

Cartel Sales Dynamics when Monitoring for Compliance is More Frequent than Punishment for Non-Compliance

Joseph E. Harrington, Jr. and Juan-Pablo Montero*

December 10, 2013

Abstract

This study investigates when a cartel that uses a sales quota allocation scheme monitors more frequently than it enforces; for example, monitoring of sales is done on a weekly basis but firms are only required to comply with sales quotas on a quarterly basis. In a simple three-period quantity game with *iid* cost and demand shocks, we show that the volatility of a cartel member's sales follows a U-shape within the compliance horizon. In comparison, sales volatility is constant over time under competition. This result offers a simple empirical test for distinguishing collusion from competition using sales data.

1 Introduction

Many cartels, especially those in intermediate goods markets, do not just coordinate on prices but also on an allocation of the market (Harrington, 2006). One method of market allocation is sales quotas which can take the form of an absolute quantity or a market share. An interesting feature of these quota-allocation schemes is that the timing of monitoring and enforcement are not necessarily the same. Members of the lysine cartel, for example, coordinated on sales quotas to be enforced on a quarterly basis although they reported their sales on a monthly basis. As explained in Harrington and Skrzypacz

*Harrington is with the Department of Business Economics and Public Policy at The Wharton School of the University of Pennsylvania (harrj@wharton.upenn.edu) and Montero is with the Department of Economics of the Pontificia Universidad Católica de Chile (jmontero@uc.cl). The first author acknowledges the financial support of the National Science Foundation (SES-1148129) and the second the support of ISCI (Milenio P05-004F & Basal FBO-16). Both authors thank Martin Pietz for his constructive comments.

(2011), one of the member companies of the lysine cartel – Ajinomoto – was assigned the task of preparing monthly "scorecards" for the cartel. Each member company telephoned or mailed their *monthly* sales volumes to an executive of Ajinomoto, who then prepared a spreadsheet that was distributed at the *quarterly* meetings of the cartel. And to enforce compliance the cartel utilized a "guaranteed buy-ins" scheme: if during the quarter a company sold more than its quota, it had to buy output from producers that were below quota.

The citric acid cartel, which also operated during the 1990s, is another example. In that cartel, monitoring of sales was done on a monthly frequency but enforcement occurred only annually. Members coordinated on a sales quota scheme in terms of market shares. On a monthly basis, each company reported its sales to an executive of Hoffmann LaRoche. The data were then assembled and reported back to the members by telephone. Enforcement was through a "buy-back system" whereby a company that exceeded its assigned quota in any one year was obliged to purchase product from the companies with sales below their quota in the following year.

It could be argued that a similar phenomenon may apply in an on-going antitrust case in which three chicken producers have been accused of having coordinated on sales quotas in Chile's wholesale chicken market. The defense argues that these were not quota allocations but rather sales projections provided by the trade association. (Without taking a position here on what they were, for our current purposes we will refer to them as quota allocations.) The companies were accused of restricting competition by allocating weekly sales quotas of total kilos of chicken in its different forms (legs, breast and whole chicken). This claim was examined in Montero et al (2013) from which we provide Figure 1 which compares companies' actual weekly sales with their weekly "quota allocations" for 2007 (figures for other years are quite similar). It is evident from Figure 1 that the companies systematically and significantly deviated from their allocations, both individually and as a group. It is common to find weeks in which their sales were 10 or 20% above their "quotas" or 10 or 20% below, and this weekly "deviation" pattern is not unique to the largest company but applies to all three firms.

From our knowledge of the operation of past cartels, however, one cannot rule out the existence of a collusive sales quota agreement because there is a systematic and significant discrepancy between actual weekly sales and purported weekly sales quotas. It is possible that the compliance horizon is not weekly but of some longer length of time. In fact, in its accusation and without being explicit about it, the National Prosecuting Authority (FNE) appears open to the idea that the compliance horizon may have not been the week but rather the calendar year based on its extensive discussion of how the companies followed rather closely sales suggestions at the annual level (FNE, 2011, pp. 8-12). Figure 2 shows that "deviations" at the annual level were indeed far smaller, which may lead

