

Discounts as a Barrier to Entry

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January 29, 2014

Abstract

We study the anti-competitive effects of discount contracts (e.g., two-part tariffs, rebates, market-share discounts) between an incumbent and a buyer in the rent-shifting environment of Aghion and Bolton (A&B, 1987). While it is known that the exclusive dealing contracts of A&B work very much like two-part tariffs, we show that such equivalence does not extend to alternative discount schemes. Contracts that only commit the buyer to a price-quantity purchasing schedule (e.g., rebates) restrict the amount of rents the incumbent and the buyer can agree to transfer. The restriction is so strong that these contracts are rarely anticompetitive, especially when the incumbent's bargaining power and outside option are large. The reason these discount contracts exist is because they can still be used to extract rents from inefficient rivals, that is, they can serve as a barrier to inefficient entry. (*JEL* L42, K21, L12, D86)

1 Introduction

The potential exclusionary effect of rebates and quantity discounts has received widespread attention from scholars and antitrust authorities in recent years. There is a long list of cases from the well known *EU Commission v. British Airways* (1999), *EU Commission v. Michelin II* (2001) and *AMD v. Intel* (2005) to very recent ones such as *FZ Meritor v. Eaton* (2012) and *Chile v. Unilever* (2013). Rebates are believed to be a cheaper and more effective way of exclusion as they allow firms to use the inelastic portion of the demand as leverage to decrease

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the price in the more elastic portion, thereby increasing buyers' switching costs (Maier-Rigaud, 2005). The granting of rebates would then generate a "lock-in" effect inducing buyers to concentrate their purchases from a single supplier, very much like exclusive dealing contracts would do. As a result, courts on both sides of the Atlantic have been particularly harsh in some of their rulings towards these non-linear contracts, especially regarding all-unit retroactive rebates (where the per-unit price falls discontinuously for *all* units purchased after a pre-specified sales threshold is reached).¹

What makes these rulings controversial is that these discounts can arise without an exclusionary motive and, more importantly, be efficient. For example, all-unit discounts may be used in a bilateral monopoly setting to avoid the double marginalization problem when demand is known to both sides or as a screening device when it is only known to downstream retailers (Kolay, Ordover and Shaffer, 2004). Long-term discount contracts can also help solve agency and hold-up problems by aligning the incentives of manufacturers and retailers, much like exclusive dealing contracts can do (Marvel, 1982; Motta, 2004; and Whinston, 2006).² Rebates may also generate efficiency gains if they help the upstream supplier to exploit its economies of scale and/or save on transaction costs (Rey et al., 2005); and they may even increase price competition between downstream firms (Ahlborn and Bailey, 2006).

According to the so-called Chicago critique (Posner 1976; Bork 1978), these efficiency gains are all that matters when evaluating these contracts because a downstream retailer would never sign an exclusive contract that reduces competition unless it is fully compensated for it, which the incumbent manufacturer cannot afford when the entrant is more efficient. We know now that the Chicago critique fails to hold when a contract signed by two parties can have some form of externality onto a third party absent at the contracting table (i.e., when the third party's payoff is not fully internalized in the signing parties' maximization problem). In the "rent-shifting" models of Aghion and Bolton (1987) and Marx and Shaffer (2004), among others,³ there is a "seller-side" externality that arises when the signing parties have imperfect information about the entrant's cost. The incumbent supplier and the retail buyer set the terms of their contract with an eye on extracting rents from the potential entrant; but in the presence of imperfect information rent extraction is not complete and exclusion of some efficient rivals emerges as a

¹For an excellent survey of some European cases see Gyselen (2003), and for an analysis of comparative law between the US and the EU see Ahlborn and Bailey (2006).

²See Conlon and Mortimer (2013) for a recent evaluation of the efficiency gains of the rebate contracts of a dominant chocolate candy manufacturer.

³See also Spier and Whinston (1995) and Marx and Shaffer (1999).

side effect.

In the “naked-exclusion” models of Rasmussen et. al. (1991) and Segal and Whinston (2000),⁴ a “buyer-side” externality arises from a contract signed by the incumbent and one of the many buyers when the potential rival requires of a minimum scale of operation due to either scale or network economies. Facing unorganized buyers, the incumbent needs to compensate and lock-in only a subset of buyers in order to prevent the rival from achieving this minimum scale.⁵ Finally, in the “downstream-competition” models of Simpson and Wickelgren (2007), Abito and Wright (2009) and Asker and Bar-Isaac (2013), a “consumer-side” externality may arise when buyers are downstream competitors. When the entry of an upstream supplier intensifies downstream competition, signing an exclusive deal with the incumbent to prevent such entry may be profitable for the downstream retailers even in the absence of coordination problems and economies of scale. As a result, final consumers end up paying higher retail prices.

While the response to the Chicago critique has been framed around exclusive dealing contracts, it is accepted that in many cases it extends to other type of contracts with apparently similar exclusionary implications, namely, rebates and quantity discount contracts (e.g., Spector 2005; Office of Fair Trading 2005). Formal connections between the different type of contracts have been made, for example, by Karlinger and Motta (2012) for the naked-exclusion models and by Asker and Bar-Isaac (2013) for the downstream-competition models. A formal equivalence between two-part tariffs (2PT) and exclusive dealing contracts has also been established for the rent-shifting models of Aghion and Bolton (A&B hereafter); first by Marx and Shaffer (1999) in a full-information environment and then by Choné and Linnemer (2012) in an imperfect-information environment. They show that 2PT contracts work just like A&B contracts in setting a tax on entry, although again, at the expense of blocking some moderately efficient competitors. Because 2PT and other non-linear schemes are generally seen as equivalent for the purposes of rent extraction, for example, in monopoly pricing,⁶ it has been tempting to extend the “exclusionary” 2PT-A&B equivalence to any discount scheme such as rebates (e.g., Rey et. al. 2005).

In this paper we find otherwise: rebates are hardly ever anticompetitive but for a reason totally unrelated to the Chicago critique rationale. Extending the model of A&B to rebate

⁴See also Fumagalli and Motta (2006) and Spector (2011).

⁵Compensations are not even required when the incumbent makes buyers to “compete” for them, for example, when bilateral negotiations are sequential (Whinston, 2006).

⁶But not necessarily in duopoly pricing as recently shown by Calzolari and Denicolo (2013).

contracts, we show that the existence of unconditional payments, or more precisely, of an ex-ante commitment (e.g., liquidated damages, up-front payments) to transfer rents from the retailer to the manufacturer is crucial for the equivalence to hold. Interestingly, such transfers are either rarely observed or have not received much attention by antitrust authorities when analyzing rebates and other forms of quantity discounts. More importantly, we find that non-linear contracts that do not rely on such transfers (e.g., rebates) are highly unlikely to deter efficient rivals from expanding/entering, especially when the incumbent’s *bargaining position*, which can be understood as some combination of a party’s outside option (i.e., payoff in the absence of contracts) and its bargaining power, is strong.⁷

Our result has evident antitrust implications for cases that are best examined through the lens of a rent-shifting model.⁸ To understand it, notice first that the optimal contract the incumbent and the buyer would like to sign is composed of two parts: a below-cost marginal price (also known as the effective price the rival must compete with) that is used to extract rents from efficient rivals, and a high infra-marginal price that is used to distribute surplus from the buyer to the incumbent. When unconditional transfers are part of the contractual arrangement, like in A&B and 2PT contracts, it is feasible for the two parties to implement the optimal contract because part of the transfer is done ex-ante, so a significant amount of those high infra-marginal prices become sunk ex-post, i.e., at the time the buyer negotiates with the expanding rival and makes his purchasing decision. However, in the absence of such commitment, any transfer of surplus must be done ex-post, i.e., at the transaction or purchasing stage. This implies that these high infra-marginal prices are fully internalized by the buyer at the time of purchase, who may refrain to buy any additional units from the incumbent once he decides to buy from the rival supplier.

By placing a cap on the price that can be charged for the infra-marginal units at the transaction stage, the buyer’s ex-post participation constraint makes rent distribution for the “incumbent-buyer coalition” much harder.⁹ It forces the coalition to move away from the optimal contract towards contracts with less or no exclusionary potential. This is particularly

⁷Note that bargaining powers and outside options have no allocative implications in A&B and 2PT contracts; they only affect how the surplus extracted from the entrant is split between the two parties.

⁸A good example where a rent-shifting analysis seems to apply is *Canada v. Nielsen*. As explained by Jing and Winter (2014), “..the incumbent (Nielsen) and the buyer (a grocery product manufacturer) in any downstream contract were implementing a transfer away from IRI (Nielsen’s competitor), contingent on the states of successful entry...”

⁹Note also that when unconditional payments are restricted, the standard approach in the literature for finding the coalition optimum –the solution that maximizes overall surplus– is not longer valid. We investigate this in Section 3.

so when the incumbent’s bargaining position is strong—for example, due to the indispensable nature of its product—because there is a need for a large (ex-post) transfer of rents from the buyer to the incumbent. Conversely, when the incumbent’s bargaining position is weak, the buyer has enough room to transfer rents to the incumbent without violating her ex-post individual rationality constraint. This implies that the presence of small potential entrants or expanding rivals, which can only contribute to strengthen the incumbent’s bargaining position, makes the implementation of an anticompetitive rebate contract less not more likely, as commonly believed.¹⁰

The reason then rebates are rarely anticompetitive in our model is not because the incumbent cannot fully compensate the buyer for the reduction in competition (the Chicago critique rationale), but because the two parties fail to maximize their joint payoff ex-ante, that is, at the time of contracting. However, even if these quantity discount contracts do not prevent efficient rivals from expanding/entering, they can still emerge in equilibrium as a way to extract rents from inefficient rivals, which limits the amount of inefficient entry/expansion that would otherwise occur in the absence of contracts.¹¹ In that sense, a rebate contract can be certainly seen as a barrier to (inefficient) entry.¹²

The rest of the paper is organized as follows. We start the next section with a brief discussion of *Chile v. Unilever* (FNE 2013), simply to illustrate what seems to make a good case for a “rent-shifting” analysis.¹³ We then present a simple model that follows A&B very closely (the only difference is that we consider entrants that at best can serve a fraction of the total demand; otherwise rebates cannot operate by construction). In this simple model, rebates are never anticompetitive and may arise only as a vehicle to extract rents from inefficient rivals. In Section 3 we work with a more general formulation that considers the full range of possible

¹⁰In fact, Motta (2006, p. 372) explains “...a better understanding of how to balance exclusionary and efficiency effects of exclusive contracts is needed but it seems safe to assume that the former might dominate the latter only if the firm using exclusive contracts has a very strong market position.” In our rent-shifting environment, we are finding the exact opposite.

