Settlements in the Presence of Leniency Programs: Costs and Benefits+

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Abstract

Over the last few decades, leniency programs have been an important component of anti-cartel policies in many jurisdictions. An extensive literature shows how such programs can destabilize cartels and even discourage their formation in the first place. Much less studied are settlement policies under which reduced fines are offered to settling parties late in the prosecution (when the probability of conviction is high). In particular there has been little attention paid to the interaction of leniency and settlement policies. This paper examines whether the availability of late-stage settlements could negatively impact the effectiveness of early-stage leniency programs. Our main finding is that an appropriately designed settlement program can make collusion more difficult: in equilibrium, the adoption of an optimal settlement program by the Antitrust Authority reduces the occurrence of cartels by decreasing the long-run gain from collusion. However, an overly generous settlement policy may undermine leniency programs and encourage the formation of more cartels.

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1. Introduction

Over the last few decades, leniency programs have been central to the design of successful anti-cartel policies by antitrust authorities. By offering incentives in the form of fine reductions, the United States saw a dramatic increase in self-reporting by cartel members after the redesign of the Corporate Leniency program in 1993.\(^1\) Following this success, many jurisdictions have adopted similar policies to protect consumers from higher prices by inducing a reduction in collusive behavior, including the European Union and Canada.\(^2\). These policies aid in the detection of collusion, but also reduce the incentive to form a cartel in the first place.

Leniency policies offer reduced fines, or in some cases complete immunity to a recipient firm that comes forward with information leading to the detection and/or prosecution of firms for cartel conduct in which the recipient was itself a participant. As enforcement policies, they work by offering firms an incentive to self-report collusion to the Antitrust Authority (AA). To be eligible for leniency firms are typically required to self-report before the onset of an investigation or, under certain circumstances, during the early stages of an investigation. In many cases, the only eligible recipient of the reduced fine (or full immunity) is the first firm to come forward.

Although leniency is an important component of anti-cartel policies, in this paper we draw attention to a relatively unexplored area of price fixing prosecutions – settlements and plea bargaining. Settlements, as we study them here, differ from leniency in that they tend to occur in

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1 See, United States of America (1993).
2 See, for example, Canada (2019) and the European Union (2006). Information on the growing experience with leniency programs internationally can be found on the websites of the International Competition Network (ICN), e.g. ICN (2014 and 2019) and the Organization for Economic Cooperation and Development (OECD), e.g. OECD (2001 and 2002). See, also Spagnolo (2008). More references to literature on leniency are provided below.
the later stages of a legal proceeding and are commonly offered to speed up final resolution and reduce all parties’ legal costs.  

Since settlements as an enforcement tool predate modern leniency programs, an important question is whether the effectiveness of the newer leniency programs is affected by the presence of settlement policies. It is a question being considered by competition authorities as well. For example, the International Competition Network (2008a at 40) observes: “… the possible reductions of fine that can be obtained from leniency and from settlement have to be carefully balanced in order to ensure consistency and reciprocal strengthening between the two instruments”. Similarly, the Organization for Economic Cooperation and Development (2008 at 4) cautions: “Depending on how leniency programs are structured, and the parameters of settlements, the relationship between both settlements and leniency programs can raise difficult questions.”

To address the question, we study how settlements impact the incentives generated by a leniency program. Specifically, we would like to explore whether the ability to settle, in the later stages of a legal proceeding, reduces the incentives to apply for leniency in the beginning stages of a price fixing trial. Put another way, we are interested in whether, and to what extent, leniency and settlements are complements or substitutes from an enforcement perspective.

We present a theoretical model that explores the interaction between leniency programs and settlement policies. Our analysis proceeds in two parts. First, we study the conditions under which settlements 

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3 To be clear, what are called settlement policies can differ across countries. On this see, e.g., Guttuso (2015). In some jurisdictions (e.g. the United States) settlements can operate more like extensions of the leniency program, offering reduced fines to defendants who come forward relatively soon after leniency has been awarded to another. Our approach of treating settlements as coming later, and with the objective of securing procedural efficiencies, is closer to that of the European Commission (Guttuso (2015) at 397). See also Schinkel (2011).

4 See generally International Competition Network (2008a at 39-40, and 2008b at 7-8).

5 It goes on, “For example, some jurisdictions expressed a concern that settlement discounts must be calibrated carefully and take account of the specificities of leniency regimes in order to avoid the risk of granting settlement discounts in a range that would reduce the incentives of companies to come forward and cooperate under leniency.”
which collusion occurs in equilibrium under a leniency program in the absence of any settlement policy (base case). Second, we introduce a settlement policy and explore the conditions that permit collusion as an equilibrium outcome in this case. Comparing the settlement policy case to the base case provides insights regarding the impact of settlement policies on the incidence and duration of collusive conduct.

The primary finding of our model demonstrates that a well-designed settlement policy can increase the effectiveness of an anti-cartel policy. We do find that settlements have both pro-collusive and anti-collusive effects. On the one hand, a settlement can increase incentives to collude by lowering the expected cost of forming a cartel, just as leniency can. On the other hand, a well-designed settlement eliminates the need for a trial, and in a sense, acts as a guaranteed conviction, ending the cartel – this reduces the incentives to collude. Furthermore, we find that leniency and settlements can be complementary law enforcement policies. Leniency acts directly on the incentives to defect from the cartel while settlements act more directly on the incentives to collude. Once the AA reaches the lower bound of leniency (full amnesty from fines) there is an additional anti-collusive effect that can be achieved through a settlement policy.

We classify the effects of a settlement policy as follows. First, the settlement can be viewed as a valuable option for colluding firms: they can take a “wait and see approach”, waiting until the conclusion of an investigation to decide if they want to self-report. This increases the value from forming a cartel and is pro-collusive. However, a generous settlement policy has the effect of guaranteeing the collapse of the cartel if an investigation is opened. In other words, in a given period, the probability of cartel collapse is higher with settlements. This decreases the value from forming a cartel and discourages collusion. If the settlement is chosen optimally, we can guarantee the second effect dominates the first, thus leading to an anti-collusive settlement policy. However,
if it is not chosen carefully, settlements can in fact undermine the effectiveness of the overall enforcement policy, leading to more collusion.

The next section of the paper briefly reviews the most relevant literature, covering the areas of leniency policies and plea-bargaining/settlements research. Section 3 lays out the model and describes the timing of our game. The main results of the paper are presented in Section 4, first for the case of leniency with no settlements policy and then for an optimal settlements policy with leniency policies in place. Section 5 considers the harm that can be done by non-optimal settlement policies. Section 6 then offers some discussion and our conclusions.

2. Literature

The aim of this paper is to fill the gap between the literature on leniency programs and settlements policies. While leniency programs of the sort we model here are unique to anti-cartel policy, settlements and plea bargaining arise in many areas of law. The defining differences between the two, as we define them for our purposes here, is timing and intent. The leniency program, in most cases, occurs either before or during the early stages of an investigation. The intent of the leniency program is to destabilize cartels and to, in that way, act as a deterrent to forming a cartel. On the other hand, settlements occur in the later stages of a legal proceeding. The main intent of settlements is typically to expedite the judicial process and free up legal resources. Settlements are usually the result of plea bargaining between the prosecution and the defendant.

Important work on the theory of leniency programs can be found in Motta and Polo (2003), Spagnolo (2004), Aubert et al. (2006), Feess and Walzl (2004), Motchenkova (2004), Chen and Harrington Jr (2007), Harrington Jr (2008), and Chen et al. (2015), among others. The consensus in the literature is that the presence of leniency generates tradeoffs. In one direction, the ability to

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6 Spagnolo (2008) provides a nice survey of the earlier literature on leniency programs.
apply for leniency is a valuable option that a firm can use under mounting pressures from the AA. This can reduce the expected punishment firms face. However, the presence of leniency also introduces the risk that other members of the cartel may self-report, raising the probability of punishment. The overall effect of the leniency program is determined by the strength of these two effects. This finding holds across the various models specified within the literature.

The seminal work on leniency programs is that of Motta and Polo (2003). They show that, on the one hand, leniency programs can indeed destabilize cartels. However, since leniency programs reduce the expected fine from colluding, a poorly designed leniency program may have a pro-collusive effect. Another important finding is that a policy which allows leniency before and after the onset of an investigation dominates a policy that offers only pre-investigation leniency. Harrington (2008) uncovers an additional channel whereby leniency impact the incentives to collude. This additional channel, the “Race to the Courthouse Effect”, helps provide a rationale for offering leniency to only the first firm that applies.\(^7\)

The second strand of relevant literature relates to legal settlements and plea bargaining. The literature can be separated into two categories. First, Landes (1971), Gould (1973), and Adelstein (1978) provide an incentives-based approach to understanding the theory of settlements.\(^8\) These models assume that the main purpose of settlements is to reduce the economic burden of going to trial. The second category builds on the work of Shavell (1982a and 1982b) which considers a single agent decision theory model of settlements. Later work by Grossman and Katz (1983), P’ng (1983), Reinganum (1988), Bebchuk (1984), Baker and Mezzetti (2001), and

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\(^7\) More recently, Chen et al. (2015) consider the inclusion of “no immunity for the instigator clauses” (NIICs) in leniency programs. Their main finding is that the inclusion of a NIIC in the leniency program can either strengthen or weaken deterrence.

\(^8\) In particular, the theoretical analysis by Landes (1971) finds that the decision to settle depends on the perceived severity of the crime (as measured by the sentence if convicted at trial), the probability of conviction, and the relative cost of a trial versus a settlement.
Kim (2010) introduce strategic interactions. This approach provides for a richer set of justifications for settlements. The literature on settlements has found that, even if the economic burden of trial is zero, settlements can insure against the risk of wrongful convictions and aide in the screening of guilty/innocent defendants. In our model, we point out that settlements can reduce the economic burden of the AA but we focus on the ability of settlements to eliminate type II judicial errors (failing to convict guilty defendants) which reduces the incentive to collude.

This introduces the two independent strands of literature that form the basis of our theoretical analysis. On the one hand, we have seen that early-stage leniency programs have been introduced as a means of detecting cartels, but also to aid in the deterrence of collusive behavior. With respect to late-stage settlements, the literature suggests that their main purpose is to reduce the prosecutorial burden of continuing with a lengthy and costly trial. Our contribution to the literature is a formal model of strategic behavior which considers the effect that settlement programs have on collusive behavior in the presence of leniency programs. Since settlements predate leniency programs and have been applied across many areas of law, it is natural to question whether settlement programs/methods should be adapted to accommodate the newer leniency programs.

3. Model Description

Our model builds on the framework developed by Motta and Polo (2003). The major modification we make is to allow for the possibility of late-stage settlements. In our model, we examine the effect of various settlement policies on the incentive to collude and compare this to a baseline case – no settlements.

For our purposes here, we define a settlement as a binding contract between the defense and prosecution that offers a lower fine in return for a guilty plea. The defendant can only accept
or reject that offer and there are no further negotiations possible. Importantly, we also assume that
the acceptance of a settlement by one party does not provide any additional information that can
impact the strength of the prosecution’s case against the other members of the cartel.⁹ Of course,
settlement agreements can in practice take many forms and some might involve situations in which
one firm’s settlement raises the probability of conviction of other participants, but in such a case
the settlement is a form of late and (potentially) less generous leniency. We make these
assumptions about settlements to provide a clearer distinction between the policy tools.