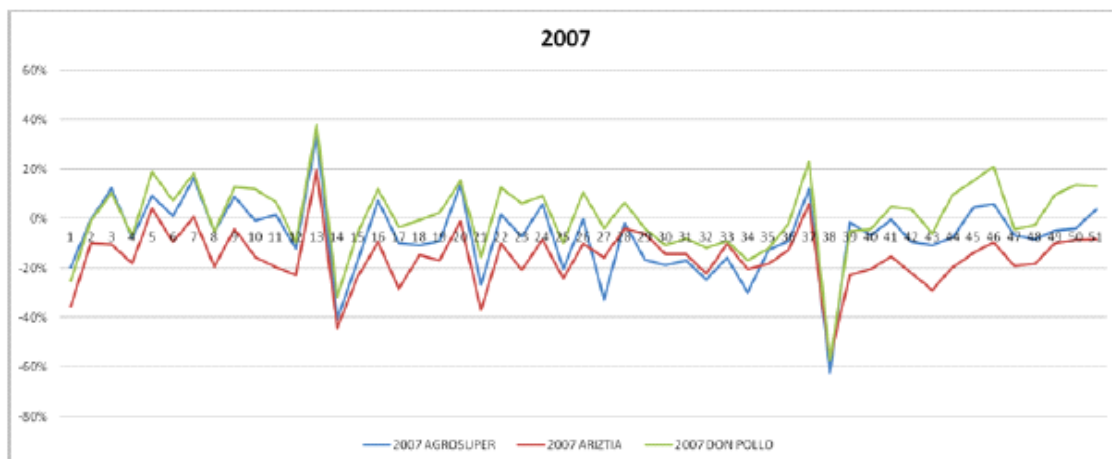


Figure 1: Weekly "deviations" of chicken producers (reproduced from Figure 3.3 of Montero et al., 2013)

one to reasonably argue that the compliance horizon with the agreement was not the week but the calendar year; that is, the arrangement was for firms to meet their sales quotas at the annual level with monitoring done at the weekly level in order to provide information to assist in making that happen. Of course, an alternative hypothesis is that the lower deviation from sales projection at the annual level is the product of the law of large numbers and not of collusion. If observed weekly sales equal weekly sales projections plus *iid* noise then the variance of annual sales will necessarily be less than the variance of weekly sales. Thus, competition could well produce this pattern. It is then important to develop other tests that could distinguish collusion from competition in the intertemporal pattern of sales, which is the contribution of this paper.

In this paper we exploit the different timing between monitoring and compliance for the purpose of deriving behavior that would allow one to empirically distinguish collusion from competition. This investigation is conducted for a simple linear quantity game with iid cost and demand shocks. We find that early in the compliance horizon (which, as described above, is conjectured to be annual for the purported chicken processing cartel), the sales of a cartel member are more sensitive to contemporaneous shocks than are later sales to contemporaneous shocks. The logic is that a cartel member will take greater advantage of favorable shocks (that is, low cost or high demand) early in the compliance horizon by producing above average, as it knows there will be ample time to offset the over-production with under-production in the remainder of the year and still hit the cumulative quota target established by the cartel. Based on this "adjustment-to-shocks"

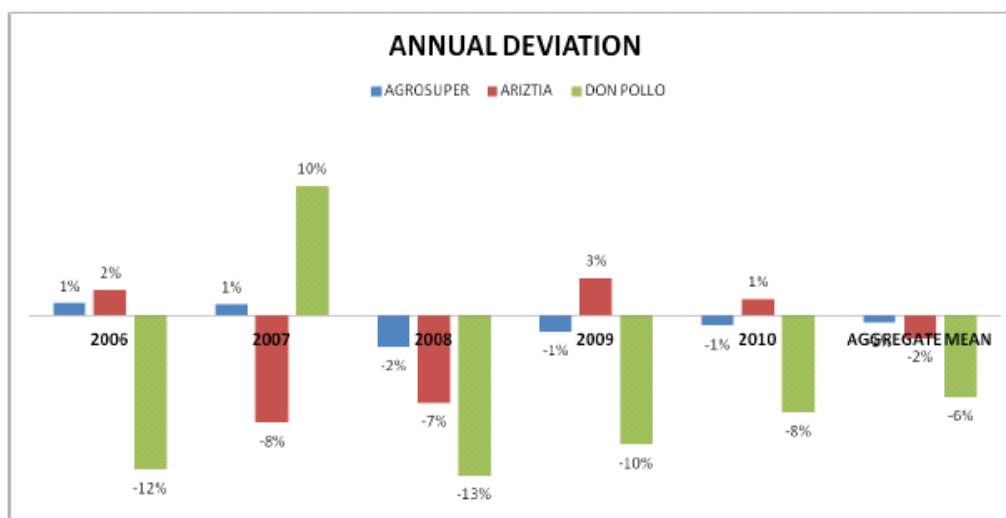


Figure 2: Annual "deviations" of chicken producers (reproduced from Figure 3.4 of Montero et al., 2013)

effect, the prediction is derived that, under collusion, the sensitivity of contemporaneous sales to contemporaneous shocks should decline over the compliance horizon. In comparison, in our stationary environment, the sensitivity of sales to shocks is stationary over time under competition.

There is a second effect which we refer to as the "propagation-of-shocks" effect. As firms move through the compliance horizon, a cartel member's residual or available quota (that is, the quota less sales already realized) is more volatile as the result of previous shocks which adds to the volatility of sales. As the "adjustment-to-shocks" and "propagation-of-shocks" effects operate in different directions, we find that the volatility of sales of a cartel member follows a U-shape as opposed to the constant pattern under competition. That the volatility of sales under collusion follows a distinct path from that under competition could prove useful as a test for the presence of a cartel that engages in sales quotas.

The remainder of the paper is organized as follows. The model is presented in Section 2 and firms' equilibrium strategies are derived in Section 3. In Section 4, we derive the equilibrium implications for the intertemporal pattern in the variance of sales and how it can allow us to distinguish collusive from competitive behavior. We conclude in Section 5.

