<u>Legal Uncertainty and Optimal Penalties</u>

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1. Preliminaries

In general we can assume that the penalty takes the form of a fixed penalty plus a proportional penalty. These reflect the twin desires to link penalty to harm and to the private benefit firms obtain from acting badly. As we will see, in general the optimal penalty can always be obtained by using just a penalty proportional to private benefit.

The idea is this. Suppose the penalty takes the form $\psi + \varphi b, \quad \psi \geq 0, \varphi \geq 0$. Let π be coverage rate and δ the delay in making decisions. Let $\beta, \quad 0 \leq \beta \leq 1$ be probability of having action banned. Then for a firm with private benefit *b* the net benefit from taking the action is

$$b\left\{\left[1-\beta\pi(1-\delta)\right]-\beta\pi\varphi\right\}-\beta\pi\psi \ . \qquad (1)$$

If $\varphi > \frac{1}{\beta\pi} - (1-\delta)$ then no firm will take the action whatever the value of ψ, so we might as well set $\psi = 0$.

If $\varphi < \frac{1}{\beta\pi} - (1-\delta)$ and ψ=0, then all firms will take the action, while if $\varphi < \frac{1}{\beta\pi} - (1-\delta)$ and $\psi > 0$ then the only firms that take the action are those with high values of private benefit. But it turns out that in all cases we either want all firms of a particular type to take the action or none of them to do so. So we can achieve the optimal fines by using ONLY proportional fines.

So then critical value of φ below which a firm would definitely take action and above which it would not is :

$$\hat{\varphi} = \left(\frac{1}{\pi\beta} - 1\right) + \delta \ . \qquad (2)$$

In all that follows we will assume that there is *procedural uncertainty* arising because not all firms are investigated - $\pi < 1$ - and there is a delay in taking decisions - $\delta > 0$ - so that, even if actions will be banned for sure - $\beta = 1$ - a positive penalty will be needed if actions are to be deterred.

2. Optimal Penalties and Welfare with Different Decision Rules and Different Degrees of Legal Uncertainty

Here we determine the optimal penalties and associated level of welfare under different decision rules – **Effects-Based** and **Per Se** – and, for **Effects-Based**, under different degrees of Legal Uncertainty. Throughout we will consider only **Effects-Based** decision rules for which the CA can **effectively discriminate.**

Consider first an **Effects-Based** decision rule and the three degrees of Legal Uncertainty.

2.1 Effects Based: No Legal Uncertainty

Here a fraction p_B resp. $1-p_H$ of firms from the benign (resp. harmful) environment know for sure that their action will be allowed. So, irrespective of the penalty, they will take the action. The remaining firms from the harmful (resp. benign) environment will know for sure that their action will be allowed. Given our assumption that private benefit is uncorrelated with harm it follows that for **any** given penalty the same fraction $F, 0 \le F \le 1$ of these firms will be deterred. Consequently welfare under any given penalty regime is

$$\begin{aligned} W = & -\left\{\gamma h_H\left(1-p_H\right)+(1-\gamma)h_B p_B\right\}+ \\ & -\left[1-\pi(1-\delta)\right]\left(1-F\right)\left\{\gamma h_H p_H-(1-\gamma)\left(-h_B\right)(1-p_B)\right\} \end{aligned} \tag{3}$$

Now we know from our previous work that if the CA's rule can **effectively discriminate** then whether an action is *Presumptively Legal* or *Presumptively Illegal* then

$$\gamma h_H p_H > (1-\gamma)\left(-h_B\right)\left(1-p_B\right). \tag{4}$$

Given (4) it follows that to maximise welfare we want to deter all the firms who know for sure that their action will be disallowed, and, from (2) to do this we set the proportional penalty at the level $\left(\frac{1}{\pi}-1\right)+\delta$. So we have:

Proposition 1
When there is *no legal uncertainty*

(i) the optimal penalty is $\hat{\varphi}^0 = \left(\frac{1}{\pi}-1\right)+\delta$; (5)

(ii) the associated maximum level of welfare is $\hat{W}^0 = -\left\{\gamma h_H\left(1-p_H\right)+(1-\gamma)h_B p_B\right\}$. (6)