In our analysis, the AA is a relatively passive player in the sense that it will set some key
policy parameters related to leniency and settlements at the start and then through the course of
the game apply those policies mechanically. The settlement and leniency policies will be set to
minimize the incidence of collusion. Our analysis culminates in a ‘comparative statics’ exercise
which compares the incentive to collude with and without settlement policies.

Firms

In our model, two symmetric firms play an infinitely repeated game. The objective of each
firm is to maximize its profits taking into consideration the strategic decisions of the other firm.
Firms discount future payoff streams with a discount factor δ. We assume that the payoffs in the
static stage game are the same in all periods. If both firms play their (static) Nash equilibrium
strategy, they each receive payoff \( \pi^N \). We allow for the possibility of collusion between the two
firms. If the firms collude and jointly set their strategic variable, they can each achieve a per-period
profit of \( \pi^M \). Once firms commit to collusion, defection can occur at some point in the future. If a
firm defects from the agreement it will receive \( \pi^D \) that period while the other firm will receive \( \pi^S \).

⁹ Leniency programs, by contrast, typically require the recipient’s cooperation to secure the conviction of other
defendants.
If both firms defect from the agreement, they each receive $\pi^B$. To ensure that the static stage game has the familiar Prisoner’s Dilemma form, we assume the following ordering of payoffs:

**Assumption 1.** The ordering of payoffs is defined as follows: $\pi^D > \pi^M > \pi^N \geq \pi^B > \pi^S \geq 0$.

The firms play a grim trigger strategy. If any of the following events happen at time $t$, both firms will revert to static Nash strategies from time $t + 1$ on forever: (i) one or both firms defect; (ii) one or both firms apply for leniency; (iii) one or both firms accepts a settlement; or (iv) a trial results in conviction(s). To make our problem interesting requires that collusion be profitable and stable in the absence of any anti-cartel enforcement. In the absence of an AA, collusion is an equilibrium strategy in an infinitely repeated game if the following condition holds:

$$\pi^D + \sum_{t=1}^{\infty} \delta^t \pi^N < \sum_{t=0}^{\infty} \delta^t \pi^M. \quad (1)$$

Collusion can be sustained in equilibrium if the discounted present value of the perpetual profit stream $\pi^M$ (right-hand side of (1)) is greater than the one-shot gain from deviating plus the discounted present value of the perpetual profit stream $\pi^N$ (left-hand side). It is convenient to express Condition (1) in terms of the discount factor:

$$\delta > \frac{\pi^D - \pi^M}{\pi^D - \pi^N} \equiv \delta_0. \quad (2)$$

From Condition (2), in the absence of an AA, if firms are sufficiently patient, that is $\delta > \delta_0$, collusion can be sustained as an equilibrium outcome. In other words, if the AA did not exist (no fine and no leniency) the minimum (critical) discount factor that would permit collusion as an equilibrium outcome in a repeated game is $\delta_0$.

In the presence of an AA, firms that collude face the prospect of being investigated and successfully prosecuted. At each period, we assume the AA opens an investigation with probability $\alpha$. Subsequent to an investigation, if there are no leniency applicants and a cartel exists, a trial
results in a successful conviction with probability $\rho$.\textsuperscript{10} If firms are convicted they are required to pay the full fine $F$. The parameters $\alpha$, $\rho$, and $F$ are exogenous and common knowledge from the beginning of the game.

The critical value for the discount factor must take into consideration the probability of cartel collapse. This will raise the critical discount factor. At this point, we assume:

**Assumption 2.** $\delta > \frac{\delta_0}{1-\alpha}$

Assumption 2 is a necessary condition for collusion to be sustainable when there is no leniency program, no trial cost and guaranteed conviction (upon investigation).\textsuperscript{11}

We also assume that there is a pre-investigation leniency program in place. Before the initiation of an investigation (early stage), firms have the choice of revealing the existence of the cartel to the AA. Firms that provide incriminating evidence to the AA will pay a reduced fine $L$. Consistent with much of the literature (and the practice in some countries), we allow only the first applicant to be eligible for leniency. As such, in cases where both firms simultaneously apply, we assume that the AA randomly awards leniency to one of them. We also make the following assumption regarding the value of the reduced fine:

**Assumption 3.** The reduced fine under leniency, $L$, is non-negative.

With Assumption 3, consistent with practice in most of the antitrust world, leniency applicants or whistleblowers cannot receive monetary rewards for their cooperation.\textsuperscript{12}

\textsuperscript{10} In the model, if both firms are on trial either both will be convicted or neither will be convicted. Also we assume there will be no false positives in the model – firms that do not collude run no risk of prosecution and/or conviction.

\textsuperscript{11} To be more specific, Assumption 2 can be derived from the counterpart of (1) for the situation where a cartel is caught and convicted with probability $\alpha$ in each period. In that situation, a firm’s expected payoff from collusion is $V^C = [\pi^N + \alpha \frac{\delta}{1-\delta} \pi^N]/[1 - (1 - \alpha) \delta]$, while its payoff from defection is $V^D = \pi^D + \frac{\delta}{1-\delta} \pi^N$. The cartel is sustainable if $V^C > V^D$. Solving this inequality for $\delta$ yields the condition in Assumption 2.

\textsuperscript{12} Though the list of countries that offer rewards for participants coming forward to reveal cartel conduct is growing, it is still relatively small, including for example, the UK, South Korea, Hungary and Pakistan. See, e.g. Financier Worldwide’s report (August 2020): https://www.financierworldwide.com/whistleblower-programmes-making-cartel-detection-more-effective.
If and when the settlement stage is reached, we assume that the AA makes a take-it-or-leave-it offer to each firm which is then either accepted or rejected. If at least one firm rejects the settlement the AA proceeds to trial (against the non-settling firms); otherwise the settlement is accepted by both firms and the game ends with Nash reversion in all future periods.

To avoid later confusion, we define a few related terms. First, we refer to the *settlement policy* as the policy tool that permits the AA to settle with the defendant.\(^{13}\) Second, we refer to the *settlement offer* as the take-it-or-leave-it offer to settle made by the AA to the defendant.\(^{14}\) Finally, we refer to the *settlement* as the amount paid by the defendant if it accepts the settlement offer.

The settlement takes the form of a reduced fine, \(S_i\), for firm \(i\).

*The Game*

The timing of the game is as follows:

In Period 0 the AA decides to adopt, or not, a *settlement policy*. It also sets the value of the reduced fine under leniency, \(L\). These decisions are observed by all firms.

In Periods 1 onward, the following stages are observed:

**Stage 1:** Firms decide whether to collude. A cartel is formed only if every firm agrees to collude. If at least one firm refuses, firms compete non-cooperatively and the game moves to the next period. If firms collude, the game proceeds to the next stage.

**Stage 2:** Firms simultaneously and independently decide whether to reveal the cartel to the AA (i.e. apply for leniency) and/or whether to defect from the collusive agreement. In this stage, firms choose from the following four actions:

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\(^{13}\) This policy, which is not specific to a particular case, may be formalized in the form of regulations or guidelines, but for our purposes it is really just a credible statement that the AA will (or will not) make settlement offers after an investigation. Settlement offers are determined on a case by case basis by the AA.

\(^{14}\) For expositional ease, we will treat the AA as if it is the prosecuting authority here. In fact, in some jurisdictions, prosecutions are led (particularly in criminal matters) by a public prosecutor independent of (though typically supported by) the AA.
1. Collude and Not Reveal (denoted C/NR)
2. Collude and Reveal (C/R)
3. Defect and Not Reveal (D/NR)
4. Defect and Reveal (D/R)

Stage 3: If the firms collude and a leniency applicant comes forward (reveals) the cartel ends and firms revert to the static Nash equilibrium indefinitely. The leniency applicant pays the leniency level of fine, $L$ while the other firm is convicted with certainty and pays the full fine $F$. However, if there is no leniency applicant, the AA opens an investigation with probability $\alpha$.

Stage 4: At the end of the investigation, the AA observes whether collusion has taken place, but this does not guarantee conviction. If collusion has not taken place, the AA simply closes the case and the game moves to the next period (i.e. there are no wrongful convictions). If collusion has taken place and the AA has a settlement policy, it makes settlement offers to both firms. These offers are not necessarily identical. If the AA does not have a settlement policy, the case proceeds to trial.

The game moves to the next period if all firms accept the settlement offers with both firms reverting to Nash play forever. Otherwise, the game proceeds to the next stage.

Stage 5: The AA begins a trial against firms that have not accepted a settlement. In this stage, firms can plead guilty and pay the full fine $F$. By pleading guilty firms avoid paying legal fees $T$. However, if firms proceed to trial they face both the full fine $F$ in addition to those legal fees $T$.15

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15 To be clear, the costs $T$ must be paid if a defendant goes to trial, whether or not it wins. Recognizing that the AA also faces a cost of going to trial, though very reasonable, would complicate the AA’s objective function, so we rule
At this point, we make assumptions on the values of $T$, $\rho$, and $F$.

**Assumption 4.** $F > \rho F + T$.

We assume that the cost of legal proceedings $T$ are relatively small when compared to the full fine $F$. Alternatively, Assumption 4 also implies that, for a given value of $T$, the probability of conviction in the absence of a leniency applicant is sufficiently low. The implication of Assumption 4 is that, in the absence of a settlement policy, firms would choose to proceed to trial rather than pleading guilty (and paying $F$).

The realized probability of conviction depends on the number of leniency applicants. If there are no leniency applicants, the probability of conviction is $\rho$. On the other hand, if there is at least one leniency applicant, we assume that the probability of conviction is 1. The rationale for the latter condition is that, as part of leniency, firms are normally expected to provide evidence that can substantially increase the probability of securing a conviction against the other firms. Finally, we assume that the probability of conviction is invariant to the number of firms that have accepted a settlement. The rationale for this derives from our assumption that there is no cooperation requirement in exchange for a settlement. As such, the acceptance of the settlement will have no impact on the proceedings against the other firms who do not accept the settlement.

As is standard with such dynamic games, we apply the method of backward induction to solve for the set of subgame perfect Nash equilibria. Since there is a possibility of multiple equilibria, when needed we apply the payoff dominance criterion as proposed by Harsanyi and Selten (1988).
4. Model Solution

We present the solution to this model in three steps. In Section 4.1 we consider the game without the possibility of settlement; in Section 4.2 we add settlements as a policy option for the AA; and in Section 4.3 we compare the results to determine whether the AA should commit to a settlement policy in Stage 0.

