1 \]

\[ n \]

\[ \phi \]
absorbs some of a price increase. Its pass-through rate is less than 100%. Consequently, the elasticity of demand is lower upstream than downstream. Such that the margin of an unconstrained supplier upstream must exceed that downstream.

This intuition is simpler than that of section II. Because with passive beliefs, down-stream firms do not react if a competitor’s input price changes. (Hence there are no feedback effects from other firms adjusting their prices.) Therefore, here less-than-full pass-through results directly from the log concavity of demand.

## B Further extensions

The results presented hitherto, Theorem 1, Corollary 1 and the test (Proposition 1) are robust to a series of extensions. Starting from the baseline model of section II, I consider each of the following extensions separately.

First, suppose the input is differentiated. (E.g. one supplier is able to produce a chip of high quality at a given unit cost, while another is able to produce a chip of lesser quality at some other given unit cost.) Because Theorem 1 and Corollary 1 relate the downstream margins to the \textit{unconstrained} upstream margin, both results are unrelated to the degree of upstream product or quality differentiation among suppliers of the given input component. As this might seem surprising, the following provides some intuition. Product differentiation only impacts the constraint. Instead of being constrained at \(c^{[2]}\), the supplier is constrained at another limit price: at the level of \(v_i\) beyond which downstream firm \(i\) would switch from the first-ranked to the second-ranked supplier.\(^{48}\)

Second, suppose the input is non-essential (i.e. that downstream firms can do without the input and still produce a valuable consumer product). Then the optimal price function of the supplier of that input contains a further limit price, beyond which a downstream firm would choose not to source the input for its consumer product.\(^{49}\) Whenever a unique equilibrium exists, then allowing for non-essential components leaves results presented hitherto unaltered – except for the further limit price and thus further explanation as to why a supplier may be constrained rather than unconstrained.

Third, the results do not change if the upstream firm has strictly positive unit costs of production, and if the upstream firm has different unit costs of production for different downstream firms. (Theorem 1 and Corollary 1 relate upstream to downstream margins – the latter are net of unit costs.)

\(^{48}\) The ranking is defined as follows. Suppose all suppliers bid a unit price equal to their unit production cost. At those offer prices, the downstream firm will have a (weak or strict) preference for one supplier. This supplier is the first-ranked supplier, with unit production cost \(c^{[1]}\). The second-ranked supplier has unit production cost \(c^{[2]}\). In the undifferentiated case, the limit price on the first-ranked is \(c^{[2]}\). In the differentiated case the limit price (greater than \(c^{[1]}\)) may be greater or smaller than \(c^{[2]}\) (for, respectively, a quality advantage or disadvantage).

\(^{49}\) The limit price, say \(v_i\), is a function of \(\sum_{j \neq i} v_j\). The limit price is unique because the downstream firm’s profit function is monotonically decreasing in \(v_i\).
Fourth, suppose the upstream supplier produces a range of different inputs (rather than quality differences I here refer to inherently different inputs). Whether or not the two downstream firms source the same input or source different ones does not impact any of the algebra presented above. The results verify whether, for whatever the upstream firm sells to a downstream firm, the supplier is constrained.

None of the generalisations outlined above alter the intuition about the relation of upstream versus downstream margins, which rests on the lower elasticity of “total” demand faced upstream versus downstream.

C Multi-product downstream firms

As is well-known, with single-product successive monopolies a vertical merger eliminates EDM and necessarily lowers the downstream price – to the benefit of consumers. Building on Edgeworth’s taxation paradox (Edgeworth, 1925), Salinger (1991) and Luco and Marshall (2020, 2021) lay out a counterintuitive result for vertical mergers. They show that if the downstream firm is a multi-product firm, vertical integration can lead all downstream prices to increase if some products benefit from EDM (“integrated products”) while others do not (“non-integrated products”). This is surprising. Costs decrease but prices increase. Intuitively, thanks to EDM it becomes more profitable to sell integrated products and, therefore, the merged entity raises the price of non-integrated products to divert sales to integrated products. The latter effect can be large, sufficiently large for all downstream prices to rise.