¹¹Although for very different reasons, Whinston (2006) also provides examples in which quantity contracts and exclusive deals may differ in their exclusionary potentials. In some cases quantity contracts may be a better choice (e.g., because they would involve no deadweight loss from monopoly pricing) while in others exclusive deals may be the only choice because quantity contracts lack any exclusionary power (e.g., when the entrant product is of higher quality) or are extremely costly to use.

¹²Absent of contracts, there is no inefficient entry in A&B because either supplier can serve the entire demand. In our setting, the rival can at best serve a fraction of the total demand which gives rise to inefficient mixed-strategy equilibria in the spot market; very much like in the slot-competition model of Jeon and Menicucci (2012). In their paper, inefficiencies disappear when firms are allowed to sell bundles while here when firms are allowed to use non-linear prices.

¹³None of us is or has been involved in the case.

bargaining powers and outside options. This generalization allows us to better understand the forces behind the results of the simple model and appreciate how general they seem to be. The general formulation is also interesting from a more technical perspective. The buyer’s ex-post participation constraint introduces non-linearities (and non-regularities) into the incumbent-buyer bargaining problem that prevents us from invoking the Coasian principle to solve the problem. In Section 4 we show that our results also extend to a downward sloping demand. We conclude in Section 5 with some antitrust implications of our results.

2 A Simple A&B Model

2.1 A motivating case

The rent-shifting model of A&B suits well for analysis of antitrust cases in which (i) buyers are relatively big, so an entrant or expanding supplier can achieve a minimum scale of operation by serving just one buyer; (ii) downstream competition is not that intense, so upstream competition does not fully permeate into the downstream market; and (iii) the incumbent and buyer are not entirely certain at the time of contracting about the value that a rival’s product can add to the market either in terms of lower costs or higher quality (or cannot discriminate against a constant flow of potential rivals with different characteristics).¹⁴

Chile v. Unilever is a case in which all three conditions seem to apply. Unilever has been accused of restricting the expansion of rivals in the wholesale market for laundry detergents by agreeing to “all-unit” retroactive discounts with supermarket and other distributors. Unilever has a market share of over 70% in the supermarket segment (its closer competitor has 13% and there are many other small suppliers, some short-lived). Supermarket sales make up for over 60% of sales, and of those supermarket sales, the vast majority is from the three largest supermarket chains with shares of 35%, 28% and 24%, respectively. Any of these supermarkets qualify as a large buyer (that could very well approach a small supplier to develop its own private label). As for the intensity of competition at the retail level, there is some evidence showing the two largest chains pricing well above wholesale prices.¹⁵ Finally, these retroactive discounts are typically negotiated on a yearly basis without any indication of discrimination

¹⁴Notice that in a rent-shifting case, the plaintiff is never the buyer, but rather alternative upstream suppliers. This also applies to the downstream-competition models.

¹⁵See, for example, Noton and Elberg (2013) for markups in instant coffee. Notice also that having information on markups for laundry detergents would not be much of a help; a high markup could be interpreted as the result of either low competition or the existence of rebates.

among potential rivals. One possible reason for this is that the quality and cost of the rivals' products are unknown at the time of contracting and likely to vary across suppliers depending on advertising and on innovation efforts. There seems to be constant innovation in production processes, especially on powder detergents (FNE 2013).

2.2 Model assumptions

There are three risk-neutral agents: a single buyer B and two suppliers I and E . Buyer B demands one unit of an infinitely divisible good at reservation value v .¹⁶ Supplier I is an incumbent firm that has unlimited capacity to produce the good at a constant marginal cost $c_I \in (0, v)$. Supplier E , on the other hand, is a small firm that can produce at most $\lambda \in (0, 1)$ units of the good at a marginal cost $c_E \leq c_I$. We can think of E either as a small firm that is already in the market looking to expand in λ units (our preferred interpretation) or as a potential entrant with capacity λ that faces no entry costs. In either case, the literal interpretation of λ is one of production capacity but an alternative and more recent interpretation is one of contestable capacity, that is, the “contestable share” of the market for which the buyer is willing and able to find substitutes (European Commission, 2009).

Note that letting $\lambda \in (0, 1)$ is our only departure from the A&B original model where $\lambda = 1$ (aside from the fact that we consider more than one type of contract). While assuming $\lambda < 1$ does not alter any of the A&B results, there are practical and technical reasons for this departure. Most antitrust cases are not about a potential entrant prevented from completely replacing the incumbent ($\lambda = 1$) but rather about a small, existing firm trying to acquire more market share ($\lambda \ll 1$). Nevertheless our analysis covers the entire range of possible values of λ , except for $\lambda = 1$ because by construction this would invalidate the use of rebates. For a rebate scheme to be operational the incumbent must sell something in both states of the world, with and without entry/expansion.

There are three periods: two contracting stages ($t = 1, 2$) followed by a spot market or transaction stage ($t = 3$). As standard in rent-shifting models, there is sequential contracting. In period 1, I makes a take-it-or-leave-it contract offer to B (as we will consider different type of contracts, the specific form of the offer is specified below).¹⁷ At this time c_E is unknown to

¹⁶The unit-demand assumption makes the model easier to handle as it eliminates firms' incentives for using non-linear contracts to avoid allocative inefficiencies. In Section 4 we allow for a downward sloping demand curve.

¹⁷In the simple model, and as in A&B, suppliers have all the bargaining power in their bilateral relations with B . In section 3 we consider a Nash bargaining model that covers the full range of bargaining powers.

both I and B but it is common knowledge that it distributes according to the cdf. $F(\cdot)$, over the interval $[0, \bar{c}_E]$, where $c_I < \bar{c}_E \leq v$, F/f is non-decreasing and $(1 - F)/f$ is non-increasing. As usual we also assume that contracts are not renegotiable ex-post, that is, after c_E is revealed.

In period 2 and after observing the contract signed between I and B , E is free to make a take-it-or-leave-it offer to B for its λ units. At this moment c_E becomes publicly known. Finally, in period 3 the transaction stage opens and B decides how much to buy from each supplier. In the absence of contracts, I and E compete in the spot market by simultaneously setting linear prices,¹⁸ otherwise parties adhere to the price conditions established in the contracts.

Definition 1. *A contract is said to be “anticompetitive” if it blocks the expansion/entry of an efficient rival, that is, of a rival with cost $c_E < c_I$.*

While closer to the antitrust practice of what constitutes an anticompetitive contract, this definition makes no precision about the welfare implications of an anticompetitive contract. Such precision is unnecessary when in the absence of a contract entry is always efficient, as in the A&B original model, in which case any anticompetitive contract reduces welfare. There can be instances, however, of too much entry in the absence of a contract, in which case an exclusive deal may indeed increase welfare by limiting inefficient entry (Whinston 2006, p. 188). While it is true that this latter possibility complicates the welfare analysis of some of these contracts, it is less relevant for our paper because our focus is to show when and why many of the non-linear contracts we observe (e.g., rebates, quantity discounts) are hardly ever anticompetitive, a sufficient condition for a contract not to be welfare decreasing.

2.3 Agents’ outside options

We begin by characterizing I and B ’s outside options, that is, their payoffs in the absence of contracts. If I and B fail to sign a contract, will E and B sign one? Given its capacity constraint, E cannot use B to shift rents from the incumbent using the quantity discounts or rebates we have in mind.¹⁹ And given our inelastic demand, the best E can do is to offer a

¹⁸The linear price assumption is not only in the spot market of A&B but also in wholesale spot markets of more recent papers (e.g., Hendricks and McAfee 2008; Jeon and Menicucci 2012). More generally, the idea is that complex non-linear contracts are usually negotiated on a long term basis (1 year or more), but otherwise parties can still engage in some negotiation on a monthly/weekly basis, although simpler due to bargaining/transaction costs. In Section 3 we relax the linear pricing assumption allowing parties to clear the spot market in other ways.

¹⁹In principle it could use contracts with breach penalties. This would leave I with no rents, which is unlikely when λ is small. We like to think that the contracts the entrant and the buyer can sign, provided I does not sign any, are either linear or non-linear purchasing schedules that are not equipped with commitments to unconditional transfers.

linear price to B at this contracting stage. Thus, the problem faced by E at $t = 2$ is to either make a price offer to B for its λ units or compete in the spot tomorrow, at $t = 3$. Suppose the latter (below we show this is what happens in equilibrium).

Lemma 1. *Let $c^* \equiv c_I + (1 - \lambda)(v - c_I)$. The equilibria of the spot-market game in which firms simultaneously set linear prices can be characterized as follows²⁰*

1. *When $c_E \geq c^*$, there is a pure strategy Nash equilibrium with prices $p_I = p_E = c_E$ and payoffs $\pi_I = c_E - c_I$, $\pi_E = 0$ and $\pi_B = v - c_E < \lambda(v - c_I)$.*
2. *When $c_E < c^*$, there are only mixed strategy equilibria in which*
 - (a) *both firms randomize over the support $[c^*, v]$*
 - (b) *and their expected payoffs are $\pi_I^e(p_I) = (1 - \lambda)(v - c_I)$, $\pi_E^e(p_E) = \lambda(c^* - c_E)$, and $\pi_B^e < \lambda(v - c_I)$.*

Proof. See the Appendix. □

Now, to understand E 's decision as to whether make a price offer at $t = 2$ or compete in the spot at $t = 3$, consider first the case in which $c_E \geq c^*$. Since the buyer will not accept any price higher than c_E from E , the latter is indifferent between selling at c_E at $t = 2$ or going to the spot market at $t = 3$; in either case he makes zero profits. Both the buyer and the incumbent are also indifferent as to what E does. The second case, $c_E < c^*$, is a bit more involved. If E sets $p_E > c^*$ at $t = 2$, it is optimal for I to undercut that price at $t = 3$ and leave E with zero profits. Alternatively, if E sets $p_E \leq c^*$ at $t = 2$, it is not longer optimal for I to price right below p_E but rather to price at v and get the residual monopoly profit, $(1 - \lambda)(v - c_I)$, which is equal to the profit she would get when competing in the spot. So the best E can do is to set $p_E = c^*$, which yields the exact same payoff than any of the mixed-strategy equilibria in the spot. Whether B accepts E 's offer $p_E = c^*$ at $t = 2$ will depend on $\pi_B^e \geq \lambda(v - c^*)$; although in any case its surplus will be less than $\lambda(v - c_I)$. Therefore, regardless of whether c_E is lower than, equal to or greater than c^* , E gains nothing by contracting at $t = 2$.²¹

²⁰Note that in the case of $c_E < c^*$, as $\lambda \rightarrow 1$, $c^* \rightarrow c_I$ and spot competition becomes efficient; moreover, E 's price distribution collapses into a singleton, $p_E^* = c_I$, yielding the standard Bertrand outcome. We do not exactly converge to the result in the $A\mathcal{E}B$ model because they assume that the entrant faces a positive, although very small, fixed cost of entry. But the difference is immaterial for $A\mathcal{E}B$ contracts because it does not affect the amount of rent extraction, only how the surplus is split.