To facilitate exposition, we begin with a result regarding leniency that holds in both our non-settlement and settlement treatments. In our model, leniency destabilizes the cartel by increasing the incentive for colluding firms to reveal themselves to the AA. In fact, in our model, the incentive to defect and reveal is strictly increasing in the degree of leniency and does not impact the incentives to collude and not reveal. As such, it is always in the best interest of the AA to offer full leniency. Therefore, we have:

**Proposition 1.** Offering full leniency by setting $L = 0$ has the greatest deterrence effect in both the settlements and no settlements case.\(^{16}\)

4.1. No Settlement Policy

4.1.1. Stage 5 – Trial (Without Settlements)

We start by solving the subgame in stage 5. Firms that have not received leniency in stage 3 will stand trial in stage 5. If the other firm was a successful leniency applicant, the firm can choose to plead not guilty and face a (negative) payoff of $-(F + T)$ or plead guilty and face payoff $-F$. In this case the firm will choose to plead guilty to avoid the cost associated with a trial. On the other hand, if there are no leniency applicants, Assumption 4 implies that the firm will choose to plead not guilty and continue with the trial.

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\(^{16}\) Proofs for all propositions and lemmas can be found in Appendix B.
4.1.2. Decision to Proceed to Trial and Open an Investigation in Stage 4/3

In stage 4, after an investigation, the AA must decide whether to proceed to trial. The AA will proceed to a trial if it finds evidence of collusive behavior. By assumption here, if there is a cartel the investigation will uncover it, though not necessarily generating enough evidence to secure a conviction. Moving back to stage 3, as part of regular monitoring, the AA will open an investigation with probability $\alpha$. The goal in this case is to exert its presence (deterrence) and catch unsuspecting cartels (prosecution).

4.1.3. Collusion/Revelation Game Stage 2

In stage 2, each firm simultaneously chooses from the following set of pure actions \{ $C/NR, C/R, D/NR, D/R$ \}. As discussed earlier, the only strategy that does not immediately trigger Nash reversion is when both firms play $C/NR$. If firms play $C/NR$, the only time we revert to the Nash equilibrium is if firms are investigated and successfully prosecuted. This situation occurs with probability $\alpha\rho$. We can express the value from colluding and not revealing as follows:

$$V^{CNR} = \pi + \alpha\rho \left( \frac{\delta}{1 - \delta} \pi^N - F \right) - \alpha T + (1 - \alpha\rho)\delta V^{CNR}.$$ (3)

We can interpret Equation (3) as follows: firms that collude and do not reveal to the AA will receive collusive profits of $\pi^M$ in the period. However, the AA may open an investigation with probability $\alpha$. In which case, the firms will proceed to trial since the expected cost of a trial and conviction is less than the cost of pleading guilty. Therefore, there is an $\alpha\rho$ probability that firms are found guilty, pay a fine $F$ and revert to the static Nash equilibrium in all future periods. Since firms pay the legal fees when going to trial (win or lose) there is an $\alpha$ probability that the firms will incur legal costs of $T$. Finally, the last term represents the possibility that the cartel will not be convicted and will continue to operate next period. We can solve Equation (3) as follows:
\[ V^{\text{CNR}} = \frac{\pi^M + \alpha \rho \left( \frac{\delta}{1-\delta} \pi^N - F \right) - \alpha T}{1 - (1-\alpha \rho)\delta}. \] (4)

To cover the other possible strategies, if one firm applies for leniency it will face a fine of \( L = 0 \) while the other firm faces the full fine \( F \) (pleading guilty to avoid paying legal fees). If both firms apply for leniency they face an expected fine of \( \frac{F}{2} \). If neither firm reveals they face an expected cost of \( \alpha (\rho F + T) \). We depict the stage game using the normal form representation in Appendix A - Table A1. When \( F \) is sufficiently small there are two Nash equilibria -- namely with both playing C/NR and with both playing D/R. However, when \( F \) is large there is a single Nash equilibrium – in which both play D/R. The payoff dominant Nash equilibria of the normal form stage game are defined by Lemma 1.

**Lemma 1.** Let \( \hat{F}(\alpha) \equiv \frac{\left( (\pi^D - \pi^N)(\delta(1 - \alpha \rho) - \delta_0) - \alpha T \right)}{\alpha \rho}. \) In the absence of a settlement policy the payoff dominant equilibria of the Stage 2 subgame can be classified as follows. If \( F \leq \hat{F}(\alpha) \) then C/NR is payoff dominant. If \( F > \hat{F}(\alpha) \) then D/R is the only Nash equilibrium.

We find that if the full fine \( F \) is sufficiently small C/NR will arise as payoff dominant equilibrium in the Stage 2 sub-game.

**4.1.4. The Decision to Collude in Stage 1**

We consider the conditions under which a cartel is formed in equilibrium. If the outcome in stage 2 is D/R the firms will not form a cartel. We can see this explicitly by comparing the value from not colluding (NC) and D/R:

\[ NC \equiv \pi^N + \frac{\delta}{1-\delta} \pi^N > \pi^B + \frac{\delta}{1-\delta} \pi^N. \] (5)

This makes intuitive sense, if the firms know that defection occurs in stage 2 there is no incentive to form a cartel in stage 1. This follows directly from the fact that \( \pi^N \geq \pi^B \). Therefore, if \( F > \)
\(\hat{p}(\alpha)\) collusion cannot occur in equilibrium. However, if \(F \leq \hat{p}(\alpha)\) then collusion can occur as an equilibrium outcome if the value from playing collude and not reveal is greater than the value from not colluding \((V_{NC})\). This is illustrated by the following condition:

\[
\begin{align*}
V_{NC} \leq V_{CNR} & \Rightarrow \pi^N + \frac{\delta}{1-\delta}\pi^N \leq \frac{\pi^M + \alpha\rho \left(\frac{\delta}{1-\delta}\pi^N - F\right) - \alpha T}{1 - (1-\alpha\rho)\delta}.
\end{align*}
\] (6)

We can express Equation (6) in terms of \(F\):

\[
F \leq \frac{(\pi^M - \pi^N) - \alpha T}{\alpha\rho} \equiv \hat{F}_{IR}(\alpha)
\] (7)

Formally, \(F \leq \hat{p}(\alpha)\) is the familiar incentive compatibility constraint (IC) and \(F \leq \hat{F}_{IR}(\alpha)\) is the individual rationality constraint (IR). As such, if the IC and the IR conditions hold, collusion can occur as an equilibrium outcome. Notice that for C/NR to occur in equilibrium, \(V_{CNR} \geq \pi^D + \frac{\delta}{1-\delta}\pi^N\). Furthermore, we know that \(\pi^D + \frac{\delta}{1-\delta}\pi^N > \pi^N + \frac{\delta}{1-\delta}\pi^N\). Based on this, if the IC condition holds, then the IR condition will also hold (i.e. the latter is redundant).\(^{17}\)

We plot illustrations of condition \(\hat{F}_{IR}(\alpha)\) and \(\hat{p}(\alpha)\) in Figure 1\(^{18}\). The highlighted region, all else being equal, are the values of \(F\) and \(\alpha\) where collusion occurs in equilibrium (with no settlement policy). The area to the left of the vertical dashed line represents values that satisfy Assumption 2, involving discount factors large enough to support collusion.

\(^{17}\) To explain this important result – the redundancy of the IR constraint – another way, we need only note that if the other firm is playing C/NR then playing D/R is always more profitable than the case of no collusion. In this case the defector gets the defector’s payoff and by revealing (and getting leniency) avoids any punishment. If C/NR is an equilibrium, it must be the case that playing C/NR would then be even more profitable for the firm than D/R. Hence, when both firms playing C/NR is an equilibrium to the subgame, it must be profitable, hence the IR constraint can not be binding. The exact same argument can be applied to the game with settlements that follows.

\(^{18}\) For the purposes of presenting this graph we have set the following parameter values: \(\pi^D = 10\), \(\pi^M = 5\), \(\pi^N = 8\), \(\delta = .9\), \(\rho = .5\), \(T = 0\).
Figure 1: Collusion Region Under No Settlements

\[ \alpha < 1 - \frac{\delta_0}{\delta} \]

**Proposition 2.** In the absence of a settlement policy, collusion will occur in equilibrium if \( F \leq \bar{F}(\alpha) \). In other words, if the full fine \( F \) is sufficiently low, then collusion can be sustained as an equilibrium outcome.

**4.2 Settlement Policy**

It is first important to recognize that whether we ever reach the settlement stage of the game depends on actions taken up to that point. If the firms do not come to a collusive agreement at Stage 1, or if they do agree and at least one firm subsequently reveals there is no need for the AA to offer settlements.\(^{19}\) Therefore, the AA and colluding parties will only find themselves playing

\(^{19}\) Technically, if one firm chose to reveal, the AA could offer a settlement to the remaining firm but this would serve no purpose. In this case the conviction is assured which means the best the other firm can do is confess to reduce its own legal costs. Offering a reduced fine via settlement then, can only reduce the deterrence effect of the anti-cartel policy.
the settlements subgame if both firms had chosen either C/NR or D/NR. With any other choices the game would proceed as the game with no settlements.

4.2.1 The Settlement Stage

In the settlement stage the AA will offer each firm $i (= 1,2)$ a settlement offer $S_i$. As with leniency, we restrict settlement offers to be non-negative; that is, defendants will not be paid to settle. In this stage the objective of the AA is to induce firms to accept the settlement offer as this will guarantee the death of the cartel. Furthermore, since we assume that there are no cooperation requirements associated with settling, inducing both firms to accept is optimal in the sense that it guarantees the collapse of the cartel while foregoing a lengthy trial.\(^{20}\) However, as they represent reductions in fines, settlements have a pro-collusive effect that is increasing in the generosity of the settlement offer: as the AA lowers the settlement amount (i.e. makes the settlement more generous) it increases the expected value from playing C/NR. The key for the AA, then, will be to make offers generous enough to be accepted, ending collusion from that point on, but not more generous than necessary as this increases the profitability of collusion.

If an investigation has been opened and there are no leniency applicants, the firms proceed to the settlement stage. Payoffs in the subgame at the settlement stage will depend on the collusion/defection decisions that preceded it. In Table 1, we present the settlement subgame payoffs for the case after both firms had decided to collude and not reveal. The remaining cases are presented in Appendix A – Table A2. Notice that there are four games, each representing a situation where there has not been a leniency applicant.

---

\(^{20}\) Strictly speaking, as we have effectively chosen the AA’s action to minimize cartel activity (and not necessarily to minimize its costs or increase fine revenues) we could be indifferent between getting one firm to settle and securing settlements from both. It seems reasonable to break this tie by assuming the AA choses the lower cost alternative.
<table>
<thead>
<tr>
<th></th>
<th>Accept</th>
<th>Reject</th>
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<tbody>
<tr>
<td><strong>Accept</strong></td>
<td>(\pi^M + \frac{\delta}{1-\delta}\pi^N - S_1)</td>
<td>(\pi^M + \frac{\delta}{1-\delta}\pi^N - S_1)</td>
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<td></td>
<td>(\pi^M + \frac{\delta}{1-\delta}\pi^N - S_2)</td>
<td>(\pi^M + \frac{\delta}{1-\delta}\pi^N - \rho F - T)</td>
</tr>
<tr>
<td><strong>Reject</strong></td>
<td>(\pi^M + \frac{\delta}{1-\delta}\pi^N - \rho F - T)</td>
<td>(V_{CNR}^R)</td>
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<tr>
<td></td>
<td>(\pi^M + \frac{\delta}{1-\delta}\pi^N - S_2)</td>
<td>(V_{CNR}^R)</td>
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Table 1: Normal Form Game in the Settlement Stage

Let \(V_{CNR}^R\) represent the value of \(C/NR\) in stage 4 if both firms reject the settlement offer.