But consider, in my model, downstream firm A to be a multi-product firm. It sells two products. If downstream firm A merges with the most efficient supplier, where one of A’s products benefits from EDM while the other does not, the insights developed in section II continue to apply. In such a vertical merger, costs decrease whereas prices increase. But here this is not surprising. It is not surprising because – with or without selling multiple products – downstream firm A has an incentive to raise its downstream price due to the stakeholder effect. In section II we already saw that a vertical merger can lead all downstream prices to increase even with EDM. Selling multiple products merely adds a further tool for downstream firm A, to raise not only demand for integrated products but also demand for the rival’s product.

Salinger (1991) as well as Luco and Marshall (2020, 2021) lay out a similar theory of harm to the one presented here: theirs does not feature foreclosure either. However, in their models the merged entity does not supply a rival. Therefore, their theory of harm entails no stakeholder effect. In contrast, the stakeholder effect is central to the present paper. The main insight from Salinger (1991) and Luco and Marshall (2020, 2021), which carries over to the present paper, is that if downstream firm A is a multi-product firm, then consumer harm can also arise from higher prices of non-integrated products.
D Other price formation mechanisms & information structures

Second-price auctions

Upstream firms might not know the price offers submitted by competing suppliers. And consumer product manufacturers often set up tenders where they ask for price offers. We can model this more realistic situation as follows.

Suppose upstream producers know their own unit cost, but not that of others. And suppose each downstream firm sets up a tender in the form of a descending-price auction (where potential suppliers submit rounds of bids) or a second-price auction (where potential suppliers submit a single bid price). (Let’s put aside the option of the downstream firm to produce in-house. Also, if there is a monopolist supplier, that supplier sets $v_i$ unconstrained by any competitor.)

Both auction formats are known to yield the same outcome. The outcome of this tender process is the same as the complete information setting considered in section II, where suppliers set prices: potential suppliers bid down the offered unit price to $\min\{v_i^{u}, c^{(2)}\}$. The results of section II are therefore robust to assuming less information on the part of potential suppliers.

Bargaining

When the supplier bargains with downstream firms over input prices, a supplier does not set the price: the concept of being “constrained” is less relevant. (However it is not superfluous – as discussed further below.) Turning away from constrained suppliers, I briefly discuss general outcomes of vertical mergers with bargaining.

Bargaining is similar to price setting regarding effects on consumers: in principle the merger effects can go either way. The new US DOJ & FTC vertical merger guidelines explain how, with bargaining, disagreement payoffs change post-merger. The change allows the merged entity’s upstream unit to demand a higher input price from non-integrated downstream firms. Such that the merger results in RRC – and can harm consumers (U.S. Department of Justice & The Federal Trade Commission, 2020, p.7). Rogerson (2019) develops a tool to estimate the size of such effects, which he calls “bargaining leverage effect”, but in a partial rather than full equilibrium model (he leaves out EDM effects). In contrast, Das Varma and De Stefano (2020) show that the merged entity can have the incentive to lower the input price of its downstream rival (and customer). Such that, in equilibrium, the average downstream consumer price decreases.

However, that suppliers are in some way constrained is still relevant to bargaining models. When suppliers are not monopolists, the EDM effect may be only small. The

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50He simplifies the exercise by assuming that downstream prices are fixed during the input price negotiation. He motivates this assumption by arguing that upstream and downstream prices may be set simultaneously.
smaller the EDM effect, the less likely that the merged entity wants to lower it’s rivals’ cost. (See Das Varma and De Stefano, 2020 and Zenger, 2020.) Hence with competition upstream, bargaining models might indeed only yield RRC.

VII Endogenous ability to raise price, and when RRC does not constitute foreclosure

In other sections of this paper, the constraint on the merged entity is exogenous. It is given by \( \min \{ c^2, c_i \} \). If the supplier is constrained pre-merger then the merged entity cannot obtain a higher price post-merger. For reasons I consider in this section, the ability to raise price post-merger can – instead – be endogenous. Thus, despite the supplier being constrained pre-merger the merged entity can raise the input price post-merger.