²¹To be precise, at times B may indeed benefit from the contract, so E may well end up signing it despite it makes no difference to her. But even in those cases B makes less than $\lambda(v - c_I)$, which is all we need for the results that follow.

We can summarize this discussion as follows

Lemma 2. *When the incumbent and the buyer fail to sign a contract at $t = 1$, their outside options, denoted, respectively, by $\bar{\pi}_I$ and $\bar{\pi}_B$, are given by*

$$\begin{aligned}\bar{\pi}_I &= (v - c_I)(1 - \lambda) + [1 - F(c^*)] \{ \mathbb{E}(c_E | c_E > c^*) - c^* \} \geq (1 - \lambda)(v - c_I) \\ \bar{\pi}_B &< \lambda(v - c_I)\end{aligned}$$

Proof. Immediate from Lemma 1. □

2.4 A&B exclusive contracts and two-part tariffs

Consider first an A&B exclusive dealing contract (w, l) according to which —if accepted at $t = 1$ — the buyer commits to buy exclusively from the incumbent at $t = 3$ at the wholesale price w . In case the buyer decides to buy some units from the expanding rival, he must pay the incumbent a per-unit penalty (or liquidated damages) l .²² In period 1, the incumbent anticipates that only those expanding rivals with costs $c_E \leq w - l$ can, at $t = 2$, induce the buyer to agree to purchase units from them because no buyer is willing to switch supplier at a unit-price higher than $w - l$. Thus, the probability of expansion at the time of contracting is $F(w - l)$.

The A&B contract the incumbent offers the buyer is then obtained from the following problem

$$\begin{aligned}\max_{w, l} \mathbb{E}\pi_I(w, l) &= [(1 - \lambda)(w - c_I) + \lambda l] F(w - l) + (w - c_I) [1 - F(w - l)] \\ \text{s.t. } v - w &\geq \bar{\pi}_B\end{aligned}\tag{1}$$

which has solution

$$w^* - l^* = c_I - \frac{F(w^* - l^*)}{f(w^* - l^*)}\tag{2}$$

$$w^* = v - \bar{\pi}_B\tag{3}$$

It is clear that $w^* - l^* < c_I$ and straightforward to verify that $\mathbb{E}\pi_I(w^*, l^*) > \bar{\pi}_I$ and $w^* > c_I$.

²²It makes no difference to consider a lump-sum penalty because the buyer will never breach, if at all, for less than λ units.

As first shown by A&B, these exclusive deals are not only profitable for both the incumbent and buyer to sign but they have anticompetitive implications in that they block the expansion of some efficient rivals, those with costs $c_E \in [w^* - l^*, c_I]$.²³ The idea of an A&B contract is not to block the expansion of all efficient rivals, but to extract rents from the expansion of the most efficient ones, those with costs $c_E < w^* - l^*$. In expectation, this efficient expansion reports the incumbent profits equal to $F(w^* - l^*)[\lambda l^* - \lambda(w^* - c_I)] = \lambda F(w^* - l^*)(c_I + l^* - w^*)$.

A&B contracts are not the only way to extract rents from efficient rivals. Consider now the two-part tariff (2PT) contract (p, T) according to which the buyer commits to buy from the incumbent at $t = 3$ at the wholesale price p and to an *unconditional* lump-sum transfer of T , that is, independent of how much B ends up buying from I (possibly T is paid at the signing of the contract). Facing this contract, the only way for E to induce B to buy some units from him is with a price equal to or lower than p , which implies that the probability of expansion as of period 1 is equal to $F(p)$. Thus, irrespective of whether E expands or not, the buyer gets $v - p - T$ in case of signing the contract.

The 2PT contract the incumbent offers the buyer is again obtained from the problem

$$\begin{aligned} \max_{p, T} \mathbb{E}\pi_I(p, T) &= T + (1 - \lambda)(p - c_I)F(p) + (p - c_I)[1 - F(p)] \\ \text{s.t. } v - p - T &\geq \bar{\pi}_B \end{aligned}$$

Denote by p^* and T^* the solution to the incumbent's 2PT problem. Relabeling p as $w - l$ and T as l , it becomes clear that the 2PT problem reduces exactly to the A&B problem and $p^* = w^* - l^* < c_I$ and $T^* = l^*$. The two problems are equivalent in all respects, including their anticompetitive implications, except for the timing over which transfers *are delivered* (which is irrelevant under risk-neutrality). In 2PT contracts the surplus transfer from the buyer to the incumbent is done immediately and regardless of whether entry occurs or not; while in A&B contracts the actual transfer takes place ex-post after the expansion has actually occurred.

This 2PT-A&B equivalence result is not new; it is already in Marx and Shaffer (1999) and Choné and Linnemer (2012). Intuitively, the optimal 2PT contract is designed to extract rents from the most efficient rivals charging a low marginal price for the contestable units $p^* < c_I$ and to distribute those rents charging high inframarginal prices via the up-front payment

²³Note that if this A&B contract is not allowed there is too much entry, so it is not entirely clear from a welfare perspective the right course of action here. Our analysis contributes to this welfare analysis by identifying contracts for which this ambiguity disappears.

T^* . Uncertainty regarding c_E at the time of contracting prevents perfect discrimination which generates the well-known side effect of blocking some efficient rivals.

Curiously, when analyzing the *Standard Fashion v. Magrane-Houston* case, Marvel (1982) argues that the emergence of up-front charges with lower marginal prices after the outlaw of exclusivity contracts, was evidence against any anticompetitive implications these exclusivity clauses might have had in the first place. The 2PT-A&B equivalence points otherwise. More importantly, because 2PT contracts and other nonlinear schedules are typically seen as equivalent for the purposes of rent extraction, for example in monopoly pricing, it is tempting to extend the 2PT-A&B equivalence to other discount contracts (see, e.g., Rey et. al. 2005). We show next that there is no such equivalence.

2.5 Rebates

Following the contractual arrangements seen in some recent cases (e.g., *Michelin II*, *Eaton*, *Unilever*), consider now the all-unit retroactive rebate contract $(r, R, \bar{Q} = 1)$, where r is the list price, \bar{Q} is a pre-specified sales threshold above which the rebate applies, in this case is equal to 1, and R is a lump-sum rebate.²⁴ Under this contractual arrangement the buyer commits to a price r when purchasing $q \in (0, \bar{Q})$ units from the incumbent and to $r - R$ when purchasing above that amount.

If B signs the contract, an expanding rival E can still induce B to buy λ units from him if he is compensated for the forgone rebate. For that to be the case, E 's price offer e must satisfy

$$v - (1 - \lambda)r - \lambda e \geq v - r + R$$

or $e \leq r - R/\lambda$. If E decides to expand/enter it will offer exactly $e = r - R/\lambda$, which sets the probability of expansion at the time of contracting equal to $F(r - R/\lambda)$.

The cutoff price $e = r - R/\lambda$ is typically known as the *effective price of the contestable demand* and represents the opportunity cost the buyer faces when purchasing from an alternative supplier. This price differs from $r - R$ because the contestable share is smaller than the total demand, which is what allows the incumbent to leverage its position. Indeed, it is relatively

²⁴In Section 3 we formally show that you cannot improve upon these two-step quantity discounts. Note also that in this inelastic-demand setting we do not need to make any distinction between own-supply discounts and market-share discounts that are a function of both how much the buyer purchases from the dominant supplier and how much from rival suppliers. In Section 4 we make this distinction when we work with a downward sloping demand.

easy to conceive a profitable, yet anticompetitive, rebate scheme when λ is particularly small

$$r - \frac{R}{\lambda} < c_I < r - R$$

Since the smaller the contestable demand, the easier is for the incumbent to deter efficient expansions (in the limit as $\lambda \rightarrow 0$, it becomes virtually costless), it is not surprising the great deal of attention and controversy around the estimation of the contestable demand that we have seen in some recent cases; notably, *EU Commission v. Intel*. Because the buyer pays $r - R$ regardless of entry, the incumbent now faces the following program

$$\begin{aligned} \max_{r,R} \mathbb{E}\pi_I(r, R) &= (1 - \lambda)(r - c_I)F(r - R/\lambda) + (r - R - c_I)[1 - F(r - R/\lambda)] & (4) \\ \text{s.t. } v - r + R &\geq \bar{\pi}_B \end{aligned}$$

The first term is the profit from selling $1 - \lambda$ units at price r , which happens with probability $F(r - R/\lambda)$, and the second term is the profit from selling all units at price $r - R$, which happens with probability $1 - F(r - R/\lambda)$. Now, relabeling r as $p + T/(1 - \lambda)$ and R as $\lambda T/(1 - \lambda)$, the rebate program becomes

$$\max_{p,T} \mathbb{E}\pi_I(p, T) = T + (1 - \lambda)(p - c_I)F(p) + (p - c_I)[1 - F(p)] \quad (5)$$

s.t. $v - p - T \geq \bar{\pi}_B$, which is the incumbent's 2PT program.