We assume that the AA does not know which strategy was played by the firms in stage 2. As such, the AA wants to induce firms to accept the settlement independent of the strategy. The value of \(C/NR\) (in the settlement stage) to each firm without the acceptance of a settlement offer is as follows:

\[
V_{CNR}^R = \pi^M + \rho \left( \frac{\delta}{1-\delta} \pi^N - F \right) - T + (1 - \rho) \delta V_{CNR}.
\]  
(8)

Solving Equation (8) we arrive at the following:

\[
V_{CNR}^R = \frac{(1 - \delta \rho (1 - \alpha))\pi^M + (1 - \delta (1 - \alpha)) \left( \rho \left( \frac{\delta}{1-\delta} \pi^N - F \right) - T \right)}{(1 - \delta (1 - \alpha \rho))}.
\]  
(9)

As noted, Equation (8) reflects the case where the AA offers a settlement that is rejected by both firms. Firms proceed to trial with probability \(\rho\) of being convicted. If they are not convicted, they will continue colluding. Therefore, the expected value from not being convicted at trial after rejecting the settlement is \(V_{CNR}^R\): if the firms were to reject the settlement offer today, they would
continue to reject the settlement in the future. Hence, the firm’s continuation payoff is the same as the one without settlements.

From Table 1 and Table A2 (in the Appendix) we see that in this case with collusion but no leniency applications, the AA can induce a single firm $i$ to accept the settlement if $\pi^M + \frac{\delta}{1-\delta} \pi^N - S_i \geq V_R^{CN}$ and $S_i \leq \rho F + T$.\(^{21}\) Since ending collusion only requires reaching a settlement agreement with one firm, the settlement offer does not have to be equally generous to the two firms. Notice that, after inducing firm $i$ to accept, the AA only needs to make a settlement offer to firm $j$ such that $S_j \leq \rho F + T$. As such, assuming that the IR constraint holds ($F < F_R(\alpha)$), if the AA does not know which case has materialized, they will offer one firm $S^*$ and the other firm $S^{**}$, defined as follows:

$$S^* = \max\{(1 - \delta(1 - \alpha))(\rho F + T) - \delta (1 - \rho)(\pi^M - \pi^N) \over 1 - \delta(1 - \alpha \rho), 0\} \text{ and } S^{**} = \rho F + T. \quad (10)$$

The max operator in the $S^*$ expression limits settlement offers to non-negative values. If the optimal settlement amount, as determined by the first term in the $S^*$ expression, would have been negative, the best the AA could do would be to offer to a settlement with a fine of zero. But this will not be generous enough in such a case – firms will try to hold on, hoping for an acquittal which would allow their cartel to continue into the next period.\(^{22}\) Positive settlements can only be helpful then if the following condition holds:

$$F > \frac{\delta(1 - \rho)(\pi^M - \pi^N) - (1 - \delta(1 - \alpha))T}{\rho(1 - \delta(1 - \alpha))} \equiv F_R(\alpha). \quad (11)$$

\(^{21}\) In fact, the condition $S_i \leq \rho F + T$ will be redundant if the individual rationality condition holds.

\(^{22}\) To be more precise, there are two Nash equilibria in the stage game in this case: Accept/Accept and Reject/Reject, with the latter payoff-dominating the former. Consistent with our earlier assumptions on equilibrium selection, we assume that the firms would reach the payoff dominant equilibrium (Reject/Reject).
To provide some additional context, let us consider the relationship between $S^*$ and $S^{**}$. Assuming that all possible settlement amounts are non-negative (Condition (11) holds) we can demonstrate that:

$$S^* = \frac{(1 - \delta(1 - \alpha))(\rho F + T) - \delta(1 - \rho) (\pi^M - \pi^N)}{1 - \delta(1 - \alpha \rho)} \quad \text{and} \quad S^{**} = \rho F + T \quad (12)$$

where it can be shown that $S^{**} - S^*$ is positive as long as the firms’ individual rationality constraint is satisfied. In this situation one firm will get the generous settlement offer $S^*$ while the other firm will have a settlement amount $S^{**}$ that is slightly less than the expected fine from rejecting the settlement and proceeding to trial. In all cases we can see that this settlement structure leads to a situation where the subgame Nash equilibrium to the settlement game is Accept/Accept.\(^{23}\)

4.2.2 Collusion/Revelation Game (With Settlements) Stage 2

If the AA makes an attractive settlement offer $S$ to a firm in stage 4, the value associated with the C/NR equilibrium for the firm is as follows:

$$V_{S}^{CNR} = \pi^M + \alpha \left( \frac{\delta}{1 - \delta} \pi^N - S \right) + (1 - \alpha) \delta V_{S}^{CNR} \rightarrow V_{S}^{CNR} = \frac{\pi^M + \alpha \left( \frac{\delta}{1 - \delta} \pi^N - S \right)}{1 - (1 - \alpha) \delta} \quad (13)$$

It is important to note that $V_{R}^{CNR}$ and $V_{S}^{CNR}$ are different. $V_{S}^{CNR}$ is the value associated with the C/NR equilibrium in stage 2 before a settlement is offered. On the other hand, $V_{R}^{CNR}$ is the value associated with rejecting the settlement in stage 4 given that the firms have chosen to play C/NR. One of the important things to note is that $V_{S}^{CNR}$ does not depend on $\rho$. In other words, under

\(^{23}\) If settlement offers were in the range $S^* < S \leq S^{**}$ defendant firms would again find themselves in a coordination settlement subgame. If one firm expected the other to settle it would be best for that firm to settle as well, but if it did not expect the other to settle it would also refuse to settle. Assuming once more that the firms will reach the payoff dominant equilibrium, they will choose not to settle. It is for this reason that at least one settlement offer must be as low as $S^*$ to make its acceptance a dominant strategy. However, if defendants have more pessimistic expectations about their co-defendant’s play there is even greater scope for a settlement policy to reduce collusion.
settlements the probability of a successful conviction is 1 (accepting a settlement). This has the
effect of increasing the probability of cartel collapse from \( \alpha \rho \) to \( \alpha \). An increase in the probability
of cartel collapse leads to a reduction in the value from forming a cartel in the first place.

Next, we consider firm decisions in stage 2. The normal form game associated with this
stage can be found in Appendix A – Table A3. As in the no settlement policy case we apply the
payoff dominance refinement. Recall that one firm is offered \( S^* \) while the other firm is offered \( S^{**} \).
To guarantee mutual acceptance at the settlements stage, the AA will make the settlement offer
suggested above: \( \{S_i = S^*, S_j = S^{**}\} \). Though the firms, at stage 2 will understand that settlement
offers will be made and accepted, they will not know which firm will be getting the more generous
offer, \( S^* \). Assuming that the most generous offer is allocated randomly (perhaps after a race to the
AA), in the stage 2 subgame, firms would make decisions based on the expected settlement
amount:

\[
S^e = \frac{(2 - \delta(2 - \alpha(\rho + 1)))(F\rho + T) - \delta(1 - \rho)(\pi_M - \pi_N)}{2 - 2\delta(1 - \alpha \rho)}. \quad (14)
\]

Since \( L = 0 \), it is easy to see that both C/R and D/NR are strictly dominated by D/R (as they were
in the game without settlements). C/NR can occur in equilibrium if the following holds:

\[
\pi^D + \delta \frac{\pi^N}{1 - \delta} \leq \frac{\pi^M + \alpha \left( \delta \frac{\pi^N - S^e}{1 - \delta} \right)}{1 - (1 - \alpha)\delta}. \quad (15)
\]

Substituting \( S^e \) into Equation (15) we get the following condition on \( F \):

\[
F \leq \frac{(\pi^D - \pi^N)[\delta(2(1 - \delta + \delta \alpha)(1 - \alpha) + \alpha(1 - \rho)] - \delta_0[2 - \delta(2 - \alpha \rho - \alpha)]}{\rho \alpha[2 - \delta(2 - \alpha \rho - \alpha)]} \frac{T}{\rho} \equiv \tilde{F}_S(\alpha) \quad (16)
\]

If \( F \leq \tilde{F}_S(\alpha) \) we can demonstrate that C/NR is a payoff dominant equilibrium. In addition, if \( F > \tilde{F}_S(\alpha) \) then D/R is a payoff dominant equilibrium. We summarize this result in Lemma 2.
We can now establish that the magnitude of $F$ relative to $\bar{F}_S(\alpha)$ will determine which of the C/NR and C/R strategies will support the payoff dominant equilibrium. This is presented in Lemma 2.

**Lemma 2.** With a settlement policy in place, the payoff dominant equilibria of the Stage 2 game are as follows: If $F \leq \bar{F}_S(\alpha)$ then C/NR is payoff dominant. If $F > \bar{F}_S(\alpha)$ then D/R is the only Nash equilibrium.

### 4.2.3. Decision to Form a Cartel in Stage 1

As noted, firms will not commit to a cartel in stage 1 if they expect defection or revelation in stage 2. As such, if the play of D/NR or D/R is expected, firms will not form a cartel. However, if firms expect to reach the C/NR equilibrium in stage 2, they will, under certain circumstances, collude in stage 1. As we did in the no settlements case, we compare the value from colluding with the value from not colluding for each firm – but here under the expected settlement. In the earlier stages of the game firms based their decision to collude on the expected settlement amount the value from the C/NR equilibrium less the no collusion equilibrium would be:24

$$V_{S}^{C/NR} - V_{NC} = \frac{\pi^M + \alpha \left( \frac{\delta}{1-\delta} \pi^N - S^e \right)}{1 - (1-\alpha)\delta} - \frac{\pi^N}{1-\delta}$$

Based on this, collusion will occur in equilibrium (the difference is non-negative) if:

$$F \leq \frac{(\pi^M - \pi^N) - \alpha T}{\alpha \rho} \equiv \bar{F}_{IR}(\alpha)$$

(18)

Notice that this is the same individual rationality condition we observed in the no-settlements case (condition (7)). As the condition will again prove to not be binding, we can show:

---

24 This is the settlements-inclusive analogue of condition (6) from the no-settlements case.
**Proposition 3.** Collusion will occur as an equilibrium outcome in the game with settlements if $F \leq F_S(\alpha)$.

In Figure 2 we illustrate the collusive region under settlements for a particular set of parameter values. The figure contains two new curves. The dark red line represents the parameter values such that the payoff from the C/NR equilibrium when settlement offers are accepted are equal to profits from not colluding in the first place. Below this curve then collusion would occur. The green curve represents the parameter values for which the settlement amount $S$ must be 0 to be accepted by firms, and below the green curve the settlement amount has to be negative to be accepted. The red and green curves cross at an important point. To the left of this point, the green curve lies below the red curve, implying the existence of a range of parameters for which (if cartel forms and is later caught) the firms will accept the settlement offers from the AA. This is the red-shaded area. To the right of the intersection, on the other hand, firms that collude would accept only a negative settlement amount. As we do not allow negative settlements, the red curve to the right of the intersection is not relevant (and is dashed in the figure). The red-shaded area in Figure 2 then illustrates the region in which collusion will still arise but in which settlements will be accepted should an investigation be conducted (ending the cartel with probability $\alpha$). The red-thatched region represents parameter values where collusion will arise and no settlements will be accepted, in which case the cartel ends with probability $\alpha p$. 
4.2.4 Settlement Policy in Period 0

In the previous sections we saw that there are two possible optimal settlement amounts that occur based on the one-shot gain from collusion relative to the expected value from being investigated and convicted. We have also shown that offering full leniency has the effect of making collusion more difficult in the stage two subgame. This holds in both the settlements and no-settlements models. What remains is to demonstrate that by offering a settlement policy at the beginning of the game, the AA can reduce the parameter space over which firms will choose to
collude. We do this by demonstrating that implementing a settlement policy (on top of a leniency policy) can decrease the range of fines that would allow C/NR as an equilibrium strategy.