For the general model I consider, Chen (2001) first showed that constraints can be endogenous to the merger. However, this remarkable finding by Chen has received little attention so far in the academic and applied Antitrust literatures. I now deal with his finding in detail and extend the discussion of its implications.

A Timing and commitment

In other sections of this paper, downstream firms choose where from to source inputs after they set consumer prices and hence after consumer demand is determined. In this section, consider a change in timing. Let downstream firms choose their suppliers after upstream producers set their prices in stage 1, but before downstream firms set consumer prices.

Pre-merger, downstream firms still choose the cheapest source of supply. The supplier, if it is constrained, sets unit price \( \min \{ c^2, c_i \} \) to downstream firm \( i \). However, post-merger the non-integrated downstream firm is willing to choose the merged entity as its supplier even if the merged entity sets a higher input price than \( \min \{ c^2, c_B \} \). To see this, consider some unit price \( v_B > 0 \). The non-integrated downstream firm strictly prefers to source the input from the merged entity at \( v_B \), rather than from an alternative supplier or in-house at cost \( v_B \), because doing so renders the merged entity less aggressive downstream. Therefore, even if the supplier is constrained pre-merger, the merged entity can raise the input price post-merger. In other words: when downstream firm \( B \) commits to its choice of supplier before consumer prices are set, it can affect downstream prices. Downstream firm \( B \) sources from the merged entity to elicit weakened horizontal competition. *Ceteris paribus*, the less aggressive the merged entity becomes through the stakeholder effect, the more the merged entity can raise the input price. In this way, with the altered timing the ability of the merged entity to raise the input price above \( \min \{ c^2, c_B \} \) is endogenous. Chen (2001) first identified and described this effect.

The unit price \( \min \{ c^2, c_B \} \) is still available to the non-integrated downstream firm
post-merger. In equilibrium, the merged entity sets $v_B$ to the level where downstream firm B is just indifferent between buying from the merged entity rather than from the next-best option. Effectively therefore, in equilibrium the post-merger increase in $v_B$ nullifies downstream firm B’s benefit from the stakeholder effect. Relative to the pre-merger case, the non-integrated downstream firm is worse off.51

In equilibrium, consumer prices can still increase or decrease (due to the EDM effect). But the “choice of supplier” effect is a further merger effect that harms consumers. The non-integrated downstream firm increases its consumer price not only due to the stakeholder effect, but also because its input cost increases.

B  This is different to what is usually thought of as foreclosure

In the academic literature as well as in Antitrust policy, RRC is referred to as foreclosure. Foreclosure of a downstream firm connotes harm to that firm. Indeed, RRC is believed to “disadvantage” (Baker et al., 2019) rival downstream firms. However, in this section the increase in cost to the non-integrated downstream firm can hardly be called foreclosure. When the supplier is constrained pre-merger, downstream firm B can still source the input at the same price as pre-merger, but does not want to. The non-integrated downstream firm is willing to pay a higher input price in exchange for weakened competition downstream, which enables it to set a higher consumer price – to the detriment of consumers.

Downstream firm B would be better off if it paid the merged entity $\min\{c^B, c_B\}$, as in other sections of this paper, but relative to alternative sources of supply downstream firm B chooses to pay a higher price to the merged entity with an anticompetitive motivation.