2 Model

The model is designed to explore the implications for sales dynamics of a cartel that implements sales quotas with a multi-period compliance horizon. The compliance horizon is three periods in length which is sufficient to draw out the main insight. In this simple model, we do not consider the collusive strategies deployed in the infinite horizon setting but rather presume it involves sales quotas and just assume that firms are constrained to satisfying the sales quota by the end of the compliance horizon.¹ The environment is enriched, however, by allowing for cost and demand shocks. After presenting the model, we then solve for equilibrium strategies for firms over the compliance horizon. In the following section, we explore the implications of equilibrium behavior for the cartel sales path.

Consider an industry with two firms, i and j , and a collusive agreement in which each firm is allocated a total output quota of $\rho^i = \rho^j = \rho$ every three periods. This is the same as to say that firms receive a quota of $\rho/3$ per period but compliance is only enforced at the end of the third period. A firm is free to allocate its sales any way it wants over the three periods as long as, at the end of period 3, its cumulative sales do not exceed ρ . We assume that the firms compete in the market by simultaneously choosing output q_t^i and q_t^j in each period, $t = 1, 2, 3$. Sales are publicly observed. There is no discounting over the three periods (which is unimportant for our results).

Firms face cost and demand shocks. Each firm has constant marginal cost where the realized cost is *iid* across firms and time. Let $f : [\underline{c}, \bar{c}] \rightarrow [0, \infty)$ be the continuous density function on cost, and $\gamma \equiv \int_{\underline{c}}^{\bar{c}} sf(s)ds$ and $\sigma_c^2 \equiv \int_{\underline{c}}^{\bar{c}} (s - \gamma)^2 f(s)ds$ denote the mean and variance of cost, respectively. At the beginning of each period t and before choosing its quantity q_t^i , firm i learns its current period cost, c_t^i , but does not observe the other firm's cost. As is standard in the Cournot model, the market clears according to the linear inverse demand function $P(Q_t) = a_t - bQ_t$, where $Q_t = q_t^i + q_t^j$ is total output, and a_t is a demand shock that is *iid* over time. Let $g : [\underline{a}, \bar{a}] \rightarrow (c, \infty)$ be its continuous density function, and $\alpha \equiv \int_{\underline{a}}^{\bar{a}} sg(s)ds$ and $\sigma_a^2 \equiv \int_{\underline{a}}^{\bar{a}} (s - \alpha)^2 g(s)ds$ denote the associated mean and variance, respectively. Firms observe a_t before setting quantities for the period.

It is evident that in the absence of shocks (that is, $a_t = a$ and $c_t^i = c$ for sure and for all t) it would be impossible for the analyst to separate competitive (Cournot) behavior from collusive behavior by just looking at the evolution of sales. With either form of conduct, production is constant over time. Our focus is on how cost and demand shocks

¹As described and analyzed in Harrington and Skrzypacz (2007, 2011), some cartels imposed a penalty to exceeding the sales quota which is continuous in the discrepancy between sales and the quota. Thus, firms could violate the quota without dire consequences. In the future, it is our intent explore the questions of this paper with such a specification but for this paper we implicitly assume that the consequences of violating the sales quota are sufficiently severe that firms ensure that the quota is satisfied.

can allow us to separate collusive from competitive behavior using sales data.

3 Equilibrium Behavior

To facilitate the exposition, we consider first the case of cost shocks and then add demand shocks.

3.1 Case of Cost Shocks

Assume for now that inverse demand is deterministic: $P(Q) = a - bQ$. Denote by $\phi_1 : [\underline{c}, \bar{c}] \rightarrow [0, \infty)$ a firm's period 1 (symmetric) equilibrium strategy - which prescribes a quantity given its period 1 cost - and by $\phi_2 : [\underline{c}, \bar{c}] \times [0, \rho]^2 \rightarrow [0, \infty)$ a firm's period 2 (symmetric) equilibrium strategy - which prescribes a quantity given its period 2 cost and the available quotas of both firms from period 1.² Setting ρ sufficiently small for the problem to be non-trivial (that is, it is a binding constraint for colluding firms), a firm's period 3 strategy is simply to produce an amount equal to its available quota: $q_3^i = \rho - q_2^i - q_1^i$. To find equilibrium strategies ϕ_1 and ϕ_2 , we proceed by backward induction.

Suppose we are in period 2 and the available quotas at the beginning of the period are $\rho_2^i \equiv \rho - q_1^i$ and $\rho_2^j \equiv \rho - q_1^j$. Note that the available quotas are observed by both firms at the beginning of each period which is the result of their quantities being observed. Firm i 's expected total payoff as of period 2 (given its period 2 cost) is:

$$\int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} [(a - bq_2^i - b\phi_2(c_2^j, \rho_2^j, \rho_2^i) - c_2^i)q_2^i + (a - b(\rho_2^i - q_2^i) - b(\rho_2^j - \phi_2(c_2^j, \rho_2^j, \rho_2^i)) - c_3^i)(\rho_2^i - q_2^i)] f(c_2^j)f(c_3^i) dc_2^j dc_3^i. \quad (1)$$

Note that the expectation is taken with respect to the other firm's period 2 cost and a firm's own period 3 cost. (The other firm's period 3 cost does not matter given the assumption that firm i will produce $\rho_2^i - q_2^i$ in period 3.) Note also that the equilibrium strategy for the other firm, $\phi_2(c_2^j, \rho_2^j, \rho_2^i)$, depends not only on the firm's own cost but may also depend on the available quotas of both firms.