To begin, recall that the individual rationality constraint in the case with settlements is the same as in the case without settlements. This means that it continues to be redundant. Turning to the incentive compatibility constraint, we find that

\[
\hat{F}(\alpha) - \hat{F}_S(\alpha) = \frac{\delta(1-\rho)(\pi^D - \pi_N)(1-\delta(1-\alpha\rho))}{\rho(2-\delta(2-\alpha\rho-\alpha))} > 0 \rightarrow \hat{F}(\alpha) > \hat{F}_S(\alpha). \tag{19}
\]

This suggests that a settlement policy – when settlements are accepted -- will further reduce the range of fines that permit collusion as an equilibrium outcome (i.e. some fines that would be too low to discourage collusion in the case with no settlement policy will still be high enough to discourage collusion when we add the settlement policy.). If we allowed settlement values to be negative, we would be largely finished here, but constraining \(S\) offers to be non-negative provides a small complication.

To help fully describe the equilibrium path, we want to compare \(\hat{F}^+_S(\alpha)\), \(\hat{F}(\alpha)\), and \(\hat{F}_S(\alpha)\). Recall that \(\hat{F}^+_S(\alpha)\) is the maximum value of \(F\) such that a settlement offer would be accepted. To identify the point of intersection between \(\hat{F}_S(\alpha)\) and \(\hat{F}^+_S(\alpha)\) – the red and green curves in Figure 2 – we find the roots \(\alpha_1\) such that \(\hat{F}_S(\alpha_1) = \hat{F}^+_S(\alpha_1)\). We demonstrate in Appendix B that one of these roots lies in the interval \((0, 1 - \frac{\delta_0}{\delta})\). Let this root be denoted by \(\bar{\alpha}_1\). Furthermore, we find that \(\hat{F}(\alpha) > \hat{F}^+_S(\alpha)\) if and only if \(\delta > \frac{\delta_0}{1-\alpha}\). which, in turn, is implied by Assumption 2. Based on this result, the equilibrium path is defined in the following proposition:

**Proposition 4.** Suppose \(F < \hat{F}(\alpha)\) so firms will play C/NR in the game without settlements. In equilibrium, the AA sets \(L = 0\) and introduces a settlement policy at period 0. The equilibrium path from stage 1 onwards depends on the values of \(\alpha\), \(\delta\) and \(F\) as follows.
1) If \( F \leq \hat{F}_+(\alpha) \) the firms will choose Collude and Not Reveal and they will reject the AA’s settlement offers.

2) If \( F > \hat{F}_+(\alpha) \) and \( \alpha > \bar{\alpha}_1 \), the firms will not collude.

3) If \( F > \hat{F}_+(\alpha) \) and \( \alpha < \bar{\alpha}_1 \), there are two additional scenarios: (i) if \( F > \hat{F}_5(\alpha) \), the firms will not collude; and (ii) if \( \hat{F}_5(\alpha) \geq F > \hat{F}_+(\alpha) \), the firms will choose Collude and Not Reveal and they will accept the AA’s settlement offers after an investigation is opened.

We illustrate Proposition 4 in Figure 3, which is obtained by adding the original collusive boundary from the model without settlements, \( \hat{F}(\alpha) \), into Figure 2. Here again, the red shaded area is the collusive region in which settlements would be accepted, the red thatched area is the collusive region in which settlement offers would be rejected, and the new blue area is the reduction in the collusive region brought about by the addition of a settlement program. From Figure 3 we can see that the settlement policy reduces cartel occurrence if \( F \) is between \( \max \{\hat{F}_5(\alpha), \hat{F}_+(\alpha)\} \) and \( \hat{F}(\alpha) \). Note that \( \hat{F}_5(\alpha) > \hat{F}_+(\alpha) \) when \( \alpha \in (0, \bar{\alpha}_1) \) and \( \hat{F}_+(\alpha) \geq \hat{F}_5(\alpha) \) when \( \alpha \in [\bar{\alpha}_1, 1 - \frac{\delta_0}{\delta}] \).

By offering a generous settlement to a single firm, the AA can induce a ‘race to settle’ and take advantage of the fact that the expected settlement amount falls somewhere between the full fine and the generous settlement amount. This raises the probability of cartel death. Hence, while, on the one hand the reduced fines coming through settlements might appear to encourage collusion, the fact that settlements end cartels that might otherwise continue (with probability \( (1-p) \)) lowers the projected gains from entering into cartel agreements in the first place. This reduces the areas in parameter space over which firms will attempt collusion – shrinking it by the blue area in Figure 3.
It is instructive to consider how generous the settlement offers must be to be effective. As is clear from the expressions for $S^*$ and $S^{**}$ (in 12), the generosity of the settlement will depend on a number of model parameters, and in an intuitive way. Since $S^*$ is an offer so generous that a defendant is willing to give up a potential future series of (expected) monopoly profits, this offer will need to be more generous (i.e. $S^*$ must be lower) when those expected profits will be higher. Specifically, the better settlement offer must be more generous when the probability of conviction ($\rho$) is lower, the per-period profit advantage of collusion ($\pi^M - \pi^N$) is higher, the probability of an investigation ($\alpha$) is lower and the discount factor ($\delta$) is greater.\footnote{These results follow from a straightforward comparative statics analysis.}
As the less generous offer, \( S^{**} \), is designed only to match the expected costs of going to trial when the defendant knows the cartel will definitely end (because the other defendant has settled), its value is determined only by the expected one-shot costs from going to trial with some probability of conviction and fines.

Under this settlement policy, the AA effectively offers a “first to settle” discount to the first defendant to come forward, just as leniency programs typically favour earlier applicants. This first to settle discount can be shown to be:

\[
S^{**} - S^* = \frac{\delta(1-\rho)(\pi^M - \pi^N) - \delta \alpha (1-\rho)(\rho F + T)}{1-\delta(1-\alpha \rho)}
\]  \hspace{1cm} (20)

Or, expressed as a percentage discount from the less generous offer \( S^{**} \):

\[
\frac{S^{**} - S^*}{S^{**}} = \frac{\delta(1-\rho)((\pi^M - \pi^N) - \alpha (\rho F + T))}{(1-\delta(1-\alpha \rho))(\rho F + T)}
\]  \hspace{1cm} (21)

It is straightforward to show that this percentage discount is decreasing in the probability of conviction \( \rho \) and that the discount goes to zero as the probability of conviction goes to 1.

Note that there is an additional anti-cartel effect in the red-shaded area of Figure 3. In this area cartels will form but, if detected, will be immediately terminated as a result of the settlements. In this region cartels that would have ended with probability \( \alpha \rho \) will now end with probability \( \alpha \) – that is we should, in equilibrium, observe cartels last for fewer periods in this region. Indeed, in this region the settlement policy decreases the life expectancy of cartels. To see this, note that the probability of a cartel ending after \( T \) periods follows a geometric distribution, specifically \( Pr(t = T) = (1-p)^{T-1}p \), where \( p \) denotes the probability of cartel death in each period. In this region of Figure 3, we have \( p = \rho \alpha \) under no settlements and \( p = \alpha \) under settlements. Since the
mean of this distribution is $E(t) = 1/p$, we can conclude that the settlement policy shortens the life expectancy of cartels.

So how do settlements compare with leniency? Here leniency allows the AA to reduce the range of fines that permit C/NR as an equilibrium outcome. In fact, this deterrence effect – firms will be less likely to collude in the first place -- is the only effect of leniency programs here as in this model (and many others in the literature) firms never apply for leniency along the equilibrium path. Settlements, by contrast, are observed along the equilibrium path under some parameter conditions. A well-designed settlement policy can expand the AA’s ability to deter collusion and it can also bring some collusion to an early end. The key reason is that settlements, when well designed, act as a guaranteed conviction (and cartel death) when firms make it to the investigation stage.

5. Exogenously Fixed Settlement Policies

In the previous section we demonstrated that an optimal settlement policy, when added to a leniency policy, could reduce cartel incidence and shorten cartel lives. A key feature of the optimal settlement policy was that settlement offers were asymmetric – one firm (perhaps the first in) receives a more generous settlement offer than the other. Since one firm’s settlement is enough to end the cartel, there is less value to securing the second settlement. In this section we show that this asymmetry is a critical reason for the success of the policy.

For this purpose, we assume now that there is an exogenous symmetric settlement policy (established at the beginning of the game) that allows any firm to plead guilty and receive a reduced fine of $\bar{S}$. Assume also that the IR constraints continue to be satisfied and $F > \bar{F}_+$ to ensure that $S^* > 0$. Everything else in the game will remain the same. For a settlement to be accepted by both firms in equilibrium, it must be the case that $\bar{S} \leq S^*$. 

We can demonstrate that, in the case where \( \bar{S} = S^* > 0 \), the value from colluding under settlements is equal to the value from colluding under no settlement \( (V^\text{CNR}_S(S^*) = V^\text{CNR}_S) \). Furthermore, note from (13) that \( V^\text{CNR}_S \) is linearly decreasing in the settlement offer \( S \). This implies:

\[
V^\text{CNR}_S = V^\text{CNR}_S(S^*) < V^\text{CNR}_S(S)
\]

for any \( \in [0, S^*] \). Hence, any exogenously set settlement \( \bar{S} \in [0, S^*] \) will lead to a value of colluding under settlements that is greater than or equal to the value of colluding under no settlements. In other words, offering an exogenously set uniform settlement program, at best, has the same deterrent effect as offering no settlements. Importantly, however, the settlement program still increases the probability of cartel collapse from \( \alpha \rho \) to \( \alpha \) and thereby shortens expected cartel lives. These results lead us to the following proposition.

**Proposition 5.** Symmetric settlements, \( \bar{S} \), that are set by an exogenous policy rule lead to the following outcomes:

1) If \( \bar{S} > S^* \) then settlements will not be accepted and firms would proceed to trial (the settlement would be ineffective).

2) If \( \bar{S} = S^* \) then the settlement will be accepted by both firms but does not confer any additional anti-collusive effects.

3) If \( \bar{S} < S^* \) the settlement will be accepted by both firms but will exert a pro-collusive effect.

Hence, Proposition 5 tells us that symmetric settlement offers cannot add to cartel deterrence.

We observe, then, that the asymmetry of our original settlements policy was a key element of its success at reducing collusion. Since only one firm gets a generous settlement offer, and the firms do not know when they collude which it will be, both are deterred by the higher expected
settlement offer. It is easy to see that, if there were more than two firms and it took only one settlement to end the cartel for all firms, the expected settlement offer would rise toward $S^{**}$ providing even greater deterrence.