C  Antitrust remedies which do not solve the whole issue

Suppose an Antitrust authority imposes a remedy on a proposed merger whereby the merged entity cannot raise $v_B$ beyond the pre-merger level. Such a remedy does not prevent all harmful merger effects. The remedy protects consumers from RRC and the harmful “choice of supplier” effect. However, the stakeholder effect still materializes – downstream competition weakens. Consumer prices can increase despite the remedy. The change in consumer prices depends on the trade-off between benefits from EDM and harm from the stakeholder effect. Conversely, with such a remedy the change in downstream firm B’s profit is determined by the trade-off between its harm from EDM and its benefit from the stakeholder effect.

\[^{51}\text{It is strictly worse off if } v_A^* > 0, \text{ its profit does not change when } v_A^* = 0.\]
D Fixed costs

The merged entity is still able to raise the input price above $\min\{c^{[2]}, c_B\}$ if, instead of being fully committed to its chosen source of supply, the non-integrated downstream firm can change its choice for some fixed cost after downstream prices are set. In Chen (2001), downstream firms choose their supplier before setting their respective consumer price, but after consumer prices are set they can change their choice for some fixed switching cost. Chen (2001) motivates this assumption through relationship-specific investments. Pre-merger the most efficient supplier can extract no higher price than $\min\{c^{[2]}, c_B\}$. But post-merger, the merged entity can raise the input price because the non-integrated firm is willing to pay a higher input price to bring about the stakeholder effect. The lower the switching costs, the lesser the ability to raise price. (The results presented in other sections of this paper can be interpreted to hold when switching costs are negligible.)

Mathematically, a fixed cost to switch to another supplier is equivalent to considering that downstream firm B has a fixed cost to switch to in-house production.52 (Such fixed costs to move production in-house are considered in Katz, 1987 and Inderst and Valletti, 2009.) Thus, with fixed costs to switch to in-house production, even if there are no relationship-specific investments in the spirit of Chen (2001) the merged entity can still endogenously raise price.

VIII Conclusion

For environments where suppliers set prices, in this paper I develop a general relation between the upstream and downstream margins. The elasticity of demand upstream is lower than downstream due to three factors: to downstream cost absorption; to the strategic response of other downstream firms; and to the profit on sales diverted to any other downstream firm the upstream firm supplies. Using this relation, I develop a test for vertical mergers.

The underlying premise of numerous theoretical models, and to their use in practice, is that the upstream supplier has pricing power (or "market power"): the ability to raise the input price post-merger. But even with much market power – measured as high margins – the merged entity may be unable to engage in RRC. The test developed in this paper can be used to determine whether the upstream firm is able to raise a rival’s cost. This has implications for theory models. Models of vertical foreclosure in the existing literature

52 On the off-equilibrium path, there is a subtle difference between switching costs to another supplier and fixed costs to produce in-house. In Chen (2001), a downstream firm incurs the switching cost only if it chooses one supplier at the end of stage 1, but then orders from another at the end of stage 2. Consequently, if firm DB decides after the merger to source from the second-most efficient firm, and does not change its choice at the end of stage 2 of the post-merger game, firm DB does not incur any switching cost. In contrast, if firm DB chooses to source inputs in-house then it does incur a fixed cost.
can be consistent or inconsistent with data on industries they seek to portray. The test can be used to identify models that are consistent with observable facts.

Vertical merger theory and Antitrust agencies have largely focussed on RRC effects – which have been deemed to harm competitors. Opposite to this view, this paper argues vertical mergers can benefit all firms – including the non-integrated downstream firms – by reducing competition, increasing prices, and harming consumers. This has important implications for empirical studies. As is recognised in the literature, a downstream firm which integrates with a rival’s supplier gains a stake in that rival, and thus behaves less aggressively. Therefore, ownership of competitors’ suppliers can also contribute to rising margins. It would be interesting for further empirical work to assess the importance of such ownership in various industries, and in this way augment the GHHI index (on this index, see Azar, Raina, and Schmalz (2019)).

There are of course limitations to the test developed herein. If downstream firms can commit to choosing a supplier before consumer prices are set, then downstream firms can affect consumer prices through their choice. The merged entity can raise rivals’ costs even if the supplier is constrained pre-merger. The non-integrated downstream firm buys inputs from the merged entity – even at a higher price than what is offered by other producers – to elicit weakened competition downstream (the stakeholder effect). This is not foreclosure in the traditional sense: the non-integrated firm pays a higher input price to the merged entity due to a self-interested anticompetitive motivation rather than due to the lack of cheaper alternatives. This merger effect, due to Chen (2001), harms consumers. Absent foreclosure, consumer prices increase not solely because of the stakeholder effect, but also because the non-integrated downstream firm chooses to pay a higher input price.
APPENDIX

Proofs

In the appendix, I use the shorthand notation \( r_i = 2 - \frac{Q_i Q_i''}{Q_i'^2} \), where \( r_i \geq 1 \) as \( Q_i \) is log concave.