²Conditioning on the available quotas is equivalent to conditioning on the period 1 quantities. In principle, the period 2 strategy could also condition on a firm's period 1 cost, but that will not be the case in equilibrium.

Recalling that γ is the expected cost, (1) is equivalent to

$$\int_{\underline{c}}^{\bar{c}} [(a - bq_2^i - b\phi_2(c_2^j, \rho_2^j, \rho_2^i) - c_2^i) q_2^i + (a - b(\rho_2^i - q_2^i) - b(\rho_2^j - \phi_2(c_2^j, \rho_2^j, \rho_2^i)) - \gamma)(\rho_2^i - q_2^i)] f(c_2^j) dc_2^j.$$

Take the first order condition (FOC), and solve for optimal q_2^i which is $\phi_2(c_2^i, \rho_2^i, \rho_2^j)$:

$$\phi_2(c_2^i, \rho_2^i, \rho_2^j) = \left(\frac{1}{4b}\right) [2b\rho_2^i + b\rho_2^j - 2bE[\phi_2(c_2^j, \rho_2^j, \rho_2^i)] + \gamma - c_2^i]. \quad (2)$$

where

$$E[\phi_2(c_2^j, \rho_2^j, \rho_2^i)] = \int_{\underline{c}}^{\bar{c}} \phi_2(c_2^j, \rho_2^j, \rho_2^i) f(c_2^j) dc_2^j.$$

Taking the expectation of (2) and using the fact that c_2^i and c_2^j are independent:

$$\begin{aligned} E[\phi_2(c_2^i, \cdot)] &= \int_{\underline{c}}^{\bar{c}} \left(\frac{1}{4b}\right) [2b\rho_2^i + b\rho_2^j - 2bE[\phi_2(c_2^j, \cdot)] + \gamma - c_2^i] f(c_2^i) dc_2^i \\ E[\phi_2(c_2^i, \cdot)] &= \left(\frac{1}{4b}\right) [2b\rho_2^i + b\rho_2^j - 2bE[\phi_2(c_2^j, \cdot)] + \gamma - \gamma] \\ E[\phi_2(c_2^i, \cdot)] &= \left(\frac{1}{4b}\right) [2b\rho_2^i + b\rho_2^j - 2bE[\phi_2(c_2^j, \cdot)]] \end{aligned} \quad (3)$$

Similarly, we have

$$E[\phi_2(c_2^j, \cdot)] = \left(\frac{1}{4b}\right) [2b\rho_2^j + b\rho_2^i - 2bE[\phi_2(c_2^i, \cdot)]]$$

We have two equations, (2) and (3), and two unknowns. Solving we obtain

$$E[\phi_2(c_2^i, \cdot)] = \frac{\rho_2^i}{2}, \quad E[\phi_2(c_2^j, \cdot)] = \frac{\rho_2^j}{2}$$

and using these values in (2) leads to

$$\phi_2(c_2^i, \rho_2^i, \rho_2^j) = \frac{\rho_2^i}{2} + \frac{\gamma - c_2^i}{4b}. \quad (4)$$

On average, a firm produces half of its available quota in period 2, leaving the other half for period 3. When its period 2 cost is favorable (in the sense that it is below the mean value for cost) then it produces above the level of $\rho_2^i/2$ in order to take advantage of the low cost realization. If instead the cost realization is unfavorable then produces below $\rho_2^i/2$ in order to save more quota for period 3 when, on average, the cost realization will

be better than that for period 2.³

Recall that (4) was derived under the conjecture that a firm would want to produce at its remaining quota $\rho_2^i - q_2^i$ in period 3. We then need to verify that is indeed optimal for a firm given its period 2 equilibrium quantity and do so for all cost realizations in periods 2 and 3 and for all available period 2 quotas. For that we just need to check what happens when firms' available quotas are at their maximum values in period 3, which occurs, in equilibrium, when both firms had the highest cost of \bar{c} in periods 1 and 2. In that case, the myopic (unconstrained) best reply in period 3 exceeds the available quota if and only if:

$$\frac{a - \bar{c}}{2b} - \left(\frac{1}{2}\right) [\rho - \phi_1(\bar{c}) - \phi_2(\bar{c}, \rho - \phi_1(\bar{c}))] > \rho - \phi_1(\bar{c}) - \phi_2(\bar{c}, \rho - \phi_1(\bar{c})) \quad (5)$$

We cannot proceed any further with the verification of this conjecture without first deriving the period 1 equilibrium strategy ϕ_1 , which we do next.