This result explains the apparent equivalence between exclusive deals and 2PT contracts, on the one hand, and between 2PT contracts and quantity discounts more generally, on the other. From (4) and (5), one can see that the rebate scheme seem to work very much like a 2PT schedule, that is, charging a very low marginal price for the contestable units and high prices for the infra-marginal units which is what allows the distribution of rents among the members of the incumbent-buyer coalition. One can also see from (1) and (4) that the rebate scheme works similarly to an A&B contract, that is, imposing a tax or penalty to the buyer when switching supplier as the breaching clause in A&B is. In A&B the penalty is λl while under the rebate contract is the price increase from $r - R$ to r .²⁵

There is a missing element in all this comparison, however, that makes this apparent equiv-

²⁵Note that if $\lambda = 1$ the "penalty" still carries through under the A&B contract but not under the rebate contract, which is why we impose $\lambda < 1$.

alence to fall apart. Unlike A&B and 2PT contracts, the rebate contract is not equipped with an ex-ante commitment to transfer rents. A rebate contract only specifies to the buyer a price-quantity purchasing schedule, so all purchasing and transfer decisions occur ex-post during the transaction stage ($t = 3$). This subtle but crucial difference imposes an additional participation constraint on the buyer side. In period 3, the buyer will not be purchasing units in equilibrium at prices above its reservation price v , from either supplier. Since this requires that $r \leq v$, it can be established

Lemma 3. *The solution to the “unrestricted” rebate program (4), which we denote by (r^u, R^u) , violates the buyer’s ex-post participation constraint, i.e., $r^u > v$.*

Proof. From the relabeling of variables in programs (4) and (5) and using

$$\bar{\pi}_B = v - p^* - T^* \text{ yields}$$

$$r^u - v = \frac{T^*}{1 - \lambda} + p^* - v = \left(\frac{1}{1 - \lambda} \right) [\lambda(v - p^*) - \bar{\pi}_B]$$

Furthermore, from Lemma 2 we know that $\bar{\pi}_B < \lambda(v - c_I)$, therefore

$$r^u - v > \left(\frac{1}{1 - \lambda} \right) [\lambda(v - p^*) - \lambda(v - c_I)] = \left(\frac{\lambda}{1 - \lambda} \right) (c_I - p^*) > 0$$

which finishes the proof. □

The rebate scheme (r^u, R^u) does not allow for both optimal rent extraction and rent distribution because of the ex-post buyer’s individual rationality, or which is the same, because of the absence of unconditional payments. Moving away from the optimal contract —either (w^*, l^*) or (p^*, T^*) — brings up three important questions: what is the best rebate contract the incumbent can offer the buyer while satisfying its ex-post participation constraint? Is it still anticompetitive? And if not, does it still emerge in equilibrium?

To answer these questions, add to the incumbent’s rebate program (4) the buyer’s ex-post constraint that $r \leq v$. Let (r^*, R^*) denote the solution to this “updated” program and $c_E^* \equiv r^* - R^*/\lambda$ the critical cost level below which a rival supplier enters/expands.

Proposition 1. *The rebate contract (r^*, R^*) is never anticompetitive, i.e., $c_E^* \geq c_I$, where $c_E^* \equiv r^* - R^*/\lambda$.*

Proof. For I to offer the rebate contract (r^*, R^*) , it must be true that $\mathbb{E}\pi_I(r^*, R^*) \geq \bar{\pi}_I$, that is

$$[(r^* - c_I)(1 - \lambda) - \bar{\pi}_I] + \lambda(c_E^* - c_I)[1 - F(c_E^*)] \geq 0 \quad (6)$$

But we know that $r^* \leq v$, so $(r^* - c_I)(1 - \lambda) \leq (v - c_I)(1 - \lambda) \leq \bar{\pi}_I$. In turn, these inequalities indicate that the first term in (6) is non-positive which requires the second term to be non-negative

$$\lambda(c_E^* - c_I)[1 - F(c_E^*)] \geq 0$$

And since $F(c_E^*) < 1$, we have that $c_E^* \geq c_I$. □

Proposition 2. *The optimal rebate contract, if it exists, is characterized by $r^* = v$ and $c_E^* \in (c_I, \bar{c}_E)$. A sufficient condition for its existence is $\bar{c}_E \approx c^*$, which holds easily as $\lambda \rightarrow 0$.*

Proof. See the Appendix. □

Propositions 1 and 2 convey two remarkable messages of the simple model: Rebates are never anticompetitive, yet, they are likely to emerge in equilibrium, especially if the entrant is very small. The two results are intimately connected. To get an intuition for the first result –that rebates are never anticompetitive–, notice that it is the buyer the one that directly appropriates the rents extracted from rivals when facing the low marginal prices set by either I or E . Hence, in the absence of unconditional payments, since it is ex-post unfeasible for B to transfer a large fraction of those rents to I , at least enough to cover this latter outside payoff, the incumbent will never agree on an anticompetitive schedule.

A helpful way to communicate this “competitive” result is by rewriting the incumbent’s rebate program (4) as a function of the list price r and the effective price $e = r - R/\lambda$ as follows

$$\max_{r,e} \mathbb{E}\pi_I(r, e) = (1 - \lambda)(r - c_I) + \lambda(e - c_I)(1 - F(e)) \quad (7)$$

Ignore for the moment the buyer’s ex-ante and ex-post participation constraints. As the incumbent faces no competition for the first $(1 - \lambda)$ units, he can sell each of them for virtually any price r . For the next λ units, however, he faces competition from the entrant. The optimal action for the incumbent is to set the price of these extra units at $e = c_E^\circ$, where

$$c_E^\circ = c_I + \frac{1 - F(c_E^\circ)}{f(c_E^\circ)} \in (c_I, \bar{c}_E) \quad (8)$$

and sell them with probability $1 - F(c_E^\circ)$, which is the probability the entrant's cost is above c_E° .

Clearly a too large r violates both of the buyer's constraints. Since the A&B and 2PT programs only pay attention to the buyer's ex-ante constraint (because transfers are committed ex-ante, whether in the form of liquidated damages or upfront payments), we will introduce them gradually. The buyer's ex-ante constraint can also be rewritten as a function of the list price r and the effective price e as follows

$$r \leq \frac{1}{1 - \lambda}(v - \lambda e - \bar{\pi}_B) \quad (9)$$

This restriction must be strictly binding (provided we are still ignoring the ex-post constraint), otherwise I would set r even higher. Thus, using (9) with equality to replace r in (7) and solving for e , we obtain, not surprisingly, the A&B solution (2), which is the solution that maximizes the joint surplus of the coalition IB .

Let us now add the buyer's ex-post constraint (i.e., $r \leq v$) that any rebate contract must also satisfy. We know from Lemma 3 that this constraint must be binding in the optimal rebate contract. This, in turn, invalidates the use of the "coalitional approach" to finding the optimal contract which consists in maximizing the joint surplus of the coalition IB . Thus, the optimal rebate contract is to be found by replacing $r = v$ in (7) and solving for e . Depending on whether the buyer's ex-ante constraint is also binding (i.e., $v(1 - \lambda) = v - \lambda e - \bar{\pi}_B$) or not, the optimal effective price (or the critical cost level above which entry/expansion is deterred) is equal to

$$c_E^* = \min\{c_E^\circ, v - \bar{\pi}_B/\lambda\} \quad (10)$$

where c_E° is given by (8). While we cannot rule out that $c_E^* < c_E^\circ$, from the incumbent's participation constraint (see proof of Proposition 1) we know that c_E^* is always above c_I . In other words, as long as the buyer's ex-post participation constraint is binding, the incumbent's problem reduces to the competition for the contestable share of the market (i.e., the λ units) which must be necessarily priced above cost.

As for the second result –that rebate contracts are likely to emerge in equilibrium–, notice that the incumbent has a choice of how to compete for these λ units; he can either sign a rebate contract or go directly to the spot market. Given that rebates cannot be used to extract rents from efficient rivals (Proposition 1) and that there are no allocative inefficiencies (inelastic de-

mand), the only reason to use rebates is to deter the expansion/entry of some, not all, inefficient rivals. And for doing that the incumbent can use either (i) two non-contingent instruments, i.e., the two-step rebate contract (r, e) , or (ii) an imperfect but contingent instrument (an ex-post linear price). When $\bar{c}_E \approx c^*$, that is, when rivals are relatively efficient in that all possible types sell with positive probability in the (mixed-strategy) spot-market equilibrium, choice (i) is preferred to choice (ii). This choice is reinforced in the presence of small entrants because in the absence of a contract any of these small entrants is almost certain to be selling with positive probability regardless of her type (i.e., $c^* \rightarrow v$ as $\lambda \rightarrow 0$).

The simple model has served to advance our main results, namely, (i) that unconditional payments are critical for the equivalence between exclusive dealing contracts and non-linear contracts to hold in a rent-shifting environment; (ii) that in the absence of unconditional payments contracts such as rebates and quantity discounts are highly unlikely to be anticompetitive; and (iii) that these contracts can still emerge in equilibrium as a mean to extract rents from inefficient rivals. In the next two sections we show how these results stand to different extensions of the simple model including the consideration of more flexible non-linear schedules, different bargaining powers and outside options, and a downward sloping demand. These extensions help not only appreciate how general the results of the simple model are but also better understand the underlying forces that explain them.

3 General Setting

3.1 Preliminaries

We extend our simple model in three important directions. First, we introduce a reduced-form approach to capture any possible but feasible payoffs agents can receive in the absence of contracts. Denote them by $\bar{\pi}_I$ and $\bar{\pi}_B$ and the sum by $\bar{W} = \bar{\pi}_I + \bar{\pi}_B$. Recall that in our simple model $\bar{\pi}_I \geq (1 - \lambda)(v - c_I)$, $\bar{\pi}_B < \lambda(v - c_I)$, so $\bar{W} < v - c_I$. Alternatively, one can imagine manufacturers and retailers interacting in the spot market on the basis of short-term (e.g., weekly) non-linear prices like two-part tariffs. In this case, the incumbent and buyer's outside options are equal to

$$\bar{\pi}_I^{NL} = (1 - \lambda)(v - c_I) + \lambda[1 - F(c_I)] \{\mathbb{E}(c_E \mid c_E > c_I) - c_I\} \quad (11)$$

$$\bar{\pi}_B^{NL} = \lambda(v - c_I) - \lambda[1 - F(c_I)] \{\mathbb{E}(c_E \mid c_E > c_I) - c_I\} \quad (12)$$

and $\bar{W} = v - c_I$ (and no production inefficiencies). Yet, one can think of other mechanisms for the clearing of the spot market.²⁶ The advantage of the reduced-form approach is that we can omit how outside options are exactly determined and focus instead on the mechanism underlying rent-shifting.

Second, we allow for arbitrary bargaining powers. We denote by $\eta \in [0, 1]$ and $\beta \in (0, 1]$ the bargaining power of I and E , respectively, in their relations with B . We disregard $\beta = 0$ to rule out a trivial case. In that case, I and B will sign a contract that achieves perfect rent extraction from each of the efficient potential entrants (those with $c_E \leq c_I$). Since this is equivalent to signing a contract contingent on c_E , there is no room for the emergence of “imperfect” contracts with the potential to block the entry/expansion of some efficient rivals.