**Proof of Observation 1.** This follows from (5) and (6), and from Assumption 2. \( \square \)

**Proof of Theorem 1.** The theorem states that \( m_i^* u_i s_i \geq m_i^* i \), i.e. that

\[
- Q_i Q_i'' dP_i^* dv_i + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i} + \frac{\partial Q_j}{\partial P_i} \frac{dP_i^*}{dv_i} \geq - \frac{Q_i}{Q_j} Q_i'
\]

The proof proceeds in three steps. First, I solve for \( dP_i^* dv_i \) and \( dP_j^* dv_i \). Second I show \( \frac{dQ_i}{dv_i} < 0 \) and \( \frac{dQ_j}{dv_i} \geq 0 \), from which it follows that

\[
- v_j Q_i' Q_i'' dP_i^* dv_i + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i} + \frac{\partial Q_j}{\partial P_j} \frac{dQ_i}{dv_i} \geq 0
\]

Third, I show

\[
- \frac{Q_i}{Q_i' Q_i''} \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i} \geq - \frac{Q_i}{Q_j} Q_i'
\]

First, totally differentiating (7) with respect to \( v_i \) yields

\[
Q_i' dP_i^* dv_i + \frac{\partial Q_i}{\partial P_j} \frac{dP_j^*}{dv_i} + (P_i^* - v_i) Q_i'' dP_i^* dv_i + \frac{dP_i^*}{dv_i} (dP_i^* dv_i - 1) Q_i' = 0
\]

\[
\Rightarrow \frac{dP_i^*}{dv_i} = \frac{1 - \frac{1}{Q_i} \frac{\partial Q_i}{\partial P_j} dP_j^* dv_i}{2 - \frac{Q_i Q_i''}{Q_i'^2}}
\]

Similarly, one can calculate the total derivative

\[
\frac{dP_j^*}{dv_i} = \frac{1}{Q_j} \frac{\partial Q_j}{\partial P_j} dP_j^* dv_i \frac{1}{2 - \frac{Q_i Q_i''}{Q_i'^2}}
\]
Solving simultaneously yields
\[
\frac{dP_i^*}{dv_i} = \frac{2 - \frac{Q_i Q''_i}{Q_i^2}}{\left(2 - \frac{Q_i Q''_i}{Q_i^2}\right)\left(2 - \frac{Q_j Q''_j}{Q_j^2}\right) - \frac{1}{Q_i} \frac{\partial Q_i}{\partial P_j} \frac{1}{Q_j} \frac{\partial Q_j}{\partial P_i}} > 0
\]
\[
\frac{dP_j^*}{dv_i} = \frac{-\frac{1}{Q_j} \frac{\partial Q_j}{\partial P_i}}{\left(2 - \frac{Q_i Q''_i}{Q_i^2}\right)\left(2 - \frac{Q_j Q''_j}{Q_j^2}\right) - \frac{1}{Q_i} \frac{\partial Q_i}{\partial P_j} \frac{1}{Q_j} \frac{\partial Q_j}{\partial P_i}} > 0
\]

Second I show that \(\frac{dQ_i}{dv_i} < 0\) and that \(\frac{dQ_j}{dv_i} \geq 0\).

\[
\frac{dQ_i}{dv_i} = Q'_i \frac{dP_i^*}{dv_i} + \frac{\partial Q_i}{P_j} \frac{dP_j^*}{dv_i}
\]