Given ρ_2^i and ρ_2^j , deriving period 1 equilibrium strategies requires that we first compute firms' payoffs in period 2 onwards as a function of the unused allocations coming out of period 1. Denote these payoffs by $V_2^i(\rho_2^i, \rho_2^j)$ and $V_2^j(\rho_2^i, \rho_2^j)$. Thus, firm i 's expected payoff in period 2 (given the available quotas) is:

$$E[V_2^i(\rho_2^i, \rho_2^j)] = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} [(a - b\phi_2(c_2^i, \cdot) - b\phi_2(c_2^j, \cdot) - c_2^i) \phi_2(c_2^i, \cdot) + (a - b(\rho_2^i - \phi_2(c_2^i, \cdot)) - b(\rho_2^j - \phi_2(c_2^j, \cdot)) - \gamma) (\rho_2^i - \phi_2(c_2^i, \cdot))] f(c_2^i) f(c_2^j) dc_2^i dc_2^j.$$

It takes a bit of algebra to show that

$$\begin{aligned} \frac{d}{d\rho_2^i} E[V_2^i(\rho_2^i, \rho_2^j)] &= \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} \left[a - \frac{1}{2}\gamma - \frac{1}{12}c_2^j - \frac{5}{12}c_2^i - \frac{1}{2}b\rho_2^j - b\rho_2^i \right] f(c_2^i) f(c_2^j) dc_2^i dc_2^j \\ &= a - \frac{1}{2}bE[\rho_2^j] - b\rho_2^i - \gamma. \end{aligned}$$

Now, firm i 's expected total payoff as of period 1 (given its period 1 cost) is:

$$\int_{\underline{c}}^{\bar{c}} [(a - bq_1^i - b\phi_1(c_1^j) - c_1^i) q_1^i] f(c_1^j) dc_1^j + E[V_2^i(\rho_2^i, \rho_2^j)]$$

where recall $\rho_2^i \equiv \rho - q_1^i$ and $\rho_2^j \equiv \rho - q_1^j$. Firm i 's problem is to choose q_1^i , given the cost

³Note that the equilibrium strategy (4) is independent of the rival's available quota which is probably due to the linearity of the problem (that is, linear demand and cost functions), and thus need not hold more generally.

realization c_1^i . The FOC for this problem is

$$a - 2bq_1^i - bE[\phi_1(c_1^j)] - c_1^i = a - \frac{1}{2}b(\rho - E[\phi_1(c_1^j)]) - b(\rho - q_1^i) - \gamma,$$

which, after rearranging, leads to the optimal period 1 quantity $\phi_1(c_1^i)$:

$$\phi_1(c_1^i) = \frac{1}{6b} (3b\rho - 3bE[\phi_1(c_1^j)] + 2\gamma - 2c_1^i).$$

Proceeding as we did in solving for the period 2 equilibrium strategy, we have

$$E[\phi_1(c_1^i)] = \frac{1}{6b} (3b\rho - 3bE[\phi_1(c_1^j)]),$$

and, using the symmetry of the problem, we get

$$E[\phi_1(c_1^i)] = E[\phi_1(c_1^j)] = E[\phi_1(c_1)] = \frac{\rho}{3}.$$

Therefore,

$$\phi_1(c_1^i) = \frac{\rho}{3} + \frac{\gamma - c_1^i}{3b} \quad (6)$$

Similar to the period 2 case, a firm produces above (below) the per period quota of $\rho/3$ when its cost realization is below (above) average.

In comparing the period 1 and period 2 equilibrium strategies in (4) and (6), respectively, we find that the period 1 quantity is more sensitive to the period 1 shock than the period 2 quantity is to the period 2 shock, which in turn are more sensitive than the period 3 quantity to the period 3 shock (which, by the assumption that the quotas are binding with probability one, is not sensitive at all to period 3 cost):

$$\frac{\partial \phi_1}{\partial c_1^i} = -\frac{1}{3b} < -\frac{1}{4b} = \frac{\partial \phi_2}{\partial c_2^i} < 0 = \frac{\partial \phi_3}{\partial c_3^i}.$$

Property 1: A firm's quantity is more sensitive to the contemporaneous cost shock when it is earlier in the compliance horizon.

A firm's quota for the compliance horizon is a scarce resource and when it produces above its quota in a given period there is less quota available for later production. Given that cost is stochastic, a firm wants to save up its quota so that it can produce more when its cost is low. If a firm produces more in period 2 in order to take advantage of low period 2 cost, there is necessarily less output that can be produced in period 3 which means a firm could not take advantage of a low period 3 cost realization. In comparison, in period 1, a firm does not have to be as frugal when it comes to using up its quota because there are two periods with which to adjust its quantity in order to comply with the quota. Thus,

even if firm 1 produces at a high level in period 1 because of a low period 1 cost, it will still be in a position to produce at a high level in period 2 if there is a low period 2 cost. In sum, early in the compliance horizon, quantities are more sensitive to cost shocks as a firm can depart from "average quantity" knowing it'll have more periods to adjust its quantity and still hit the cumulative quantity target. As a firm gets near the end of the compliance horizon, its quantity becomes less sensitive to contemporaneous cost shocks and more sensitive to the available quota as it has a shorter time to manage its quantities in order to ensure that its sales do not exceed the its sales quota.