2.2 Effects Based: Partial Legal Uncertainty

For any given penalty scheme fewer firms from the benign environment will be deterred than from the harmful environment. That is, if the fraction deterred from environment $e = H, B$ is F_e, $0 \le F_e \le 1$, then $F_H > F_B$ and so welfare under any penalty is:

$$W = (1-F_H)\gamma(-h_H)[1-\pi p_H(1-\delta)] + (1-F_B)(1-\gamma)(-h_B)[1-\pi(1-p_B)(1-\delta)] \quad (5)$$

The optimum is to set $F_H = 1, F_B = 0$ and, from (2) we can do this by setting a penalty $\left(\frac{1}{\pi p_H} - 1\right) + \delta$. So we have:

Proposition 2
When there is *partial legal uncertainty* then

(i) the optimal penalty is $\hat{\varphi}^p = \left(\frac{1}{\pi p_H} - 1\right) + \delta$, and (7)

(ii) the associated maximum level of welfare is
$\tilde{W}^p = (1-\gamma)(-h_B)[1-\pi(1-p_B)(1-\delta)] > 0$. (8)

2.3 Effects-Based: Complete Legal Uncertainty

When there is complete legal uncertainty then each firm sees the risk of having their action disallowed as just the average probability $\bar{p} = \gamma p_H + (1-\gamma)p_B < p_H$, and, given our assumption of zero correlation between the harm and private benefit, under any penalty regime the same fraction of firms will be deterred from each of the two environments. So welfare is just $W = (1-F)\tilde{W}$ where:

$$\tilde{W} = \gamma(-h_H)[1-\pi p_H(1-\delta)] + (1-\gamma)(-h_B)[1-\pi(1-p_B)(1-\delta)] \quad (9)$$

Notice that if we let $x = \pi(1-\delta)$, $0 \le x \le 1$ then we can think of $\tilde{W}$ as being a function of *x*, and, moreover, it is strictly increasing function since

$$\frac{d\tilde{W}}{dx} = \gamma h_H p_H - (1-\gamma)(-h_B)(1-p_B) > 0 \quad (10)$$

where the inequality follows from (4).

Notice also that

$$\tilde{W}(0) = -\bar{h} \text{ and } \tilde{W}(1) = \tilde{W}^0 = (1-\gamma)(-h_B)p_B - \gamma h_H(1-p_H). \quad (11)$$

It follows from (11) and (10) that if an action is *Presumptively Legal* $\left(\bar{h} < 0\right)$ then $\tilde{W} > 0 \quad \forall x, 0 \le x \le 1$.

On the other hand if an action is *Presumptively Ilegal* $\left(\bar{h} > 0\right)$ then clearly $\tilde{W} < 0$ if $\pi(1-\delta) \approx 0$ but, provided the CA's **Effects-Based** rule can **effectively discriminate –** which requires that $(1-\gamma)\left(-h_B\right) p_B - \gamma h_H \left(1-p_H\right) > 0$ - then $\tilde{W} > 0$ if $\pi(1-\delta) \approx 1$

So we have:

Proposition 3
If there is *complete legal uncertainty* then
(a) if $\tilde{W} > 0$ - for which a sufficient but not necessary condition is that the action is *Presumptively Legal* - then:

(i) the optimal penalty is $\hat{\varphi}^c = 0$, and, (12)

(ii) the associated maximum level of welfare is $\tilde{W}^c = \tilde{W} > 0$; (13)

(b) if $\tilde{W} < 0$ - for which a necessary but not sufficient condition is that the action is *Presumptively Illegal* - then:

(i) the optimal penalty is $\hat{\varphi}^c = \left[\frac{1}{\pi \bar{p}} - 1\right] + \delta$, and, (14)

(ii) the associated level of welfare is $\tilde{W}^c = 0$. (15)

2.4 Per Se

If the action is *Presumptively Legal* all firms will take the action whatever the penalty and the associated level of welfare is $-\bar{h} > 0$.

If the action is *Presumptively Ilegal* under any penalty regime the same fraction of firms will be deterred from the harmful and benign environments and so welfare is just $W = -(1-F)\bar{h}$ and the optimal penalty is the same as with *no legal uncertainty*.