Plugging in \(\frac{dP_i^*}{dv_i}\) and \(\frac{dP_j^*}{dv_i}\) yields

\[
Q'_i \frac{dP_i^*}{dv_i} - \frac{\partial Q_i}{P_j} \frac{1}{Q_j} \frac{\partial Q_j}{\partial P_i}
\]

The fraction above has a positive denominator, because \(r_j \geq 1, r_i \geq 1\) and, by Assumption 1, \(\frac{\partial Q_i}{Q_i} \frac{\partial Q_j}{\partial P_i} < 1\). Therefore, the sign of the fraction above is equal to the sign of the nominator. Evaluating the sign of the nominator yields

\[
\text{sign}\{\text{nominator}\} = -\text{sign}\{r_j - \frac{1}{Q_i} \frac{\partial Q_i}{P_j} \frac{1}{Q_j} \frac{\partial Q_j}{\partial P_i}\} < 0
\]

Therefore \(\frac{dQ_i}{dv_i} < 0\). Now I evaluate

\[
\frac{dQ_j}{dv_i} = Q'_j \frac{dP_j^*}{dv_i} + \frac{\partial Q_j}{P_i} \frac{dP_i^*}{dv_i}
\]

Plugging in \(\frac{dP_i^*}{dv_i}\) and \(\frac{dP_j^*}{dv_i}\) yields

\[
-\frac{\partial Q_j}{P_i} + \frac{\partial Q_j}{Q_j} r_j
\]

\[
\frac{2 - \frac{Q_j Q''_j}{Q_j^2}}{\left(2 - \frac{Q_j Q''_j}{Q_j^2}\right)\left(2 - \frac{Q_i Q''_i}{Q_i^2}\right) - \frac{1}{Q_j} \frac{\partial Q_j}{\partial P_i} \frac{1}{Q_i} \frac{\partial Q_i}{\partial P_j}}
\]

\[53\text{We have } \left|\frac{dP_i^*}{dv_i}\right| > \left|\frac{dP_j^*}{dv_i}\right| \text{ because the nominator of the former (weakly) exceeds 1 (due to log concavity of demand) while the nominator of the latter lies in }(0, 1)\text{ (as the consumer products are gross substitutes, Assumption 1).}\]
The sign of the above expression is equal to the sign of the nominator.

\[
\text{sign}\{\text{nominator}\} = \text{sign}\{(r_j - 1) \frac{\partial Q_j}{\partial P_i}\} \geq 0
\]

Therefore \(\frac{dQ_j}{dv_i} \geq 0\).

It follows that

\[
-\frac{v_j}{\frac{dQ_j}{dv_i}} \geq 0
\]

because the denominator \(\frac{dQ_j}{dv_i} < 0\) and the nominator \(\frac{dQ_j}{dv_i} \geq 0\).

Third and finally, I show:

\[
-Q_i \frac{dP^*_i}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP^*_j}{dv_i} \leq -\frac{Q_i}{Q_j} R
\]

Rearranging,

\[
Q'_i \leq Q'_i \frac{dP^*_i}{dv_i} + \frac{\partial Q_i}{\partial P_j} \frac{dP^*_j}{dv_i}
\]

Plugging in \(\frac{dP^*_i}{dv_i}\) and \(\frac{dP^*_j}{dv_i}\), and simplifying,

\[
\iff r_j Q'_i - \frac{\partial Q_i}{\partial P_j} \frac{1}{Q'_j} \frac{\partial Q_j}{\partial P_i} \leq r_j Q'_i - \frac{\partial Q_i}{\partial P_j} \frac{1}{Q'_j} \frac{\partial Q_j}{\partial P_i}
\]

\[
\iff r_j (r_i - 1) \leq 0
\]

because \(r_j \geq 1\), \(Q'_i < 0\) and \(r_i - 1 \geq 0\). Therefore \(m^u_{Si} \geq m^*_i\).

**Proof of Corollary 1.** By Theorem 1, if \(v_i^* = v_i^{u*}\) then \(m^u_{Si} \geq m^*_i\). Therefore \(m^u_{Si} < m^*_i \Rightarrow v_i^* \neq v_i^{u*}\).