Our third extension is to consider the full family of discount contracts that I and B can signed in period 1, not just the two-step contracts of the simple model. In principle, these contracts can take the form $T(q_I, q_E)$, which tells how much the buyer pays the incumbent when buying $q_I > 0$ units from the incumbent and q_E units from the entrant ($T(0, q_E) = 0$). In our inelastic demand setting, where $q_I + q_E = 1$, these “market-share” contracts reduce to “own-supplied” contracts of the form $T(q_I)$ with $T(0) = 0$. We return to the distinction between market-share and own-supplied contracts in Section 4, when we consider a downward-sloping demand curve. Finally, and provided that $q_E \leq \lambda$ and c_E is constant and known in period 2, for the family of discount contracts that E and B can sign in period 2, we can restrict attention to linear schedules of the form $p_E(c_E)q_E(c_E)$. This is without loss of generality.

Under these generalizations, the determination of the optimal contract reduces to a non-linear and non-regular control problem, where I chooses the quantity and the price at which to sell to B , taking into account the effect of this decision on the next period negotiation between B and E . We tackle this problem following the mechanism design approach of Choné and Linnemer (2012), which in turn builds upon Baron and Myerson (1982). There is an important difference with our paper, however, that results from the transfer restriction that B ’s ex-post participation constraint may introduce to the problem. In that case, the bargaining problem between I and B cannot be solved invoking the Coasian principle. It requires to go back to the “primitive” bargaining problem of maximizing the parties’ Nash product.

²⁶For example, the mechanism in Krasteva and Yildirim (2012) that with probability $\alpha > 0$ one of the parties makes the take-it-or-leave-it offer and with probability $1 - \alpha$ the other party does.

3.2 Rewriting the coalition's problem

The optimal schedule $T(q_I)$ to be signed by I and B is found by backward induction. Given schedules $T(q_I)$ and $p_E(c_E)q_E(c_E)$, the buyer's problem at the transaction stage is to decide whether to buy everything from I or from both suppliers, that is,

$$\max_{q_I, q_E} \pi_B = vq_I - T(q_I) + (v - p_E) \min\{\lambda, q_E\}$$

where $q_I + q_E \leq 1$. For the buyer to buy units in equilibrium the marginal prices in both schedules must be equal to or smaller than v , i.e., $T'(q_I) \leq v$ for all $q_I \in [0, 1]$ and that $p_E \leq v$. At the margin the buyer is not going to buy units at a price above v , so any schedule that violates this “marginal-price” condition is weakly dominated by one that does not. This latter together with $T(0) = 0$ imply that in equilibrium $T(q_I) \leq vq_I$ for all $q_I \in [0, 1]$.

In equilibrium we also know that $q_I + q_E = 1$, so we can write, without loss of generality, that $T(q_I) = T(1 - q_E)$. What is then the optimal schedule $p_E(c_E)q_E(c_E)$ that E and B negotiate in period 2 having both observed $T(q_I)$? Since in period 2 both parties are bargaining under full information, the program they confront is

$$\max_{p_E, q_E} \Pi_{EB} = \pi_E^\beta (\pi_B - v + T(1))^{1-\beta} \quad (13)$$

s.t. $\pi_B \geq v - T(1)$, $\pi_E \geq 0$, and where $\pi_E = q_E(p_E - c_E)$ and $\pi_B = v - p_E q_E - T(1 - q_E)$. Notice that disagreement payoffs are $v - T(1)$ for B (the surplus B obtains if it purchases the entire production from I according to $T(\cdot)$) and 0 for E . By the Coase bargaining principle, problem (13) can be conveniently rewritten as follows

Lemma 4. *Problem (13) is equivalent as to choosing the quantity $q_E \leq \lambda$ that maximizes the joint surplus of the EB coalition*

$$W_{EB} = \pi_B + \pi_E = v - c_E q_E - T(1 - q_E) \quad (14)$$

and the price p_E that splits the surplus so that

$$\begin{aligned} \pi_E &= \beta \Delta W_{EB}^*(c_E) \\ \pi_B &= (1 - \beta) \Delta W_{EB}^*(c_E) + v - T(1) \end{aligned} \quad (15)$$

where $\Delta W_{EB}^*(c_E) = W_{EB}^*(c_E) - v + T(1)$ and $W_{EB}^*(c_E) = \max_{q_E \leq \lambda} W_{EB}(q_E; c_E)$.

Proof. See the Appendix. □

Finally, I and B negotiate in period 1 anticipating the optimal schedule E and B will negotiate in period 2. Their problem is to determine the schedule $T(\cdot)$ that maximizes the Nash product of their expected payoffs, that is

$$\max \Pi_{IB} = (\mathbb{E}\pi_I - \bar{\pi}_I)^\eta (\mathbb{E}\pi_B - \bar{\pi}_B)^{1-\eta} \quad (16)$$

s.t. $\mathbb{E}\pi_I \geq \bar{\pi}_I$, $\mathbb{E}\pi_B \geq \bar{\pi}_B$, $T(1 - q_E) \leq v(1 - q_E)$ for all q_E , and $q_E(c_E) \leq \lambda$, where

$$\mathbb{E}\pi_I = \int_0^{\bar{c}_E} \{T(1 - q_E(c_E)) - c_I(1 - q_E(c_E))\} f(c_E) dc_E \quad (17)$$

$$\mathbb{E}\pi_B = \mathbb{E}[(1 - \beta)\Delta W_{EB}^*(c_E)] + v - T(1) \quad (18)$$

To solve this problem we follow the mechanism design approach of rewriting (16) only in terms of $q_E(c_E)$. By the envelope theorem we have $\partial W_{EB}/\partial c_E = -q_E$, so

$$\mathbb{E}W_{EB}^* = W_{EB}^*(0) - \int_0^{\bar{c}_E} [1 - F(c_E)] q_E(c_E) dc_E$$

Imposing the usual monotonicity implementation constraint $q_E'(c_E) \leq 0$,²⁷ we arrive at the following lemma:

Lemma 5. *The problem faced by the IB coalition can be rewritten as to choosing the allocation rule $q_E(c_E) : [0, c_E^\circ] \rightarrow [0, \lambda]$, where $c_E^\circ \in (c_I, \bar{c}_E)$ is implicitly given by (8), and the value $T(1 - q_E(0))$ that solve*

$$\max \Pi_{IB} = (\mathbb{E}\pi_I - \bar{\pi}_I)^\eta (\mathbb{E}\pi_B - \bar{\pi}_B)^{1-\eta} \quad (19)$$

²⁷It is easily verified that the single crossing property is satisfied in this setting:

$$\frac{\partial}{\partial c_E} \left(\frac{\partial W_{EB}/\partial q_E}{\partial W_{EB}/\partial T(1 - q_E)} \right) = 1 > 0, \quad \forall c_E$$

s.t.

$$\mathbb{E}\pi_B = v - T(1 - q_E(0)) - \int_0^{c_E^\circ} \left\{ \frac{1 - (1 - \beta)F(c_E)}{f(c_E)} \right\} q_E(c_E) f(c_E) dc_E \geq \bar{\pi}_B \quad (20)$$

$$\mathbb{E}\pi_I = T(1 - q_E(0)) - c_I - \int_0^{c_E^\circ} \left\{ c_E - c_I - \frac{[1 - F(c_E)]}{f(c_E)} \right\} q_E(c_E) f(c_E) dc_E \geq \bar{\pi}_I \quad (21)$$

$$q'_E(c_E) \leq 0 \quad (22)$$

$$T(1 - q_E(0)) \leq v(1 - q_E(0)) \quad (23)$$

Proof. It is clear that the optimal allocation rule $q_E(c_E)$ is equal to zero for all $c_E > c_E^\circ \in (c_I, \bar{c}_E)$, so we only need concentrate our analysis on when $c_E \leq c_E^\circ$. For that see the Appendix. \square

Conditions (20) and (21) are the ex-ante participation constraints of B and I respectively, (22) is the sorting condition, and (23) is a transfer constraint consistent with B 's ex-post participation constraint.

3.3 Revisiting the (non-)equivalence

We define the relaxed problem as the IB problem without constraint (23). Then,

Proposition 3. *The solution of the relaxed problem is Coasian: The optimal allocation rule $q_E^*(c_E)$ maximizes the joint surplus $\mathbb{E}W_{IB} = \mathbb{E}\pi_B + \mathbb{E}\pi_I$, and transfers are determined by $T^*(1 - q_E^*(0)) = T^*(1 - \lambda)$, as prescribed by a Nash Bargaining solution. Formally,*

$$q_E^*(c_E) = \begin{cases} \lambda & \text{if } c_E \in [0, \tilde{c}_E] \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

where $\tilde{c}_E < c_I$ satisfies

$$f(\tilde{c}_E)(\tilde{c}_E - c_I) + \beta F(\tilde{c}_E) = 0 \quad (25)$$

and

$$\mathbb{E}\pi_I(c_E; \tilde{c}_E) = \bar{\pi}_I + \eta[\mathbb{E}W_{IB}^*(c_E; \tilde{c}_E) - \bar{W}] \quad (26)$$

$$\mathbb{E}\pi_B(c_E; \tilde{c}_E) = \bar{\pi}_B + (1 - \eta)[\mathbb{E}W_{IB}^*(c_E; \tilde{c}_E) - \bar{W}] \quad (27)$$

Proof. The existence of a solution to this relaxed problem is guaranteed given that $q_E(c_E)$ is a weakly decreasing function of c_E , and by our assumption that $\mathbb{E}W_{IB}^*(c_E; \tilde{c}_E) > \bar{W}$. This

latter implies $\mathbb{E}\pi_I(c_E; \tilde{c}_E) - \bar{\pi}_I = \eta[\mathbb{E}W_{IB}^*(c_E; \tilde{c}_E) - \bar{W}] \geq 0$ and $\mathbb{E}\pi_B(c_E; \tilde{c}_E) - \bar{\pi}_B = (1 - \eta)[\mathbb{E}W_{IB}^*(c_E; \tilde{c}_E) - \bar{W}] \geq 0$, so all the other constraints, beside $T(1 - q_E(0)) \leq v(1 - q_E(0))$, are met. For the rest see the Appendix. \square

This result is a generalization of the A&B model to arbitrary bargaining powers. A similar result is found in Choné and Linnemer (2012), who directly consider the maximization of the joint surplus, therefore implicitly assuming that the buyer's ex-post participation constraint is not binding. The optimum prescribes an schedule that charges an implicit penalty of $c_I - \tilde{c}_E$ for each unit B decides to purchase from E . As in the A&B model, entry is inefficiently deterred as moderately more efficient rivals do not enter.