Importantly, the Proposition also tells us – at part 3 -- that an exogenously-settlement policy can actually lead to more collusion. By offering very generous settlement terms ($\bar{S} < S^*$) to all participants equally, the AA reduces considerably the expected costs of collusion more than it reduces the expected benefits, encouraging cartel formation.

6. Conclusion

We have provided a model that adds the possibility of late-stage settlements into a cartel policy with fines and leniency. We find that offering an appropriate settlement program can further deter collusion relative to the deterrence provided by a system of fines and leniency. By offering the highest (i.e. least generous) settlement that would induce all firms to accept, the AA can guarantee the breakdown of the cartel. Even though it reduces the expected punishment for detected cartels, this early termination can lead to enhanced cartel deterrence.

Of course, this means that, beyond deterrence, the policy has a second important effect: it shortens the expected lives of cartels that are nevertheless formed. In areas of the parameter space in which cartels would still form but where settlement offers would be accepted, cartels will end upon detection without the need to go to trial where the AA could lose (and the cartel survive).

Importantly, these positive effects derived from the settlement policy add to the deterrence provided by the leniency policy – suggesting a complementarity between the policies when properly designed. We did see, however, that a particular feature of the optimal settlement policy is key: settlement offers here were asymmetric. One firm would receive a very generous settlement offer – after all only one settlement is needed to end the cartel -- but the other a much
less generous offer. It is, in part, the probability of getting the less generous offer at settlement time that discourages cartel formation. If, for some reason, the settlements offers made to the two firms must be the same, the added benefit of the settlement policy with respect to deterrence disappears – and in fact overly generous settlement offers may undo some of the benefits of the leniency program and encourage collusion.

This is the first work we know of exploring, in a formal way, the relationship between settlements and leniency policies. There is clearly much more to be done here. Future work could study, for example, the comparative statics around the equilibrium, test the generality/robustness of various results of the model in the face of some alternative modelling assumptions, and consider the implications of offering rewards at either the leniency or settlement stages.
References


Appendix

A. Payoff Matrices of Various Subgames

Here we present the payoff matrices associated with several subgames in our model. Specifically, Table A1 is the payoff matrix of a stage-2 subgame under the assumption that there is no settlement policy. Table A3 is the counterpart to Table A1 under the alternative assumption that there is a settlement policy in place. The four payoff matrices in Table A2 are the normal form representation of subgames at the settlement stage of the model (with a settlement policy in place).

Table A1: Stage 2 – Payoff Matrix Without a Settlement Policy

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>C/NR</th>
<th>C/R</th>
<th>D/NR</th>
<th>D/R</th>
</tr>
</thead>
<tbody>
<tr>
<td>C/NR</td>
<td>$\nu_{\text{CNR}}$</td>
<td>$\nu_{\text{CNR}}$</td>
<td>$\pi^M + \delta \frac{\pi^N}{1 - \delta} - L$</td>
<td>$\pi^M + \delta \frac{\pi^N}{1 - \delta} - F$</td>
</tr>
<tr>
<td>C/R</td>
<td>$\pi^M + \delta \frac{\pi^N}{1 - \delta} - L$</td>
<td>$\pi^M + \delta \frac{\pi^N}{1 - \delta} - F + L$</td>
<td>$\pi^M + \delta \frac{\pi^N}{1 - \delta} - L$</td>
<td>$\pi^M + \delta \frac{\pi^N}{1 - \delta} - F + L$</td>
</tr>
<tr>
<td>D/NR</td>
<td>$\pi^D + \delta \frac{\pi^N}{1 - \delta} - \alpha(\rho F + T)$</td>
<td>$\pi^D + \delta \frac{\pi^N}{1 - \delta} - F$</td>
<td>$\pi^D + \delta \frac{\pi^N}{1 - \delta} - \alpha(\rho F + T)$</td>
<td>$\pi^D + \delta \frac{\pi^N}{1 - \delta} - F$</td>
</tr>
<tr>
<td>D/R</td>
<td>$\pi^S + \delta \frac{\pi^N}{1 - \delta} - \alpha(\rho F + T)$</td>
<td>$\pi^S + \delta \frac{\pi^N}{1 - \delta} - L$</td>
<td>$\pi^S + \delta \frac{\pi^N}{1 - \delta} - \alpha(\rho F + T)$</td>
<td>$\pi^S + \delta \frac{\pi^N}{1 - \delta} - L$</td>
</tr>
</tbody>
</table>
Table A2: Normal Form Representation of Subgames in the Settlement Stage

<table>
<thead>
<tr>
<th>Case 1: {C/NR, C/NR}</th>
<th>Case 2: {D/NR, D/NR}</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Firm 2</strong></td>
<td><strong>Firm 2</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Accept</strong></td>
<td><strong>Accept</strong></td>
</tr>
<tr>
<td>(\pi^M + \frac{\delta}{1 - \delta} \pi^N - S_1)</td>
<td>(\pi^M + \frac{\delta}{1 - \delta} \pi^N - S_1)</td>
</tr>
<tr>
<td>(\pi^M + \frac{\delta}{1 - \delta} \pi^N - S_2)</td>
<td>(\pi^M + \frac{\delta}{1 - \delta} \pi^N - \rho F - T)</td>
</tr>
<tr>
<td><strong>Reject</strong></td>
<td><strong>Reject</strong></td>
</tr>
<tr>
<td>(V^CNR_R)</td>
<td>(V^CNR_R)</td>
</tr>
<tr>
<td>(\pi^M + \frac{\delta}{1 - \delta} \pi^N - \rho F - T)</td>
<td>(\pi^M + \delta \pi^N - S_2)</td>
</tr>
<tr>
<td>(\pi^M + \frac{\delta}{1 - \delta} \pi^N - S_2)</td>
<td>(\pi^M + \delta \pi^N - \rho F - T)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 3: {D/NR, C/NR}</th>
<th>Case 4: {C/NR, D/NR}</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Firm 2</strong></td>
<td><strong>Firm 2</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Accept</strong></td>
<td><strong>Accept</strong></td>
</tr>
<tr>
<td>(\pi^D + \frac{\delta}{1 - \delta} \pi^N - S)</td>
<td>(\pi^D + \frac{\delta}{1 - \delta} \pi^N - S)</td>
</tr>
<tr>
<td>(\pi^S + \frac{\delta}{1 - \delta} \pi^N - S)</td>
<td>(\pi^S + \frac{\delta}{1 - \delta} \pi^N - \rho F - T)</td>
</tr>
<tr>
<td><strong>Reject</strong></td>
<td><strong>Reject</strong></td>
</tr>
<tr>
<td>(\pi^D + \frac{\delta}{1 - \delta} \pi^N - \rho F - T)</td>
<td>(\pi^D + \frac{\delta}{1 - \delta} \pi^N - S)</td>
</tr>
<tr>
<td>(\pi^S + \frac{\delta}{1 - \delta} \pi^N - S)</td>
<td>(\pi^S + \frac{\delta}{1 - \delta} \pi^N - \rho F - T)</td>
</tr>
</tbody>
</table>
Table A3: Stage 2 – Payoff Matrix with a Settlement Policy

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>C/NR</th>
<th>C/R</th>
<th>D/NR</th>
<th>D/R</th>
</tr>
</thead>
<tbody>
<tr>
<td>C/NR</td>
<td>$V_{S}^{CNR}$</td>
<td>$\pi^{M} + \delta \frac{\pi^{N}}{1-\delta} - F$</td>
<td>$\pi^{D} + \delta \frac{\pi^{N}}{1-\delta} - \alpha S^{e}$</td>
<td>$\pi^{S} + \delta \frac{\pi^{N}}{1-\delta} - F$</td>
</tr>
<tr>
<td></td>
<td>$V_{S}^{CNR}$</td>
<td>$\pi^{M} + \delta \frac{\pi^{N}}{1-\delta} - L$</td>
<td>$\pi^{D} + \delta \frac{\pi^{N}}{1-\delta} - F$</td>
<td>$\pi^{S} + \delta \frac{\pi^{N}}{1-\delta} - L$</td>
</tr>
<tr>
<td>C/R</td>
<td>$\pi^{M} + \delta \frac{\pi^{N}}{1-\delta} - L$</td>
<td>$\pi^{M} + \delta \frac{\pi^{N}}{1-\delta} - \frac{F + L}{2}$</td>
<td>$\pi^{D} + \delta \frac{\pi^{N}}{1-\delta} - \frac{L}{2}$</td>
<td>$\pi^{D} + \delta \frac{\pi^{N}}{1-\delta} - \frac{F + L}{2}$</td>
</tr>
<tr>
<td></td>
<td>$\pi^{D} + \delta \frac{\pi^{N}}{1-\delta} - \alpha S^{e}$</td>
<td>$\pi^{S} + \delta \frac{\pi^{N}}{1-\delta} - F$</td>
<td>$\pi^{B} + \delta \frac{\pi^{N}}{1-\delta} - \alpha S^{e}$</td>
<td>$\pi^{B} + \delta \frac{\pi^{N}}{1-\delta} - F$</td>
</tr>
<tr>
<td>D/NR</td>
<td>$\pi^{D} + \delta \frac{\pi^{N}}{1-\delta} - \frac{L}{2}$</td>
<td>$\pi^{S} + \delta \frac{\pi^{N}}{1-\delta} - L$</td>
<td>$\pi^{B} + \delta \frac{\pi^{N}}{1-\delta} - \frac{L}{2}$</td>
<td>$\pi^{B} + \delta \frac{\pi^{N}}{1-\delta} - \frac{F + L}{2}$</td>
</tr>
<tr>
<td></td>
<td>$\pi^{S} + \delta \frac{\pi^{N}}{1-\delta} - F$</td>
<td>$\pi^{S} + \delta \frac{\pi^{N}}{1-\delta} - \frac{F + L}{2}$</td>
<td>$\pi^{B} + \delta \frac{\pi^{N}}{1-\delta} - F$</td>
<td>$\pi^{B} + \delta \frac{\pi^{N}}{1-\delta} - \frac{F + L}{2}$</td>
</tr>
</tbody>
</table>

B. Proofs of Lemmas and Propositions

While Proposition 1 is placed before Lemmas 1 and 2 in the main text, here we will present the proof of the proposition after those of the two lemmas. This is because the former is built on (a more general version of) Lemmas 1 and 2.

Proof of Lemma 1

Here we derive the payoff dominant equilibria of the Stage 2 subgame in the absence of a settlement policy. The payoff matrix associated with this subgame is presented in Table A1. Even though Lemma 1 in the main text is stated for the case of $L = 0$ only, we will demonstrate below
a more general version of the lemma that holds for $L \in [0, F)$. We do so because the more general
version of Lemma 1 is needed for the proof of Proposition 1.

As the first step of this analysis, we note that both firms playing D/R is a Nash equilibrium
in this subgame. To see this, consider a firm’s payoffs associated unilateral deviations from D/R.
Comparing the payoffs in Table A1, we see that deviating to C/NR, C/R or D/NR is not profitable
because $\frac{F+L}{2} < F$ and $\pi^B > \pi^S$. This confirms that \{D/R, D/R\} is indeed an equilibrium in this
subgame.