Notice, however, that equilibrium quantities are more sensitive to cost shocks in period 1 than in period 2 does not necessarily imply that the sales path of a cartel firm is more volatile at the beginning of the compliance window than at the end. There is a second force that operates in the opposite direction: the quantity in period 2 is not only subject to period 2 cost shocks but also to period 1 cost shocks which operate indirectly through the available quota (i.e., ρ_2^i). In the next section, we study how the combination of these two forces determines the evolution of sales volatility.

Before closing the section, we return to the verification we initiated in equation (5), which can be rewritten as

$$\begin{aligned} \frac{a - \bar{c}}{2b} &> \left(\frac{3}{2}\right) (\rho - \phi_1(\bar{c}) - \phi_2(\bar{c}, \rho - \phi_1(\bar{c}))) \\ \frac{a - \bar{c}}{3b} &> \left[\rho - \left(\frac{\rho}{3} + \frac{\gamma - \bar{c}}{3b}\right) - \left(\frac{\rho}{3} + \frac{\gamma - \bar{c}}{12b}\right)\right] \\ \frac{a - \bar{c}}{3b} &> \frac{\rho}{3} + \frac{5(\bar{c} - \gamma)}{12b} \Rightarrow \frac{a - \bar{c}}{3b} - \frac{\rho}{3} > \frac{5(\bar{c} - \gamma)}{12b} \end{aligned} \quad (7)$$

Recall that we want to ensure that firm i finds it optimal to produce in period 3 so that its sales quota is binding; that is, it produces $\rho_2^i - q_2^i$ for any cost realizations for both firms in periods 1 and 2 and for any cost realization for this firm in period 3. Let us assume the per period collusive allocation $\rho/3$ is less than the average static Nash equilibrium quantity,

$$\frac{a - \gamma}{3b} > \frac{\rho}{3},$$

so that, on average, the sales quota is less than the competitive supply. In that case, (7) holds as $\bar{c} \rightarrow \gamma$. Hence, if the support on cost is not too large then (7) holds, and the equilibrium strategies found above, (4) and (6), are valid for any cost realizations in $[\underline{c}, \bar{c}]$.

3.2 Case of Cost and Demand Shocks

It is straightforward to add *iid* demand shocks to the model.⁴ As before, equilibrium strategies are found by backward induction. Suppose we are in period 2 and $\rho_2^i \equiv \rho - q_1^i$ and $\rho_2^j \equiv \rho - q_1^j$. Then a firm's payoff as of period 2 is

$$\int_{\underline{a}}^{\bar{a}} \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} [(a_2 - bq_2^i - b\phi_2(c_2^j, a_2, \cdot) - c_2^i) q_2^i + (a_3 - b(\rho_2^i - q_2^i) - b(\rho_2^j - \phi_2(c_2^j, a_3, \cdot)) - c_3^i) (\rho_2^i - q_2^i)] f(c_2^j) f(c_3^i) g(a_3) dc_2^j dc_3^i da_3.$$

Proceeding as before to derive the optimal q_2^i , we obtain the equilibrium strategy

$$\phi_2(c_2^i, a_2, \rho_2^i, \rho_2^j) = \frac{\rho_2^i}{2} + \frac{a_2 - \alpha}{6b} + \frac{\gamma - c_2^i}{4b} \quad (8)$$

where recall $\alpha = \int_{\underline{a}}^{\bar{a}} sg(s) ds$.

We next turn to period 1 quantities given the derived equilibrium quantity rules for period 2. Firm i 's expected total payoff as of period 1 (given period 1 demand and cost) is:

$$\int_{\underline{c}}^{\bar{c}} [(a_1 - bq_1^i - b\phi_1(c_1^j, a_1, \cdot) - c_1^i) q_1^i] f(c_1^j) dc_1^j + E[V_2^i(\rho_2^i, \rho_2^j)]$$

where $E[V_2^i(\rho_2^i, \rho_2^j)]$ is the expected equilibrium payoff in the continuation game with available quotas ρ_2^i and ρ_2^j and it is equal to

$$\int \dots \int [(a_2 - b\phi_2^i - b\phi_2^j - c_2^i)\phi_2^i + (a_3 - b(\rho_2^i - \phi_2^i) - b(\rho_2^j - \phi_2^j) - \gamma) (\rho_2^i - \phi_2^i)] f(c_2^i) f(c_2^j) g(a_3) g(a_2) dc_2^i dc_2^j da_3 da_2$$

where $\phi_2^i \equiv \phi_2(c_2^i, a_2, \rho_2^i, \rho_2^j)$.