Proposition 4
Under **Per Se**,
(a) if the action is *Presumptively Legal* then

(i) the optimum penalty is $\hat{\varphi}^{PSL} = 0$, and (16)

(ii) the associated maximum level of welfare is $\tilde{W}^{PSL} = -\bar{h} > 0$; (17)

(b) if the action is *Presumptively Illegal* then

(i) the optimum penalty is $\hat{\varphi}^{PSI} = \hat{\varphi}^{0} = \left[\frac{1}{\pi} - 1\right] + \delta$, and (18)

(ii) the associated maximum level of welfare is $\tilde{W}^{PSI} = 0$. (19)

3. Comparisons

Here we compare the outcomes in terms of welfare and penalties.

*3.1 **Effects-Based**: Partial vs No Legal Uncertainty*

From (8), notice that $\tilde{W}^{p}$ is strictly decreasing in $\pi(1-\delta)$ so, using (6)

$$\begin{aligned} \tilde{W}^{p} &> (1-\gamma)(-h_B)\left[1-(1-p_B)\right] = (1-\gamma)(-h_B)\,p_B > \\ &-\left[\gamma h_H(1-p_H) + (1-\gamma)h_B p_B\right] = \tilde{W}^{0} \end{aligned} \tag{20}$$

Notice also that from (5) and (7) it is clear that

$$\hat{\varphi}^{p} > \hat{\varphi}^{0}. \tag{21}$$

Thus we have:

Proposition 5
If the CA sets the optimum penalty then *partial legal uncertainty* is unambiguously welfare superior to *no legal uncertainty* and entails a higher penalty.

The reason is straightforward, with *partial legal certainty* the CA can exploit its ability to discriminate (albeit imperfectly) between harmful and benign acts and set a penalty that deters ALL harmful acts. However when there is no legal uncertainty then, although it can deter firms from taking action when they know their actions will be disallowed, there will still be some harmful actions taken. It has to use a higher penalty to deter harmful actions because firms still only have a chance of having their acts disallowed.

This is a stronger result that in our earlier work where we could show that *partial legal certainty* welfare-dominated *no legal uncertainty* only in certain cases.

*3.2 **Effects-Based:** No Legal Uncertainty vs Complete Legal Uncertainty*

From (6), (10) and (11) we know that $\tilde{W}^0 = \tilde{W}(1) \geq \tilde{W}(x) \quad \forall x = \pi(1-\delta), 0 \leq x \leq 1$, while from (13) and (15) we know that $\tilde{W}^c = MAX\left[\tilde{W}, 0\right] \geq 0$. We also know that provided the CA's **Effects-Based** rule can **effectively discriminate** then it is certainly the case that $\tilde{W}(1) = \tilde{W}^0 > 0$. Taken together this implies that

$$\tilde{W}^0 = \tilde{W}(1) \geq \tilde{W}^c, \tag{22}$$

with equality iff $\pi(1-\delta) = 1 \Leftrightarrow \pi = 1$ and $\delta = 0$.

However from (5) (12) and (14) we see that:

$$\text{if} \quad 0 < \tilde{W}^c < \tilde{W}^0 \text{ then } \hat{\varphi}^c = 0 < \hat{\varphi}^0 = \left[\frac{1}{\pi} - 1\right] + \delta, \tag{23}$$

while

$$\text{if} \quad \tilde{W}^c = 0 < \tilde{W}^0 \text{ then } \hat{\varphi}^c = \left[\frac{1}{\pi \bar{\bar{p}}} - 1\right] + \delta > \hat{\varphi}^0 = \left[\frac{1}{\pi} - 1\right] + \delta. \tag{24}$$

So we have:

Proposition 6
If the CA sets the optimum penalty then:
(a) Welfare under *No Legal Uncertainty* is at least as great as that under *Complete Legal Uncertainty.*
(b) Welfare under *No Legal Uncertainty* is identical to that under *Complete Legal Uncertainty* if and only if there is no procedural uncertainty – i.e. all cases are investigated and decisions reached without delay.
(c) Optimal penalties under *No Legal Uncertainty* may be higher than those under *Complete Legal Uncertainty* - certainly the case if the action is *Presumptively Legal* – but may also be lower – which will be the case only if action is *Presumptively Illegal*.