Moreover, there is one or two additional Nash equilibria in this subgame. The specific
number of equilibria depends on the magnitude of $L$. Accordingly, the rest of the analysis is divided
into two cases: $L \in [0, \alpha(\rho F + T))$ and $L \in [\alpha(\rho F + T), F)$.

Case 1. $L < \alpha(\rho F + T)$

In this case, \{C/NR, C/NR\} is an additional Nash equilibrium in the subgame if

$$F \leq \frac{(\pi^D - \pi^N)(\delta(1 - \alpha \rho) - \delta_0) + (1 - (1 - \alpha \rho)\delta)L - \alpha T}{\alpha \rho} \equiv \hat{F}(L, \alpha). \quad (A1)$$

To show this, note that $\pi^D > \pi^M$ and $\pi^B > \pi^S$ imply that C/R is dominated by D/R. Moreover,
$L < \alpha(\rho F + T)$ and $\frac{F+L}{2} < F$ entail that D/NR is dominated by D/R. Hence, for \{C/NR, C/NR\} to
be an equilibrium, we only need to show that a firm earns a larger payoff by playing C/NR than
playing D/R, i.e.,

$$V_{CNR}^C > \pi^D + \delta \frac{\pi^N}{1 - \delta} - L. \quad (A2)$$

Substituting (4) for $V_{CNR}^C$ in (A2) and rearranging terms, we obtain (A1).

Furthermore, note that

$$\pi^D + \delta \frac{\pi^N}{1 - \delta} - L > \pi^B + \delta \frac{\pi^N}{1 - \delta} - \frac{F + L}{2}. \quad (A3)$$
Then (A2) and (A3) imply that each firm earns a larger payoff in the C/NR equilibrium than in the D/R equilibrium. In other words, the C/NR equilibrium payoff dominates the D/R equilibrium if $F \leq \hat{F}(L, \alpha)$. In the case where $F > \hat{F}(L, \alpha)$, the D/R equilibrium is the only Nash equilibrium.

Recall that Lemma 1 in the main text presents the result for the case $L = 0$, which is a special case of $L < \alpha(\rho F + T)$. The $\hat{F}(\alpha)$ function in Lemma 1 is obtained by setting $L = 0$ in $\hat{F}(L, \alpha)$ defined in (A1).

Case 2. $L \in [\alpha(\rho F + T), F)$

In this case, there may be two Nash equilibria in this subgame in addition to \{D/R, D/R\}: \{D/NR, D/NR\} and \{C/NR, C/NR\}. To see that \{D/NR, D/NR\} is a Nash equilibrium, we consider a firm’s payoffs associated unilateral deviations from D/NR. Comparing the payoffs in Table A1, we find that deviating to C/NR, C/R or D/R is not profitable because $\pi^B > \pi^S$ and $F \geq \alpha(\rho F + T)$. This confirms that \{D/NR, D/NR\} is indeed an equilibrium in this subgame.

To see that \{C/NR, C/NR\} may be an equilibrium in this subgame, note again that $\pi^D > \pi^M$ and $\pi^B > \pi^S$ imply that C/R is dominated by D/R. Moreover, given its rival’s choice of C/NR, each firm earns a larger payoff by playing D/NR than playing D/R because $L > \alpha(\rho F + T)$. Hence, for \{C/NR, C/NR\} to be an equilibrium, we only need to show that a firm earns a higher payoff by playing C/NR than playing D/NR, i.e.,

$$V^{CNR} \geq \pi^D + \delta \left( \frac{\pi^N}{1 - \delta} - \alpha(\rho F + T) \right). \quad (A4)$$

Substituting (4) for $V^{\pi_2}$ in (A4) and rearranging terms, we obtain

$$F \leq \frac{(\pi^D - \pi^N)(\delta(1 - \alpha \rho) - \delta_0) - \alpha \delta(1 - \alpha \rho)T}{\alpha \rho \delta(1 - \alpha \rho)} \equiv \hat{F}_{NL}(\alpha). \quad (A5)$$

Therefore, \{C/NR, C/NR\} is also an equilibrium if (A5) holds.
To apply the criterion of payoff dominance, note that \( L \geq \alpha(\rho F + T) \) implies that \( \frac{F + L}{2} > \alpha(\rho F + T) \). Then from Table A1 we see that both firms earn a larger payoff in the D/NR equilibrium than in the D/R equilibrium. Therefore, if \( F > \bar{F}_{NL}(\alpha) \) (i.e., if (A5) fails to hold), the payoff dominant equilibrium is \{D/NR, D/NR\}.

On the other hand, if (A5) holds, \{C/NR, C/NR\} is the third Nash equilibrium. Since

\[
\pi^D + \delta \frac{\pi^N}{1 - \delta} - \alpha(\rho F + T) > \pi^B + \delta \frac{\pi^N}{1 - \delta} - \alpha(\rho F + T) > \pi^B + \delta \frac{\pi^N}{1 - \delta} - \frac{F + L}{2}, \quad (A6)
\]

we conclude from (A4) and (A6) that \{C/NR, C/NR\} is the payoff dominant equilibrium when \( F \leq \bar{F}_{NL}(\alpha) \).

\[ \blacksquare \]

**Proof of Lemma 2**

This proof is based on comparisons of the firms’ payoffs in Table A3, where \( S^e \) denotes the *ex-ante* expected settlement amount. As in the case of Lemma 1, we will demonstrate below a more general version of lemma 2 that holds for \( L \in [0, F] \) because it is needed for the proof of Proposition 1. Accordingly, the following analysis is conducted for \( L \) in two separate ranges: \( L < \alpha S^e \) and \( L \in [\alpha S^e, F) \).

Recall from the discussions in section 4.2 of the main text that

\[
S^e = \frac{S^* + S^{**}}{2} = \frac{[2 - \delta(2 - \alpha(\rho + 1))] (F \rho + T) - \delta(1 - \rho)(\pi^M - \pi^N)}{2 - 2\delta(1 - \alpha\rho)}. \quad (A7)
\]

While the discussion in the main text is conducted in the context of \( L = 0 \), (A7) is applicable even if \( L > 0 \) because the settlement stage of the game is reached only if there has been no leniency applicant. As a result, the magnitude of \( L \) has no impact on \( S^* \) and \( S^{**} \).

**Case 1.** \( L < \alpha S^e \)
First, we demonstrate that \( \{D/R, D/R\} \) is a Nash equilibrium. Note that \( L < F \) entails \( \frac{F+L}{2} < F \). This, along with \( L < \alpha S^e \), imply that D/NR is strictly dominated by D/R. Moreover, \( \frac{F+L}{2} < F \) and \( \pi^B > \pi^S \) (by Assumption 1) ensure that each firm has no incentive to deviate unilaterally from D/R to C/NR or C/R. This confirms that \( \{D/R, D/R\} \) is indeed a Nash equilibrium.

Second, we show that \( \{C/NR, C.NR\} \) is a Nash equilibrium if \( F \leq \hat{F}_S(L, \alpha) \), where

\[
\hat{F}_S(L, \alpha) \equiv [\alpha \rho(2 - \delta(2 - \alpha - \alpha \rho))]^{-1} \left( 2L(1 - \delta + \alpha \delta)(1 - \delta + \delta \alpha \rho) + \delta(2(1 - \delta + \alpha \delta \rho)(1 - \alpha) + \alpha(1 - \rho)) - \delta_0 (2 - \delta(2 - \alpha - \alpha \rho)) \right) (\pi^D - \pi^N) \]

\[-\frac{T}{\rho}. \quad (A8)\]

Observe from Table A3 that, given its rival’s choice of C/NR, a firm earns a larger payoff from playing D/R than playing either C/R or D/NR. Hence, a firm will have no incentive to deviate from C/NR if its payoff is no less than that from playing D/R, i.e., if

\[
V_c^{CNR} \geq \pi^D + \delta \frac{\pi^N}{1 - \delta} - L. \quad (A9)\]

Rewriting (A9) using (13) and (A7), we obtain \( F \leq \hat{F}_S(L, \alpha) \). Therefore, when \( L < \alpha S^e \) and \( F \leq \hat{F}_S(L, \alpha) \), there are two Nash equilibria in this subgame.

To apply the payoff dominance criterion, we note that

\[
\pi^D + \delta \frac{\pi^N}{1 - \delta} - L > \pi^B + \delta \frac{\pi^N}{1 - \delta} - \frac{F + L}{2}. \quad (A10)\]

Then (A9) and (A10) imply that the C/NR equilibrium payoff dominates the D/R equilibrium. In the case that \( F > \hat{F}_S(L, \alpha) \), the D/R equilibrium is the only Nash Equilibrium.
Recall that Lemma 2 in the main text presents the result for the case $L = 0$, which is a special case of $L < \alpha S^e$. The $\hat{F}_S(\alpha)$ function associated with Lemma 2 is obtained by setting $L = 0$ in $\hat{F}_S(L, \alpha)$ defined in (A8).

Case 2. $L \in [\alpha S^e, F)$

In this case, \{D/R, D/R\} is still a Nash equilibrium. To see this, note from Table A3 that any unilateral deviation by a firm from D/R to C/NR, C/R or D/NR would lead to a lower payoff because $\frac{F + L}{2} < F$ and $\pi^B > \pi^S$ (by Assumption 1). Moreover, \{D/NR, D/NR\} is also a Nash equilibrium: any unilateral deviation by a firm from D/NR to C/NR, C/R or D/R would lead to a lower payoff because $\pi^B > \pi^S$ and $L \geq \alpha S^e$.

To find the condition under which C/NR occurs in equilibrium, observe from Table A3 that, given its rival’s choice of C/NR, a firm earns a larger payoff from playing D/NR than playing either C/R or D/R. Hence, a firm will have no incentive to deviate from C/NR if its payoff is no less than that from playing D/NR, i.e.,

$$V_{S}^{CNR} \geq \pi^D + \delta \frac{\pi^N}{1 - \delta} - \alpha S^e. \quad (A11)$$

Rewriting (A11) using (13) and (A7), we obtain

$$F \leq \frac{(\delta(2 - \alpha - 2\delta + 2\alpha\delta - \alpha(1 - 2\delta + 2\alpha\delta)\rho) - \delta_0(2 - \delta(2 - \alpha - \alpha\rho)))(\pi_D - \pi_N)}{\alpha\delta\rho[2 - \alpha - 2\delta + 2\alpha\delta - \alpha(1 - 2\delta + 2\alpha\delta)\rho]}$$

$$- \frac{T}{\rho} \equiv \hat{F}_{S,NL}(\alpha). \quad (A12)$$

In other words, \{C/NR, C/NR\} is the third Nash equilibrium in this subgame if (A12) holds.

To apply the payoff dominance criterion, we first compare a firm’s payoffs under the D/R equilibrium and the D/NR equilibrium. Because $L \geq \alpha S^e$ implies $\frac{L + F}{2} > \alpha S^e$, we conclude from
Table A3 that each firm earns a larger payoff in the D/NR equilibrium than in the D/R equilibrium. In other words, the D/NR equilibrium payoff dominates the D/R equilibrium.