We now consider how the optimal allocation rule $q_E^*(c_E)$ can be indirectly implemented by some schedule $T(\cdot)$. Such schedule deters entry if and only if the EB coalition finds it optimal to buy the λ units from I instead of E , that is if $T(1) < T(1 - \lambda) + \lambda c_E$ or equivalently if

$$c_E > \frac{T(1) - T(1 - \lambda)}{\lambda}$$

That is, the effective price faced by the EB coalition for the last λ units must be smaller than the marginal cost of producing it by themselves, which is c_E . More formally,

Lemma 6. *Denote by $e(q_E)$ the effective price for q_E units*

$$e(q_E) \equiv \frac{T(1) - T(1 - q_E)}{q_E}$$

1. *If $T(\cdot)$ implements $q_E^*(c_E)$, as given by (24), then $e(\lambda) = \tilde{c}_E$.*
2. *If $e(q_E)$ is non-decreasing in q_E for all $q_E \leq \lambda$ and satisfies $e(\lambda) = \tilde{c}_E$, then $T(\cdot)$ implements $q_E^*(c_E)$ as in (24).*

Proof. See the Appendix \square

To give an example, consider an all-unit retroactive discount with list price r and rebate $R = \lambda(r - \tilde{c}_E)$, provided the buyer purchases only from the incumbent. This schedule satisfies the conditions in the lemma; therefore, it implements $q_E^*(c_E)$. As seen in Lemma 3, the problem with this or any other schedule that implements $q_E^*(c_E)$ in the relaxed or unrestricted problem is that it might violate the buyer's ex-post participation constraint. Our next proposition establishes the condition for this not to happen, which turns out to be quite restrictive.

Proposition 4. *There exists a schedule $T^*(\cdot)$ that implements $q_E^*(c_E)$, as given by (24), and satisfies the buyer's ex-post participation constraint*

$$T^*(1 - \lambda) \leq v(1 - \lambda)$$

if and only if $\bar{\pi}_I \leq \bar{\pi}_I^*(\eta)$, where

$$\bar{\pi}_I^*(\eta) \equiv (v - c_I)(1 - \lambda) + \lambda(\tilde{c}_E - c_I)[1 - F(\tilde{c}_E)] - \eta[\mathbb{E}W_{IB}^*(c_E; \tilde{c}_E) - \bar{W}] \in (0, (1 - \lambda)(v - c_I))$$

Proof. See the Appendix □

A corollary of the proposition is that when I 's outside option is below the cut-off $\bar{\pi}_I^*(\eta)$, $q_E^*(c_E)$ is indeed the solution to the general problem. It is easy to show (and understand) that the cut-off $\bar{\pi}_I^*(\eta)$ is decreasing in η , I 's bargaining power. A jump in η increases the rents that need to be transferred to I unless that jump is offset by a drop in his outside option. Proposition 4 has a key implication then. The solution of the general problem (60) coincides with A&B exclusive dealing contract outcome only when I 's outside option is sufficiently small (and more so the larger his bargaining power). To illustrate how restrictive the cut-off $\bar{\pi}_I^*(\eta)$ can be, notice that outside payoffs in two standard games are above this cutoff, namely, when I and B simultaneously set linear prices in the spot (see Lemma 2) and when they use non-linear prices (see (11)).

3.4 The bite of the transfer restriction

Let us now characterize the optimal schedule $T(\cdot)$ when $\bar{\pi}_I > \bar{\pi}_I^*(\eta)$. In this region $T(1 - q_E(0)) = v(1 - q_E(0))$ (see Lemma 7 and Proposition 5), so the IB coalition's problem becomes

$$\max_{q_E(c_E) \leq \lambda} \Pi_{IB} = (\mathbb{E}\pi_I - \bar{\pi}_I)^\eta (\mathbb{E}\pi_B - \bar{\pi}_B)^{1-\eta} \quad (28)$$

s.t.

$$\begin{aligned} \mathbb{E}\pi_B &= vq_E(0) - \int_0^{c_E^\circ} \left\{ \frac{1 - (1 - \beta)F(c_E)}{f(c_E)} \right\} q_E(c_E) f(c_E) dc_E \geq \bar{\pi}_B \\ \mathbb{E}\pi_I &= v(1 - q_E(0)) - c_I - \int_0^{c_E^\circ} \left\{ c_E - c_I - \frac{1 - F(c_E)}{f(c_E)} \right\} q_E(c_E) f(c_E) dc_E \geq \bar{\pi}_I \end{aligned}$$

and $q'_E(c_E) \leq 0$. We define the *admissible set* of the problem, as all allocation rules $q_E(c_E) : [0, c_E^\circ] \rightarrow [0, \lambda]$ that satisfy all aforementioned constraints.

In finding the solution to the coalition's problem, it helps to take first a look at the objective function and ask for the schedule that maximizes B 's net surplus, $\mathbb{E}\pi_B - \bar{\pi}_B$, without paying any attention to the sorting and participation constraints. The schedule that does that is $q_E(c_E) = 0$ for all $c_E > 0$, which almost completely prevents entry. This is not because B wants to restrict competition among manufacturers, but because such schedule offers her with the lowest possible effective price ($e(\lambda) \approx 0 < c_I$), which increases the buyer's surplus. Analogously, the schedule that maximizes I 's net surplus, $\mathbb{E}\pi_I - \bar{\pi}_I$, is $q_E(c_E) = \lambda$ for $c_E \leq c_E^\circ$ and 0 otherwise. Since blocking an expansion is costly (because it requires a low effective price) and appropriating a larger fraction of the rents extracted by B from E is not possible (because $T(1 - \lambda) \leq v(1 - \lambda)$ must hold at all times), it is optimal for I to charge an effective price above its marginal cost ($e(\lambda) = c_E^\circ > c_I$). In other words, the incumbent would rather behave as a monopoly over the contestable share and allow for some inefficient entry (see also discussion below Proposition 2).

A&B's key insight was that this bargaining situation is not a zero-sum game. They show that there is an schedule that could potentially make both parties better-off, one that extracts rents from more efficient rivals. Critical for that arrangement to work, however, is the existence of unrestricted transfers from B to I because it is B who first receives the rents extracted from E . So, when there is a cap on the total amount of surplus that can be transferred, the Coasian principle behind A&B's insight breaks down. Then, it is not longer possible to solve the coalition problem by following two independent steps: first, finding the schedule that maximizes joint surplus, and then, sharing the surplus in a way that that is approved by both parties. A cap on transfers not only modifies the form of the optimal schedule but may also destroy enough (ex-ante) rents that signing the agreement becomes not longer optimal for one or the two parties.

We now tackle the problem formally. From the unconstrained maximization of $\mathbb{E}\pi_I$ shown above, we can rewrite the maximum value $\mathbb{E}\pi_I$ can take in this transfer-restricted setting as

$$(1 - \lambda)(v - c_I) + \lambda(c_E^\circ - c_I)[1 - F(c_E^\circ)] \equiv \bar{\pi}_I^{***} > \bar{\pi}_I^*(\eta)$$

We restrict attention then to $\bar{\pi}_I$ belonging to the non-empty interval $(\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***}]$, since otherwise the solution clearly does not exist as no schedule will be acceptable to I .

Requiring $\bar{\pi}_I \leq \bar{\pi}_I^{***}$ is only a necessary, but not a sufficient condition, since we still have to

check whether: (1) the Nash product is well-behaved; and (2) there exists a feasible allocation rule that simultaneously satisfies B 's participation ex-ante (implying that the admissible set is non-empty). We defer this discussion however and, assuming existence, concentrate first into characterize the shape of the optimal schedule when $\eta = 1$ and $\eta = 0$. As we will see, because of the simple nature of the solution, the remaining conditions to ensure existence will then follow.

Lemma 7. *Fix \bar{W} and consider some $\bar{\pi}_I \in (\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***}]$.*

1. *Suppose that $\eta = 1$ and let $\hat{c}_E(\bar{\pi}_I) = \min \{c_E^\circ, \delta(\bar{\pi}_I)\}$, where δ is strictly increasing in $\bar{\pi}_I$ and implicitly given by*

$$\lambda(v - \delta) + \lambda(1 - \beta)F(\delta)[\delta - \mathbb{E}(c_E \mid c_E \leq \delta)] = \bar{W} - \bar{\pi}_I$$

If a solution exists, then it is given by (i) $T^(1 - \lambda) = v(1 - \lambda)$ and (ii) $q_E^*(c_E) = \lambda$ if $c_E \leq \hat{c}_E(\bar{\pi}_I)$, and 0 otherwise.*

2. *Suppose that $\eta = 0$ and let $\check{c}_E(\bar{\pi}_I)$ be given by the solution to*

$$(1 - \lambda)(v - c_I) + \lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)] = \bar{\pi}_I$$

*which implies that $\check{c}_E(\bar{\pi}_I)$ is strictly increasing in $\bar{\pi}_I$, with $\check{c}_E(\bar{\pi}_I^{***}) = c_E^\circ$. If a solution exists, then it is given by (i) $T^*(1 - \lambda) = v(1 - \lambda)$ and (ii) $q_E^*(c_E) = \lambda$ if $c_E \leq \check{c}_E(\bar{\pi}_I)$, and 0 otherwise.*

Proof. See the Appendix. □

Two issues are worth highlighting of this lemma. First, independently of whether $\eta = 0$ or $\eta = 1$, the solution is simple in the sense that the optimal allocation $q_E^*(c_E)$ is entirely described by a cutoff value for c_E over which entry is completely blocked. In both cases E sells either λ or 0, depending on its cost realization. And second, the optimal cutoff in each case is usually²⁸ chosen to barely satisfy the counter-party ex-ante participation constraint. That is, when $\eta = 0$ the cutoff makes I 's participation binding and vice-versa.