Proceeding as in Section 3, it is not difficult to show that the period 1 equilibrium strategy is

$$\phi_1(c_1^i, a_1) = \frac{\rho}{3} + \frac{2(a_1 - \alpha)}{9b} + \frac{\gamma - c_1^i}{3b}. \quad (9)$$

Since $2/9 > 1/6$, we find a similar relationship to that in Section 3.1 in that period 1 quantities are more sensitive to period 1 demand shocks than period 2 quantities are to period 2 demand shocks, which in turn are more sensitive than period 3 quantities to period 3 demand shocks. In fact, the equilibrium strategy for any period $t = 1, 2, 3$ can

⁴Given that demand shocks are market-wide, we also expect the same implications to emerge from a common cost shock to firms as would occur, for example, from a common change in an input price.

be written more generally as

$$\phi_t(c_t^i, a_t, \rho_t^i) = \frac{\rho_t^i}{T-t+1} + k_t \left(\frac{2}{9b}(a_t - \alpha) + \frac{1}{3b}(\gamma - c_t^i) \right) \quad (10)$$

where $T = 3$ is the number of periods within the compliance window, $k_1 = 1$, $k_2 = 3/4$ and $k_3 = 0$. Note how the "adjustment" coefficients k_t 's go down overtime. Furthermore, from looking at the regularity in (10), it should be possible to find the equilibrium strategies for any T recursively (something we don't do here).

Property 2: A firm's quantity is more sensitive to the contemporaneous demand shock when it is earlier in the compliance horizon.

4 Volatility of Sales

The volatility of period 1 sales depends only on the volatility of period 1 shocks. From the equilibrium strategy (9), we obtain that the variance of period 1 sales is

$$\text{Var}[q_1^i] = \frac{1}{9b^2} \left(\frac{4}{9}\sigma_a^2 + \sigma_c^2 \right) \quad (11)$$

where σ_a^2 and σ_c^2 are the variances of demand and cost shocks, respectively. Furthermore, given perfect compliance with the cartel agreement (that is, $q_3^i = \rho - q_1^i - q_2^i$), it is evident that period 3 sales are more volatile than sales in any previous period. In fact, the unconditional variance of period 3 sales is simply

$$\text{Var}[q_3^i] = \text{Var}[q_1^i] + \text{Var}[q_2^i].$$

Much less evident is how the volatility of period 2 sales compares to the volatility of period 1 sales because of the operation of the two effects identified earlier. Equilibrium strategies show that firms adjust less to contemporaneous shocks as we move towards the end of the compliance window - this is the adjustment-to-shocks effect - but at the same time these later sales are not only subject to contemporaneous shocks but also to previous shocks through changes in the available quota left to cover future sales - this is the propagation-of-shocks effect. Using the equilibrium strategy (8) and the fact that $\rho_2^i = \rho - \phi_1(c_1^i, a_1)$ along the equilibrium path, the unconditional variance of period 2 sales is

$$\text{Var}[q_2^i] = \frac{1}{4}\text{Var}[q_1^i] + \frac{1}{16b^2} \left(\frac{4}{9}\sigma_a^2 + \sigma_c^2 \right).$$

Using (11), we have

$$\text{Var}[q_2^i] = \frac{13}{16}\text{Var}[q_1^i].$$

In sum,

$$\left(\frac{13}{16}\text{Var}[q_1^i] = \right) \text{Var}[q_2^i] < \text{Var}[q_1^i] < \text{Var}[q_3^i] \left(= \frac{29}{16}\text{Var}[q_1^i]\right)$$

There is then the implication that period 2 sales are less volatile than both period 1 and period 3 sales. This lower volatility is the result of the adjustment-to-shocks effect dominating the propagation-of-shocks effect. Thus, early in the horizon, one would expect a high adjustment effect to dominate a low propagation effect in that sales would be mostly sensitive to contemporaneous cost shocks. As the firm moves forward, its sales becomes less sensitive to contemporaneous cost shocks and more sensitive to previous shocks as reflected in the remaining quota. In the intermediate time period, a lower adjustment effect dominates a higher propagation effect which results in overall less volatility. When the firm gets near the end of the compliance horizon, there is virtually no room for adjustment to contemporaneous shocks; its sales are nevertheless highly volatile because of the propagation effect. This implies that the volatility of sales of a cartel member follows a U-shape as opposed to the volatility of a firm's sales in the absence of collusion which exhibit a stable path.

Property 3: Sales volatility is minimized in the middle of the compliance horizon.

That the volatility of sales under collusion follows a distinct path from that under competition is a useful finding that has a simple intuition and a clean prediction that one can take to sales data. For example, if a fraction of firms of an industry are suspected of forming a cartel one can use this theory to test for differences between the volatility of sales of those firms suspected of being part of the cartel and those that are not. Provided that one can get access to monthly series of sales data of both cartel and non-cartel firms, the citric acid cartel is a potentially good example to run such a test as it comprised the five largest producers which together controlled about 60 percent of global production and 67 percent of production in the European Union (Harrington, 2006).