*3.3 **Effects-Based** with Complete Legal Uncertainty vs **Per Se***

If the action is *Presumptively Illegal* then from (13), (15) and (19) we see that

$$\tilde{W}^c = MAX\left[\tilde{W}, 0\right] \geq 0 = \tilde{W}^{PSI} \tag{23}$$

whereas

$$\hat{\varphi}^{c} = \begin{cases} \left[\dfrac{1}{\pi \bar{p}} - 1\right] + \delta > \left[\dfrac{1}{\pi} - 1\right] + \delta = \hat{\varphi}^{PSI} & \text{as } \tilde{W} < 0 \\ 0 < \left[\dfrac{1}{\pi} - 1\right] + \delta = \hat{\varphi}^{PSI} & \text{as } \tilde{W} > 0 \end{cases} \tag{24}$$

On the other hand, if the action is *Presumptively Legal* then from (13) (19) and (9) we have:

$$\begin{aligned} \tilde{W}^{c} &= -\bar{h} + \pi(1-\delta)\left[p_H \gamma h_H - (1-p_B)(1-\gamma)(-h_B)\right] \\ &= \tilde{W}^{PSL} + \pi(1-\delta)\left[p_H \gamma h_H - (1-p_B)(1-\gamma)(-h_B)\right] \end{aligned} \tag{25}$$

and from (4) we know that the second term is positive as long as the CA's Rule can **effectively discriminate.**

In addition we know from (12) and (16) we know that

$$\hat{\varphi}^{c} = \hat{\varphi}^{PSL} = 0. \tag{26}$$

So we have

Proposition 7

(a) If an action is *Presumptively Illegal* then an ***Effects-Based*** *Rule with Complete Legal Uncertainty* is no worse and may sometimes be better than a **Per Se** Rule. In cases where it is welfare superior the optimal penalty is higher, otherwise the optimal penalty is lower – indeed zero.

(b) If an action is *Presumptively Legal* and if the CA's rule can **effectively discriminate** then an ***Effects-Based*** *Rule with Complete Legal Uncertainty* welfare dominates a **Per Se** Rule but requires exactly the same penalty – zero.

From Propositions 5, 6 and 7 we get:

Proposition 8

Provided the CA can **effectively discriminate** and provided it sets optimal penalties then there is a clear welfare ranking of decision environments: an **Effects-Based** rule with *Partial Legal Uncertainty* dominates that with *No Legal Uncertainty* which in turn dominates that with *Complete Legal Uncertainty* which in turn dominates a **Per Se** Rule.

Put differently a **Per Se** Rule is never better than an **Effects –Based** Rule - and is in many cases worse - however great the degree of *Legal Uncertainty*.

However while in many cases a higher welfare ranking is associated with the imposition of tougher penalties, this s not always the case.

Optimal Penalties and Appeals

Consider now what happens when firms can appeal. This affects the analysis in two ways:

(i) There are two decisions that the authorities can influence – the decision to take the action and the decision to appeal.
(ii) There are potentially different instruments since the penalties available to CA may be different from those available to court.

Case 1. Partial Legal Uncertainty

Let $\beta_e, \quad e = H, B$ be probability that action from environment *e* will be banned by CA, and $\beta_e^a, \quad e = H, B$ the probability that it will be banned on appeal. Assume $\beta_H > \beta_B$ and $\beta_H^a > \beta_B^a$.

Let $\pi, \quad 0 < \pi < 1$ be fraction of actions investigated by CA, $\delta, \quad 0 \le \delta < 1$ be the delay in CA's taking a decision and $\delta^a, \quad 0 \le \delta^a \le 1 - \delta$ the further delay in making decision on appeal.

Let $F_e, \quad 0 \le F_e \le 1$ be fraction of firms from environment *e* that are deterred from taking the action and let $F_e^a, \quad 0 \le F_e^a \le 1$ be the fraction of firms from environment *e* whose actions have been banned by CA who decide to appeal.