A sufficient condition for a non-empty admissible set then arises naturally. Consider the set Θ of all allocation rules $q_E(c_E)$ with that simple threshold shape: $q_E(c_E) = \lambda$ if $c_E \leq x$

²⁸This is always true when $\eta = 0$. When $\eta = 1$ however, there may exist cases in which, if B outside is sufficiently small, a cutoff equal to c_E° (I 's "unrestricted" optimal cut-off) leaves some slackness on B 's ex-ante participation. This explains why $\hat{c}_E = \min \{c_E^\circ, \delta\}$. This is analogous to equation (10) in the simple model.

and $q_E(c_E) = 0$, otherwise. A sufficient condition for the existence of an admissible set, is the existence of a non-empty subset of Θ consisting in those threshold allocation rules that also satisfy both participation constraints. However, due to its shape, this is equivalent to

$$\begin{aligned}\mathbb{E}\pi_B(c_E; x) &= \lambda(v - x) + \lambda(1 - \beta)F(x)[x - \mathbb{E}(c_E \mid c_E \leq x)] \geq \bar{\pi}_B \\ \mathbb{E}\pi_I(c_E; x) &= (1 - \lambda)(v - c_I) + \lambda(x - c_I)[1 - F(x)] \geq \bar{\pi}_I\end{aligned}$$

But using the definition of $\check{c}_E(\bar{\pi}_I)$ and $\hat{c}_E(\bar{\pi}_I)$ from Lemma 7, exists some $x \in [\check{c}_E(\bar{\pi}_I), \hat{c}_E(\bar{\pi}_I)]$. Therefore, a sufficient condition for the existence of a feasible allocation rule is $\check{c}_E(\bar{\pi}_I) \leq \hat{c}_E(\bar{\pi}_I)$. It turns out that this is actually a necessary condition as well:

Lemma 8. *Fix \bar{W} and consider $\bar{\pi}_I \in (\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***}]$. Then, for all $\eta \in [0, 1]$ the admissible set of the general problem is non-empty if and only if $\check{c}_E(\bar{\pi}_I) \leq \hat{c}_E(\bar{\pi}_I)$.*

Proof. See the Appendix. □

It is straightforward to check furthermore, that $\check{c}_E(\bar{\pi}_I) \leq \hat{c}_E(\bar{\pi}_I)$ is also necessary and sufficient for the existence of solution in the boundary cases $\eta = \{0, 1\}$.

Corollary 1. *A solution to the boundary problems $\eta = 0$ and $\eta = 1$ exists if and only if $\check{c}_E(\bar{\pi}_I) \leq \hat{c}_E(\bar{\pi}_I)$*

Proof. Immediate from above. □

We can now fully characterize the solution to the general problem for arbitrary bargaining powers η and $1 - \eta$. It turns out that the simple cutoff nature of the solution actually extends to bargaining powers strictly between 0 and 1:

Proposition 5. *Fix \bar{W} , consider $\bar{\pi}_I \in (\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***}]$, and suppose $\check{c}_E(\bar{\pi}_I) \leq \hat{c}_E(\bar{\pi}_I)$. Then, the solution to the general problem exists, and is given by $T^*(1 - \lambda) = v(1 - \lambda)$ and*

$$q_E^*(c_E) = \begin{cases} \lambda & \text{if } c_E \leq c_E^*(\eta, \bar{\pi}_I) \\ 0 & \text{otherwise} \end{cases} \quad (29)$$

where $\check{c}_E(\bar{\pi}_I) < c_E^*(\eta, \bar{\pi}_I) < \hat{c}_E(\bar{\pi}_I)$.

Proof. See the Appendix □

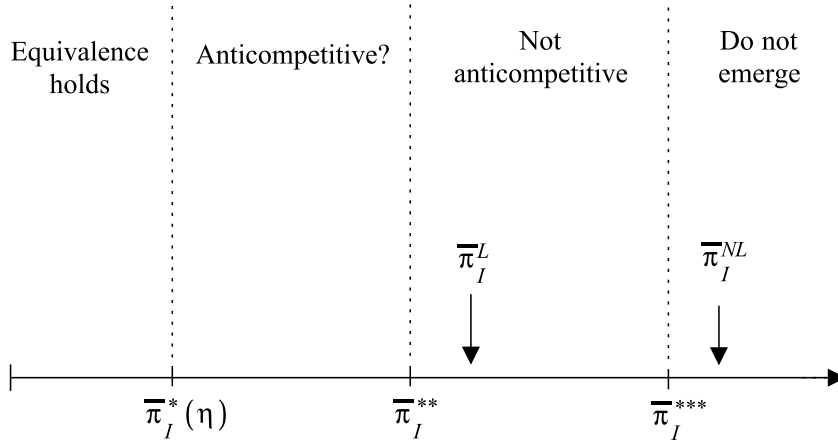
Given the simple nature of the solution, then the “anticompetitive” nature of a discount contract is entirely summarized in the optimal cutoff $c_E^*(\eta, \bar{\pi}_I)$. Moreover, it implies that the rebate schemes (r, R) analyzed in Section 2.5 were constraint-optimal, so results in the simple model of Section 2, were not driven by selecting a particular subset of schedules without unconditional payments.

This simple solution allows us to characterize very neatly sufficient conditions as to when contracts are anticompetitive and when they are not. In particular, given the optimal allocation rule, payoffs are given by

$$\begin{aligned}\mathbb{E}\pi_B(c_E; c_E^*) &= \lambda(v - c_E^*) + \lambda(1 - \beta)F(c_E^*)[c_E^* - \mathbb{E}(c_E \mid c_E \leq c_E^*)] \\ \mathbb{E}\pi_I(c_E; c_E^*) &= (1 - \lambda)(v - c_I) + \lambda(c_E^* - c_I)[1 - F(c_E^*)]\end{aligned}$$

Then, if $\bar{\pi}_I \geq (1 - \lambda)(v - c_I) \equiv \bar{\pi}_I^{**} \in (\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***})$, which is equivalent to $\check{c}_E = c_I$, then “anticompetitive” contracts cannot arise in equilibrium, as they do not satisfy I ’s participation constraint (i.e. $\lambda(c_E^* - c_I)[1 - F(c_E^*)] \geq 0$ necessarily). Hence, $\bar{\pi}_I \geq \bar{\pi}_I^{**}$ is sufficient to ensure that anticompetitive contracts are not signed. Contracts with $c_E^* \geq c_I$ however, can emerge if $\check{c}_E = c_I < \hat{c}_E$.

Figure 1: Regions where rebates may or may not be anticompetitive



The above discussion help us partition the set of potential $\bar{\pi}_I$ in different regions, as shown in Figure 1. In particular, if $\bar{\pi}_I$ is between $(0, \bar{\pi}_I^*(\eta))$, even without unconditional payments, the optimal schedule is equivalent to the A&B exclusive dealing contract. On the other hand,

when $\bar{\pi}_I \in [\bar{\pi}_I^*(\eta), \bar{\pi}_I^{**}]$ the sufficient conditions already derived do not apply, so we need a finer characterization. If $\bar{\pi}_I \in [\bar{\pi}_I^{**}, \bar{\pi}_I^{***}]$ then discount contracts cannot be anticompetitive, but may still emerge if $\check{c}_E \leq \hat{c}_E$. And finally if $\bar{\pi}_I > \bar{\pi}_I^{***}$ then with certainty no contract is signed at $t = 1$, as I rejects any proposal.

The figure also includes the payoffs $\bar{\pi}_I^L$ and $\bar{\pi}_I^{NL}$ which correspond, respectively, to I 's outside payoff in the simpler model of Section 2 (see Lemma 2) and to his payoff in the non-linear spot pricing game (see (11)). As $\bar{\pi}_I^L \geq \bar{\pi}_I^{**}$, anticompetitive rebates can not emerge in equilibrium in this case. Moreover, when $\bar{\pi}_I = \bar{\pi}_I^L$ it is straightforward to prove that $\bar{c}_E \leq c^*$ is sufficient to ensure that $\check{c}_E = c_I < \hat{c}_E$, and hence, contracts would in fact be used to extract rents from more inefficient rivals. This latter does not extend to the case when $\bar{\pi}_I = \bar{\pi}_I^{NL} > \bar{\pi}_I^{***}$. In this case, no rebate contract will ever be signed ex-ante as $\check{c}_E > \hat{c}_E = c_E^o$. Intuitively, when transfers are restricted the distortion over the ex-ante schedule needed to pay for the incumbent's outside option is so great, that the total surplus generated is not sufficient to satisfy both participation constraints.

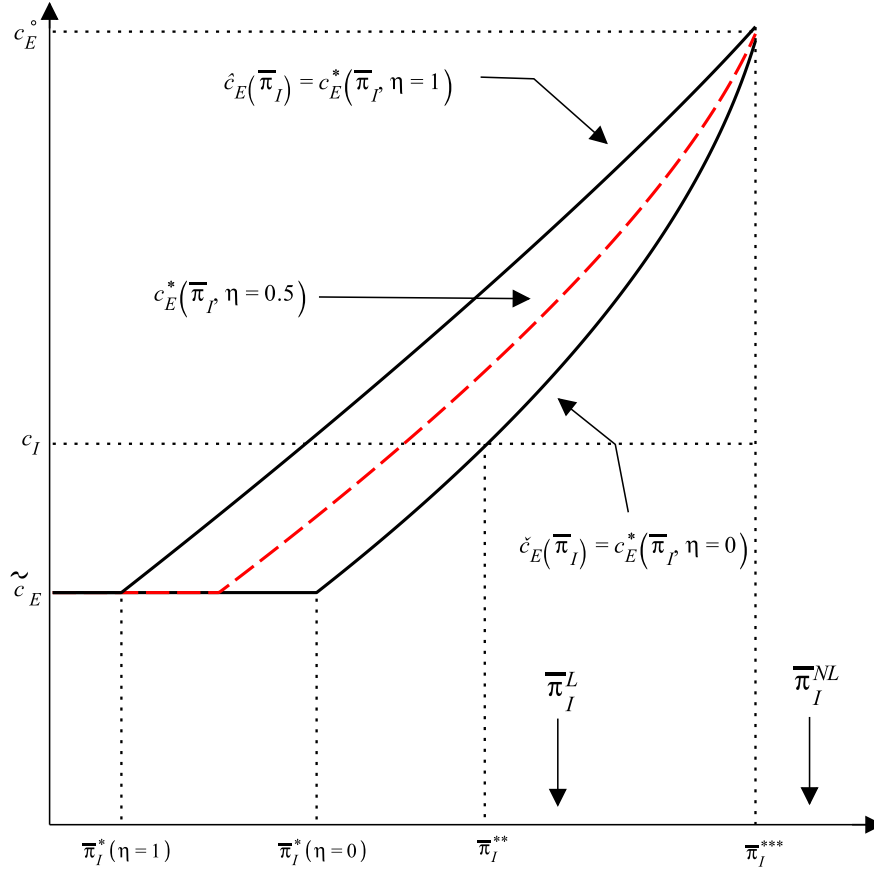
Returning to the general formulation, we still need to characterize the optimal rebate contract when $\bar{\pi}_I \in [\bar{\pi}_I^*(\eta), \bar{\pi}_I^{**}]$. It is not possible to determine exactly when a contract in this region is anticompetitive, at least without further specifying the distribution of E 's costs. With the help of the following proposition, we can nevertheless indicate when that is more likely to be the case:

Proposition 6. *Let the pair $(\eta, \bar{\pi}_I)$ define the incumbent's bargaining position. As the optimal cutoff $c_E^*(\eta, \bar{\pi}_I)$ is increasing in both η and $\bar{\pi}_I$, the stronger the incumbent's bargaining position the less likely a rebate contract prevents the entry/expansion of efficient rivals (i.e., the more likely $c_E^*(\eta, \bar{\pi}_I) > c_I$).*

Proof. It requires to work out the comparative statics of the cutoff function, which is done in the Appendix □

Figure 2 shows how c_E^* varies with η and $\bar{\pi}_I$ for a uniform distribution of E 's costs. As grasped from the figure, the main implication of the general model is that without unconditional payments, a strong bargaining position (large values of η and $\bar{\pi}_I$) is not consistent with anticompetitive rebate contracts. Those contracts may still exist but would only result in a barrier to inefficient entry.