**Proof of Proposition 1.** If \(m^u_{SB} < m^*_B\), then by Corollary 1 the supplier is constrained pre-merger. If the supplier is constrained it has \(v^*_B = \min\{c^{[2]}, c_B\}\) (rather than \(v^*_{B} = v^{u*}_{B}\)). Therefore, it cannot raise the price beyond \(\min\{c^{[2]}, c_B\}\).

**Proof of Observation 2.** If the supplier is constrained at \(c_B\), stopping to supply leads to no increase in price. If \(c^{[3]} = c^{[2]}\) then either \(c_B \leq c^{[2]}\), in which case the merged entity cannot raise price, or \(c_B > c^{[2]}\), in which case the second-most efficient supplier cannot raise price beyond \(c^{[3]} = c^{[2]}\). Thus stopping to supply does not raise the price paid by the non-integrated downstream firm.
Proof of Proposition 2. The following proves that the post-merger change in consumer welfare effect can go either way. Consumer welfare increases if the only merger effect is the EDM. This is the case when \( c_A > 0 \) (the merger results in an EDM effect) and \( c_B = 0 \) (there is no weakening of downstream competition. Conversely, consumer welfare decreases if the only merger effect is the weakening of downstream competition. This is the case when \( c_A = 0 \) (no EDM) and \( c_B > 0 \).

Proof of Proposition 3. To prove this Proposition, it suffices to show one example where the profit of downstream firm B rises by more than the combined profits of the supplier and downstream firm A. I provide such an example below.

Let \( Q_i = 1 - P_i + \frac{1}{2} P_j \), for \( i = \{A, B\} \). For concise workings, let \( c_A = 0 \). Let \( c_B = 0.1 \), with \( c^2 \geq c_B \).

Pre-merger, the supplier has profit \( \Pi_S = v_B Q_B \). And each downstream firm has profit \( \Pi_i = (P_i - v_i) Q_i \). The game is solved by backwards induction. The supplier’s monopoly price is 0.71. This unconstrained price exceeds \( c_B \), hence the supplier is constrained in the price it charges to downstream firm B: \( v_B = 0.1 \).

Post-merger, the merged entity unambiguously has an incentive to raise its rival’s cost (because there is no EDM). But the merged entity cannot raise \( v_B \), because it is constrained. Thus \( v_B \) remains at 0.1.

The table below provides pre- and post-merger figures on prices, quantities, sales (price times quantity), and profits.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-merger</th>
<th>Post-merger</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_A )</td>
<td>0</td>
<td>0</td>
<td>none</td>
</tr>
<tr>
<td>( v_B )</td>
<td>0.1</td>
<td>0.1</td>
<td>none</td>
</tr>
<tr>
<td>( P_A )</td>
<td>0.68</td>
<td>0.71</td>
<td>increase</td>
</tr>
<tr>
<td>( P_B )</td>
<td>0.62</td>
<td>0.73</td>
<td>increase</td>
</tr>
<tr>
<td>( Q_A )</td>
<td>0.68</td>
<td>0.66</td>
<td>decrease</td>
</tr>
<tr>
<td>( Q_B )</td>
<td>0.62</td>
<td>0.63</td>
<td>increase</td>
</tr>
<tr>
<td>( \Pi_S )</td>
<td>0.062</td>
<td>0.063</td>
<td>+0.001</td>
</tr>
<tr>
<td>( \Pi_A )</td>
<td>0.462</td>
<td>0.464</td>
<td>+0.002</td>
</tr>
<tr>
<td>( \Pi_B )</td>
<td>0.384</td>
<td>0.393</td>
<td>+0.008</td>
</tr>
</tbody>
</table>

From the table, \( \Pi_M - \Pi_S - \Pi_A \) is smaller than the post-merger increase in downstream firm B’s profit.