Then Social Welfare is:

$$W = \gamma(-h_H)(1 - F_H)\left\{[1 - \pi\beta_H(1-\delta)] + \pi\beta_H F_H^a\left[\beta_H^a\delta^a + (1-\beta_H^a)(1-\delta)\right]\right\} + (1-\gamma)(-h_B)(1 - F_B)\left\{[1 - \pi\beta_B(1-\delta)] + \pi\beta_B F_B^a\left[\beta_B^a\delta^a + (1-\beta_B^a)(1-\delta)\right]\right\} \qquad (1)$$

If we could freely choose the various fractions we would set $F_H = 1; \quad F_B = 0; \quad F_B^a = 1$. If $F_H = 1$ then society indifferent to F_H^a.

Now suppose CA sets penalty: $\psi + \varphi b, \quad \psi \ge 0, \varphi \ge 0$ while court can set penalty $\psi^a + \varphi^a b, \quad \psi^a \ge 0, \varphi^a \ge 0$. Suppose it costs $m > 0$ to mount an appeal, and that if appeal is turned down it is the penalty imposed by the court which firm has to pay.

Then for a firm from environment *e* whose private benefit is *b* mounting an appeal will be beneficial if

$$b\left[(1+\varphi)-\beta_e^a\left(1+\varphi^a-\delta^a\right)\right]\geq m+\beta_e^a\psi^a-\psi\ . \tag{2}$$

To ensure that everyone appeals we need

$$m+\beta_e^a\psi^a-\psi\leq 0 \tag{3}$$

and

$$(1+\varphi)>\beta_e^a\left(1+\varphi^a-\delta^a\right) \tag{4}$$

In what follows let us assume that $\psi\geq m$ and $\psi^a=0$ - so court uses only deterrence as a criterion in setting penalty.

Decision to take the action. If a firm knows that it will definitely appeal if it takes the action and has it banned by CA, then the net benefit from taking the action is:

$$b\left\{\left[1-\pi\beta_e\beta_e^a\left(1-\delta-\delta^a\right)\right]-\pi\beta_e\beta_e^a\varphi^a\right\}. \tag{5}$$

So if

$$\psi^a=0,\quad \varphi^a=\frac{1}{\pi\beta_H\beta_H^a}-\left(1-\delta-\delta^a\right); \tag{6}$$

and

$$\psi=m,\quad \varphi=MAX\left[0,\beta_H^a\left(1+\varphi^a-\delta^a\right)-1\right] \tag{7}$$

then we will have

$$F_H=1,\quad F_B=0,\quad F_H^a=F_B^a=1\ . \tag{8}$$

Substitute (6) into (7) to get

$$\varphi=\frac{1}{\pi\beta_H}-(1-\delta)>0\ . \tag{9}$$

So CA sets penalties that reflect both harm and deterrence. The harm component is set ti match cost of mounting an appeal while the deterrence component is set at the level that would be optimal if there is no appeal. The court sets a higher proportional penalty than CA because it has to counter effects of longer delay and lower probability of action's being banned.

Case 2 Complete Legal Uncertainty

Here firms do not know their type and base decisions on the average probability of having actions banned

So now, from (1) social welfare is:

$$W = (1-F)\tilde{W} \tag{10}$$

where F, $0 \leq F \leq 1$ is the common fraction of firms from each environment that are deterred; thus:

$$\begin{aligned}\tilde{W} = \gamma\left(-h_H\right)\left\{\left[1-\pi\beta_H(1-\delta)\right]+\pi\beta_H F^a\left[\beta_H^a\delta^a+\left(1-\beta_H^a\right)(1-\delta)\right]\right\}+ \\ (1-\gamma)\left(-h_B\right)\left\{\left[1-\pi\beta_B(1-\delta)\right]+\pi\beta_B F^a\left[\beta_B^a\delta^a+\left(1-\beta_B^a\right)(1-\delta)\right]\right\}\end{aligned} \tag{11}$$

where F^a, $0 \leq F^a \leq 1$ is the common fraction of firms from each environment that appeal.