In applying our theory to less than all inclusive cartels, however, it is important to keep in mind that the volatility of sales of a non-cartel firm is also affected by the way cartel firms price their products and adjust their quantities to shocks.⁵ This interaction makes for an interesting extension to our theory; but one that should not undermine the main result that collusive and competitive behavior can be distinguished by looking at

⁵In a model in which monitoring and enforcement follow the same timing, Montero and Guzman (2010) study how a cartel that faces a fringe of competitive suppliers adjust its overall quota to changes in demand.

the volatility of sales. Take for example a positive demand shock early on the compliance horizon. Cartel members would increase their production to take advantage of the favorable shock while non-cartel firms would not react as much to the positive shock because of the strategic substitutability of quantities. This would make cartel sales look even more volatile relative to those of non-cartel firms. This is just one example illustrating why our theory should still apply in the case of a "partial" cartel, but obviously a more formal extension is required before any conclusions are drawn.

In the case of an all-inclusive cartel, which probably better applies to the case of the allegedly chicken cartel mentioned in the Introduction and which controlled about 90% of domestic production (Montero et al., 2013), the test may require a time series of sales data long enough to cover periods where the cartel was not in operation. This data may correspond to the period before the cartel was formed or after the accusation was initiated (though provided the cartel would no longer be able to maintain cooperation). In the absence of such "counterfactual" sales data, one can still run the test if shocks are expected to follow a pattern that can be explained by other observables (e.g., prices of relevant inputs and prices of other meat products).

5 Conclusions

In a simple model, we have explored collusive behavior in the context of a sales quota scheme when monitoring occurs at a higher frequency than is enforcement imposed for non-compliance. There are two main contributions. The first contribution is identifying two dynamic forces at work with regards to a firm's quantity. The adjustment-to-shocks effect is that a cartel member's quantity is less responsive to contemporaneous cost and demand shocks as we move towards the end of the compliance (or enforcement) horizon. Early on in the compliance horizon, a firm takes greater advantage of low cost shocks by producing at a higher rate, knowing that it has more periods to adjust its sales in order to ensure compliance by the end of the compliance horizon. The propagation-of-shocks effect is that, as a cartel member moves towards the end of the compliance horizon, its available quota is more volatile as it is the cumulative product of more cost and demand shocks. While the adjustment-to-shocks effect results in greater sales volatility *earlier* in the compliance horizon, the propagation-of-shocks effect results in greater sales volatility *later* in the compliance horizon. The second contribution is showing that, when both of these effects are taken account of, sales volatility is initially decreasing as a cartel member moves through the compliance horizon and is then increasing so that sales volatility is minimized at an intermediate stage of the compliance horizon. This finding provides a test for a cartel with a sales quota scheme since, under competition, sales volatility is

stationary and thus does not exhibit this intertemporal pattern.

Under standard models of competition, if cost and demand are stationary stochastic processes then firms' quantities are stationary stochastic processes, too. However, when firms collude using sales quotas and the timing of monitoring and compliance do not coincide then firms' quantities are no longer stationary. The distribution on firms' quantities will depend on where they are in the compliance window. It was this non-stationarity property that was exploited in deriving the collusive markers for distinguishing collusion from competition. While our analysis assumed iid processes for cost and demand shocks, the general approach can be used to derive collusive markers as long as the cost and demand processes are stationary. Thus, the analysis could be extended in a straightforward way to allow cost and demand shocks to be serially correlated.

The model used for this analysis assumes a three-period compliance horizon and that firms produce subject to the constraint of satisfying the three-period sales quota. While useful for gaining some initial insight into how a sales quota scheme with an enforcement horizon exceeding the monitoring horizon impacts sales dynamics, a proper approach would set the problem up in an infinite horizon setting, endogenize the penalty from exceeding the sales quota (rather than presuming it is so severe that a firm would never exceed it), and derive equilibrium behavior given that penalty function. Of particular interest is to use that structure to solve for the optimal frequency of monitoring and enforcement. Understanding how market conditions - such as the number of firms and cost and demand volatility - impact the length of the enforcement horizon could deliver additional properties on collusive behavior and thus allow for a richer set of tests for assessing whether a cartel is present in a market.

References

- [1] Fiscalía Nacional Económica (FNE, 2011), Requerimiento contra Agrosuper Agrícola S.A. y otros (FNE's complaint against Agrosuper and others), Santiago (the document can be accessed from Chile's Competition Tribunal web site: www.tdlc.cl).
- [2] Harrington, J.E. (2006), "How Do Cartels Operate?," *Foundations and Trends in Microeconomics*, 2(1): 1–108.
- [3] Harrington, J.E., and A. Skrzypacz (2007), "Collusion under Monitoring of Sales," *RAND Journal of Economics*, 38(2): 314–31.
- [4] Harrington, J.E., and A. Skrzypacz (2011), "Private Monitoring and Communication in Cartels: Explaining Recent Collusive Practices," *American Economic Review* 101(6): 2425–49.

- [5] Montero, J.-P., and J.I. Guzman (2010), “Output-Expanding Collusion in the Presence of a Competitive Fringe,” *Journal of Industrial Economics* 58(1): 106-26.
- [6] Montero, J.-P., J.E. Harrington and F. Gallego (2013), “A Competition Analysis of Chile’s Wholesale Chicken Market” (the document can be accessed from Chile’s Competition Tribunal web site: www.tdlc.cl).