Remark. In the example above, the quantity-based and sales-based market shares of the merged entity’s downstream unit, \( \frac{Q_A}{Q_A + Q_B} \) and \( \frac{P_A Q_A}{P_A Q_A + P_B Q_B} \) respectively, decrease – hence the respective market shares of the non-integrated downstream firm increase.
Proof of Proposition 4. A diagonal merger does not lead to the EDM effect. If the merged entity continues to supply and if \( m^*_B < m^*_SB \), then by Proposition 1 the merged cannot engage in direct RRC. Therefore, the only merger effect is the weakening of downstream competition. It follows that a diagonal merger then benefits the downstream rival, and harms consumers (strictly so if and only if \( m^*_SB > 0 \)).

Proof of Proposition 5. The first part of the proof shows that \( \frac{dP^*_i}{dv_i} < 1 \). The second part shows \( m^*_u Si \geq m^*_i \).

Given the equilibrium pass-through rate \( \frac{dP^*_i}{dv_i} \) decreases with \( n \), I only need to show that for \( n = 2 \) and for any \( \phi \in \mathbb{R}^+ \) we have \( \frac{dP^*_i}{dv_i} < 1 \). Using \( n = 2 \),

\[
\frac{dP^*_i}{dv_i} = \left(\frac{2(2 + \phi)^2}{(4 + 3\phi)(4 + \phi)}\right)
\]

This is strictly smaller than 1. This is verified below:

\[
2(2 + \phi)^2 \leq (4 + 3\phi)(4 + \phi)
\]
\[
2(4 + 4\phi + \phi^2) \leq 16 + 16\phi + 3\phi^2
\]
\[
8 + 8\phi + 2\phi^2 \leq 16 + 16\phi + 3\phi^2
\]
\[
0 \leq 8 + 8\phi + \phi^2 \quad \text{which holds.}
\]

The second part of the proof proves \( m^*_u Si \geq m^*_i \). The FOC of downstream firm \( i \) yields

\[
\frac{\partial \Pi_i}{\partial P_i} = Q_i - (P_i - v_i)(1 - \frac{1}{n} + \phi)
\]
\[
\Rightarrow \frac{P^*_i - v_i}{1 - \frac{1}{n} + \phi}
\]

The optimal unconstrained (or monopoly) price of a supplier to downstream firm \( i \) solves

\[
\frac{\partial \Pi_S}{\partial v_i} = Q_i + v_i(Q'_i \frac{dP^*_i}{dv_i} + \sum_{j \neq i} \frac{\partial Q_j}{\partial P_j} \frac{dP^*_j}{dv_i}) = 0
\]
\[
\Rightarrow v^*_i = \frac{Q_i}{-Q'_i \frac{dP^*_i}{dv_i} + \sum_{j \neq i} \frac{\partial Q_j}{\partial P_j} \frac{dP^*_j}{dv_i}}
\]

To show

\( m^*_u Si \geq m^*_i \)

thus requires to show

\[
-Q'_i \frac{dP^*_i}{dv_i} - \sum_{j \neq i} \frac{\partial Q_j}{\partial P_j} \frac{dP^*_j}{dv_i} < 1 - \frac{1}{n} + \phi
\]
First note that

\[ Q_i' = 1 - \frac{1}{n} + \phi \]

Second, recall that the equilibrium pass-through rate \( \frac{dP^*_i}{dv_i} \) is always smaller than one (and positive) – see Proposition 5. Consequently,

\[ -Q_i \frac{dP^*_i}{dv_i} < 1 - \frac{1}{n} + \phi \]

Third and finally, the expression

\[ \sum_{j \neq i} \frac{\partial Q_j}{\partial P_j} \frac{dP^*_j}{dv_i} \]

is strictly positive. Because \( \frac{\partial Q_j}{\partial P_j} > 0 \) (this can be seen from the Shubik demand function), and because \( \frac{dP^*_j}{dv_i} > 0 \) (this is intuitive, it is shown in equation (31) in Zimmerman and Carlson, 2010).

\[ \square \]

References

Azar, José, Sahil Raina, and Martin C Schmalz (2019). “Ultimate ownership and bank competition”. In: Available at SSRN 2710252.