To understand whether or not it is socially desirable to have appeals, consider two sub-cases.

(a) Long Delay in Appeals.

Here $\delta^a = 1-\delta$ and (11) becomes:

$$\begin{aligned}\tilde{W} = \left\{\gamma\left(-h_H\right)\left[1-\pi\beta_H(1-\delta)\right]+(1-\gamma)\left(-h_B\right)\left[1-\pi\beta_B(1-\delta)\right]\right\}+ \\ \pi\delta^a F^a\left[\gamma\left(-h_H\right)\beta_H+(1-\gamma)\left(-h_B\right)\beta_B\right]\end{aligned} \tag{12}$$

But we know from our previous work that, as long as the CA can effectively discriminate then:

$$\gamma\left(-h_H\right)\beta_H+(1-\gamma)\left(-h_B\right)\beta_B < 0 \tag{13}$$

that is the average harm caused by actions banned by the CA is positive, and so the optimal value of $F^a = 0$ that is we want to stop firms from appealing. This is because the decision taken by the court is irrelevant and all that having an appeals process does is to prolong the time for which actions that have been banned by the CA and that are on balance harmful are taken.

But then

$$\overline{W} = \gamma\left(-h_H\right)\left[1-\pi\beta_H(1-\delta)\right]+(1-\gamma)\left(-h_B\right)\left[1-\pi\beta_B(1-\delta)\right] \quad (14)$$

Now have some discussion of the sign of this term and hence optimal deterrence.

(b) No Delay in Appeals

Here $\delta^a = 0$ and (11) becomes

$$\overline{W} = \left\{\gamma\left(-h_H\right)\left[1-\pi\beta_H(1-\delta)\right]+(1-\gamma)\left(-h_B\right)\left[1-\pi\beta_B(1-\delta)\right]\right\}+$$
$$\pi(1-\delta)F^a\left[\gamma\left(-h_H\right)\beta_H\left(1-\beta_H^a\right)+(1-\gamma)\left(-h_B\right)\beta_B\left(1-\beta_B^a\right)\right]$$

i.e.

$$\overline{W} = \left\{\gamma\left(-h_H\right)\left[1-\pi\beta_H(1-\delta)\right]+(1-\gamma)\left(-h_B\right)\left[1-\pi\beta_B(1-\delta)\right]\right\}+$$
$$\pi(1-\delta)F^a\left\{\left(1-\beta_H^a\right)\left[\gamma\left(-h_H\right)\beta_H+(1-\gamma)\left(-h_B\right)\beta_B\right]+(1-\gamma)\left(-h_B\right)\beta_B\left(\beta_H^a-\beta_B^a\right)\right\} \quad (15)$$

If we look at the coefficient on F^a then the first term in the expression in curly brackets is negative and the second positive.

Sufficient conditions for this overall this expression to be positive are:

b.1 $\quad 1 \approx \beta_H^a \gg \beta_B^a$

or

b.2 $\quad \gamma\left(-h_H\right)\beta_H+(1-\gamma)\left(-h_B\right)\beta_B \approx 0$

So either the court is extremely good at determining harm or the CA’s rule can only just effectively discriminate. These are intuitively reasonable conditions for society to want to have an appeals process.

Case 3. <u>No Legal Uncertainty</u>

Here welfare will be as without appeals (equation (3) above) except that, with appeals, from the fraction (1 – F) of those firms who know for sure that their actions will be disallowed but are not deterred and take the action – to take advantage of incomplete coverage and the delay in reaching decisions by the CA – if a fraction F^a appeals – to take advantage now of the delay involved in the appeals process – the welfare expression will contain an extra term, as follows:

$$-(1-F)F^a\delta^a\left[\gamma\left(h_H\right)\beta_H-(1-\gamma)\left(-h_B\right)\beta_B\right]<0$$

Which is negative given that, by (4), the term in square brackets is positive for effectively discriminating rules. It follows that with no-LU welfare is decreasing in the fraction of firms appealing, that is it is optimal not to have an appeals process. This is intuitively plausible since in this case the only firms that would appeal are those that would certainly be disallowed, so there is no gain to society of allowing them to appeal.