In the case where (A12) holds, we also need to compare a firm’s payoff in the C/NR equilibrium with that in the D/NR equilibrium to determine payoff dominance (or the lack of). Note that

\[ \pi^D + \delta \frac{\pi^N}{1 - \delta} - \alpha S^e > \pi^R + \delta \frac{\pi^N}{1 - \delta} - \alpha S^e. \]  

(A13)

Then (A11) and (A13) imply that \{C/NR, C/NR\} payoff dominates \{D/NR, D/NR\}. Hence, we conclude that \{C/NR, C/NR\} is payoff dominant among the three equilibria in this case.

\[ \square \]

Proof of Proposition 1

Here we want to prove that it is optimal for the AA to set \( L = 0 \) with or without a settlement policy. We start with the case where there is no settlement policy and show that deviating from \( L = 0 \) will increase cartel occurrence. Note from (A1) that

\[ \frac{\partial \hat{F}(L, \alpha)}{\partial L} = \frac{1 - (1 - \alpha \rho) \delta}{\alpha \rho} > 0. \]  

(A14)

In other words, for \( L \in [0, \alpha(\rho F + T)] \), a larger \( L \) expands the range of \( F \) over which collusion occurs in equilibrium. On the other hand, we see from (A5) that \( \hat{F}_{NL}(\alpha) \) is independent of \( L \), implying that a change in \( L \) has no impact on cartel occurrence for \( L \in [\alpha(\rho F + T), F] \). Moreover, we can demonstrate that

\[ \hat{F}_{NL}(\alpha) - \hat{F}(0, \alpha) = \frac{(1 - (1 - \alpha \rho) \delta)(\pi^D - \pi^N)((1 - \alpha \rho) \delta - \delta_0)}{\alpha \rho \delta (1 - \alpha \rho)} > 0. \]  

(A15)
Together (A14) and (A15) imply that in the absence of a settlement policy, cartel occurrence is minimized at $L = 0$. Therefore, it is optimal for the AA to set $L = 0$ in the absence of a settlement policy.

Now we consider the case with a settlement policy. Note from (A8) that

$$\frac{\partial \tilde{F}_S(L, \alpha)}{\partial L} = \frac{2(1 - (1 - \alpha)\delta)(1 - \delta(1 - \alpha\rho))}{\alpha\rho(2 - \delta(2 - \alpha - \alpha\rho))} > 0. \ (A16)$$

In other words, for $L \in [0, \alpha S^e)$, a larger $L$ expands the range of $F$ over which collusion occurs in equilibrium. On the other hand, we see from (A12) that $\tilde{F}_{S,NL}(\alpha)$ is independent of $L$, implying that a change in $L$ has no impact on cartel occurrence for $L \in [\alpha S^e, F)$. Moreover, we can demonstrate that

$$\tilde{F}_{S,NL}(\alpha) - \tilde{F}(0, \alpha) = \frac{2(1 - (1 - \alpha)\delta)(1 - \delta(1 - \alpha\rho))(\pi^D - \pi^N)}{\alpha\rho}\left[\frac{1}{A} - \frac{\delta_0}{\delta B}\right] > 0. \ (A17)$$

In (A17), $A \equiv 2 - \delta(2 - \alpha - \alpha\rho)$ and $B \equiv 2 - \alpha - 2\delta + 2\alpha\delta - \alpha\rho(1 - 2\delta + 2\alpha\delta)$. Assumption 2 implies that $\frac{1}{A} > \frac{\delta_0}{\delta B}$. Hence the sign of (A17) is positive. Taken together, these observations mean that when the AA adopts a settlement policy, cartel occurrence is minimized at $L = 0$. Therefore, it is optimal for the AA to set $L = 0$.

**Proof of Propositions 2**

This result is established through the analysis in Section 4.1.

**Proof of Proposition 3**

This result is established through the analysis in section 4.2.

**Proof of Proposition 4**

It has been established in Proposition 1 that the AA sets $L = 0$ with or without a settlement policy. The rest of the proof proceeds in three steps. First, we prove the results in each
of the three components in this proposition taking as given that the AA implements a settlement policy and that \( \bar{\alpha}_1 \) is in the interval \((0, 1 - \delta_0/\delta)\). Second, we show that the AA indeed introduces a settlement policy in equilibrium. Third, we demonstrate that \( \bar{\alpha}_1 \in (0, 1 - \delta_0/\delta) \).

To prove part 1) of the proposition, recall from the definition of \( \hat{F}_+ (\alpha) \) that a firm would reject any settlement offer with \( S_i \ge 0 \) for \( F < \hat{F}_+ (\alpha) \). As a result, a settlement policy is ineffective in this case and the equilibrium is the same as that in the absence of a settlement policy. Since \( \hat{F}(\alpha) > \hat{F}_+ (\alpha) \), Proposition 2 implies that the firms will choose Collude and Not Reveal in equilibrium.

Regarding part 2) of the proposition, it is shown below that \( \hat{F}_S (\alpha) < \hat{F}_+ (\alpha) \) for \( \alpha > \bar{\alpha}_1 \). Then \( F > \hat{F}_+ (\alpha) \) entails \( F > \hat{F}_S (\alpha) \) and hence Proposition 3 implies that the firms will not collude in equilibrium.

Turning to part 3) of the proposition, we note that \( \hat{F}_S (\alpha) > \hat{F}_+ (\alpha) \) for \( \alpha < \bar{\alpha}_1 \). (This will be shown in the final part of this proof). In the case where \( F > \hat{F}_S (\alpha) \), Proposition 3 implies that the firms will not collude in equilibrium. If \( \hat{F}_S (\alpha) > F > \hat{F}_+ (\alpha) \), on the other hand, Proposition 3 suggests that the firms will choose Collude and Not Reveal in equilibrium. Moreover, by the definition of \( \hat{F}_+ (\alpha) \) we know that they will accept the AA’s settlement offers after an investigation is opened.

Next, we show that the AA indeed introduces a settlement policy in equilibrium. Using the definitions of \( \hat{F}(\alpha) \) and \( \hat{F}_+ (\alpha) \), we find

\[
\hat{F}(\alpha) - \hat{F}_+ (\alpha) = \frac{[1 - \delta(1 - \alpha \rho)](\pi^b - \pi^N)[\delta(1 - \alpha) - \delta_0]}{\alpha \rho [1 - \delta(1 - \alpha)]} > 0. \tag{A18}
\]

by Assumption 2. Moreover, we have shown in (19) that \( \hat{F}(\alpha) > \hat{F}_S (\alpha) \). Using these observations to compare the equilibria we have derived above with those in the absence of a settlement policy,
we find that the settlement policy reduces cartel occurrence for $F$ between $\max \{\bar{F}_S(\alpha), \bar{F}_+(\alpha)\}$ and $\bar{F}(\alpha)$ and it does not change cartel occurrence otherwise. Therefore, it is optimal for the AA to adopt a settlement policy in equilibrium.

Finally, we derive $\bar{\alpha}_1$ and demonstrate that $\bar{\alpha}_1 \in (0, 1 - \delta_0/\delta)$. Recall that $\bar{\alpha}_1$ is one of the roots to the equation $\bar{F}_S(\alpha) - \bar{F}_+(\alpha) = 0$. Denote this equation by $\Theta(\alpha) = 0$. Using the definitions of $\bar{F}_S(\alpha)$ and $\bar{F}_+(\alpha)$, we find

$$\Theta(\alpha) = \frac{(1 - \delta + a\delta\rho)(\pi^D - \pi^N)[2\delta^2\alpha^2 + (3\delta - 4\delta^2 - \delta\rho + \delta\delta_0 + \delta\rho\delta_0)\alpha - 2(1 - \delta)(\delta - \delta_0)]}{\alpha(1 - \delta + a\delta)\rho(2 - 2\delta + a\delta + a\delta\rho)}. \quad (A20)$$

In (A20), note that $1 - \delta + a\delta\rho > 0$, $\pi_D - \pi_N > 0$, $1 - \delta + a\delta > 0$, and $2 - 2\delta + a\delta + a\delta\rho > 0$. Therefore, $\Theta(\alpha) = 0$ entails $\Gamma(\alpha) = 0$ where

$$\Gamma(\alpha) \equiv 2\delta^2\alpha^2 + (3\delta - 4\delta^2 - \delta\rho + \delta\delta_0 + \delta\rho\delta_0)\alpha - 2(1 - \delta)(\delta - \delta_0). \quad (A21)$$

Using the quadratic formula, we find the two roots of this equation:

$$\alpha_1 = \frac{-3\delta + 4\delta^2 + \delta\rho - \delta\delta_0 - \delta\rho\delta_0 \pm \sqrt{(3\delta - 4\delta^2 - \delta\rho + \delta\delta_0 + \delta\rho\delta_0)^2 + 16\delta^2(1 - \delta)(\delta - \delta_0)}}{4\delta^2}. \quad (A22)$$

Since the discriminant in (A22) is positive, there are two distinct real roots. Moreover, it is easy to verify that one root is positive and the other is negative. Discard the negative root and define the positive root as $\bar{\alpha}_1$, that is,

$$\bar{\alpha}_1 = \frac{-3\delta + 4\delta^2 + \delta\rho - \delta\delta_0 - \delta\rho\delta_0 \pm \sqrt{(3\delta - 4\delta^2 - \delta\rho + \delta\delta_0 + \delta\rho\delta_0)^2 + 16\delta^2(1 - \delta)(\delta - \delta_0)}}{4\delta^2}. \quad (A22)$$

To show that $\bar{\alpha}_1 \in (0, 1 - \delta_0/\delta)$, we evaluate (A21) at $\alpha = 0$ and $1 - \delta_0/\delta$ to find $\Gamma(0) = -2(1 - \delta)(\delta - \delta_0) < 0$, and
\[ \Gamma \left( 1 - \frac{\delta_0}{\delta} \right) = (1 - \rho)(\delta - \delta_0)(1 - \delta_0) > 0. \quad (A23) \]

By the intermediate value theorem, there must exist a \( c \in (0, 1 - \delta_0 / \delta) \) such that \( \Gamma(c) = 0 \). Since \( \bar{\alpha}_1 \) is the only positive root, we conclude that \( \bar{\alpha}_1 \in (0, 1 - \delta_0 / \delta) \). Moreover, note from (A20)-(A21) that \( \Theta(\alpha) \) has the opposite sign of \( \Gamma(\alpha) \), the preceding analysis also implies that \( \tilde{F}_S(\alpha) > \tilde{F}_+(\alpha) \) for \( \alpha < \bar{\alpha}_1 \).

\[ \Box \]

**Proof of Proposition 5**

We only need to demonstrate \( V_{S}^{\text{CNR}}(S^*) = V^\text{CNR} \). The rest of the proposition follows from the analysis in the main text.

Substituting \( S^* \) in (12) for \( S \) in (13) and rearranging, we obtain

\[ V_{S}^{\text{CNR}}(S^*) = \frac{\pi^M + \alpha \rho \left( \frac{\delta}{1 - \delta} \pi^N - F \right) - \alpha T}{1 - (1 - \alpha \rho) \delta}. \quad (A24) \]

Comparing (A24) with (4), we find that \( V_{S}^{\text{CNR}}(S^*) = V^\text{CNR} \).

\[ \Box \]