Figure 2: Optimal entry cutoff as a function of the bargaining positions



4 Extension: Downward Sloping Demand

The introduction of a downward sloping demand gives rise to the possibility of signing more complex discount contracts that not only depend on the incumbent's own sales (own-supply discounts) but also on the entrant's sales (market-share discounts); for example, like the ones considered by Calzolari and Denicolo (2013). The coalition IB is ready to use these more complex contracts to avoid introducing allocative inefficiencies that would otherwise destroy coalitional surplus. In this section we briefly show how our results extend to this setting.

Let $D(p)$ be B 's demand function (and $P(q)$ the corresponding inverse demand), satisfying all regularity conditions, and $S(q) \equiv \int_0^q P(s)ds$ be B 's benefit from consuming q units. We keep the perfect substitutability between units offered by the two suppliers. We also maintain the reduced-form approach of the previous section, so $\bar{\pi}_I$ and $\bar{\pi}_B$ are I and B outside options, respectively.

Suppose the incumbent offers the buyer the three-part exclusive-dealing contract (K, p, t) ,

where K is a lump-sum transfer at the signing of the contract, p is the wholesale unit price, and t is the per-unit penalty for each unit B decides to buy elsewhere.²⁹ To keep things simple, go back to the A&B assumption that $\eta = \beta = 1$, though it should become clear that the argument easily generalizes to arbitrary bargaining powers.

In this setting, E expands/enters and sells all its units whenever $c_E \leq p - t$. B 's payoff, on the other hand, is in any event $S(D(p)) - pD(p) - K$, and therefore the program that determines the optimal contract offered by I is

$$\max_{(K,p,t)} \mathbb{E}\pi_I = F(p-t)[\lambda t + (p-c_I)(D(p)-\lambda)] + [1-F(p-t)](p-c_I)D(p) + K \quad (30)$$

$$\text{s.t. } S(D(p)) - pD(p) - K \geq \bar{\pi}_B.$$

The solution to this problem is $p^* = c_I$, $K^* = S(D(c_I)) - c_I D(c_I) - \bar{\pi}_B$, and

$$t^* = \frac{F(c_I - t^*)}{f(c_I - t^*)}$$

As with the exclusive dealing contracts for the case of unit demand (see eq. (2)), only types with costs

$$c_E \leq p_I^* - t^* = c_I - \frac{F(p_I^* - t^*)}{f(p_I^* - t^*)} \equiv \tilde{c}_E \quad (31)$$

enter and sell their λ units.

Analogous to the analysis in Section 2, it is easy to see that a market-share discount equipped with unconditional payments of the form $T(q_I, q_E) = K^* + c_I q_I + (\mathbf{1}_{q_I > 0})[t^* q_E]$, where $(\mathbf{1}_{q_I > 0})$ is the indicator function and K^* is an up-front payment from B to I , achieves the exact same outcome as the exclusive dealing above. It remains to see whether this outcome can also be achieved with an equivalent market-share discount but not equipped with unconditional payments, say, a market-share rebate. Since $T(0, q_E) = 0$ must hold in equilibrium, we must rewrite this ‘‘rebate’’ contract as $T(q_I, q_E) = c_I q_I + (\mathbf{1}_{q_I > 0})[K^* + t^* q_E]$. And since K^* is paid conditional on B buying a positive amount from I , we now need to check whether B 's ex-post participation constraint (for all admissible (q_I, q_E) pairs)

$$T(q_I, q_E) \leq S(q_I + q_E) - S(q_E)$$

²⁹Whether the liquidated damages is paid lump-sum (independently of the amount breached) or by per-unit basis is again immaterial in this setting.

indeed holds. It suffices to check this for two events: the no-entry event $(D(c_I), 0)$ and the full-entry event $(D(c_I) - \lambda, \lambda)$. It is easy to see that $T(D(c_I), 0) \leq S(D(c_I)) - S(0)$ if and only if $\bar{\pi}_B \geq 0$. As for the entry event, and using that $\bar{\pi}_B = \bar{W} - \bar{\pi}_I$, we get

$$T(D(c_I) - \lambda, \lambda) \leq S(D(c_I)) - S(\lambda) \iff \bar{\pi}_I \leq \bar{W} - S(\lambda) + \lambda \tilde{c}_E$$

That is, the “rebate” contract can deliver the exclusive-dealing discount outcome only if the incumbent’s outside option is sufficiently small.

We finish this section providing a general characterization of the best market-share rebate contract I and B can agree upon. It turns that it is actually analogous to the case of the unit demand so all qualitative results remain unchanged.

Lemma 9. *The optimal market-share rebate when demand is downward sloping implements the allocation rule*

$$(q_I^*(c_E), q_E^*(c_E)) = \begin{cases} (D(c_I) - \lambda, \lambda) & \text{if } c_E \leq c_E^*(\bar{\pi}_I, \eta) \\ (D(c_I), 0) & \text{otherwise} \end{cases}$$

with

$$c_E^*(\bar{\pi}_I, \eta) = \begin{cases} \tilde{c}_E & \text{if } \bar{\pi}_I \leq \bar{\pi}_I^{D^*}(\eta) \\ z(\bar{\pi}_I, \eta) & \text{otherwise} \end{cases}$$

where $\bar{\pi}_I^{D^*}(\eta)$ is strictly decreasing in η , $z(\bar{\pi}_I, \eta) > \tilde{c}_E$ increasing in both its arguments, and \tilde{c}_E is given by (25).

Proof. See the Appendix. □

5 Conclusion

We have studied the anti-competitive effects of discount contracts (e.g., 2PT, rebates) between an incumbent and a buyer in the rent-shifting environment of A&B. We show that the existence of unconditional payments, or more precisely, of an ex-ante commitment —liquidated damages in the case of A&B contracts and up-front payments in the case of 2PT contracts— to transfer rents from the retailer to the manufacturer is crucial for the A&B anticompetitive result to hold. Rebate contracts that only commit the buyer to a price-quantity purchasing schedule restrict the amount of rents the incumbent and the buyer can transfer ex-ante so much that these contracts are rarely anticompetitive. This result is only reinforced as the incumbent’s

bargaining position, which can be understood as some combination of bargaining power and outside option, becomes stronger. The reason these discount contracts exist is because they can still be used to extract rents from inefficient rivals, that is, they can serve as a barrier to (inefficient) entry.

Our results have important antitrust implications for cases that are best examined through the lense of a rent-shifting model. Before ruling out a rebate contract as anticompetitive, an antitrust authority should still check whether the contract scheme is in fact not equipped with some form of unconditional transfers from the retailer to the manufacturer.³⁰ If the contract does involve such “up-front” payments there are good reasons to believe that an A&B anticompetitive result may apply because it is hard to see efficiency reasons to add up-front payments on top of a rebate scheme.³¹ Interestingly, in the particular case of rebates, such payments are either rarely observed or have not been documented by antitrust authorities in making their cases.

Even in the absence of up-front payments, the antitrust authority should still analyze the relative *bargaining position* of the contracting parties. In particular, the stronger the buyer *vis-a-vis* the incumbent, the greater the chance a rebate contract may have anticompetitive implications. This result is important for two reasons. First, it goes against the common belief that highly dominant incumbents are the ones more likely to foreclose the entry of efficient rivals. And second, it seems specially relevant today given the recent trend towards a greater consolidation of downstream retailers and the consequent increase of buyer’s bargaining power (see, for example, European Commission 1999 or FTC 2001).³²

³⁰Notice however that such transfers may not need to be explicit in the form of “all-event” monetary payments, but can be masked behind principal-agent efficiency rationales; for example as payments for relationship-specific investments.

³¹For example, no such payments are documented in the rebate contracts of the chocolate-candy giant Mars (Conlon and Mortimer 2013).

³²And for a more theoretical perspective, Inderst and Mazzarotto (2008)

APPENDIX

Proof of Proposition 2

The proof of $r^* = v$ is immediate: Provided that $v < r^u$, if $r^* < v$ the incumbent could increase his payoff by slightly increasing r and R so as to maintain e constant, so $r^* < v$ cannot be optimal. Using $r = v$, I 's expected payoff becomes $\pi_I^R \equiv \mathbb{E}\pi_I(e) = (v - c_I)(1 - \lambda) + \lambda(e - c_I)[1 - F(e)]$ and B 's participation constraint $\lambda(v - e) \geq \bar{\pi}_B$. We also know from Proposition 1 that “anticompetitive” rebates cannot emerge in equilibrium.

It remains to see whether non-exclusionary contracts can emerge in equilibrium, that is, whether exists $c_E^* > c_I$ such that both (ex-ante) participation constraints are met simultaneously. We next show that a sufficient condition for that to be the case is $\bar{c}_E \leq c^*$. Using $r^* = v$ and the expression for $\bar{\pi}_I$ in Lemma 2, I 's participation constraint can be rewritten as

$$\left[\frac{1 - F(c^*)}{1 - F(c_E^*)} \right] \{ \mathbb{E}(c_E \mid c_E > c^*) - c^* \} \leq \lambda(c_E^* - c_I)$$

which holds easily because the left side of the inequality is zero when $\bar{c}_E \leq c^*$. On the other hand, the buyer's participation constraint can be rearranged as $\lambda(c_E^* - c_I) \leq \lambda(v - c_I) - \bar{\pi}_B$, which is satisfied for any c_E^* greater but sufficiently close to c_I .

Finally and regardless of whether $\bar{c}_E \leq c^*$ holds, it can also be shown that $c_E^* < \bar{c}_E$. Differentiating π_I^R with respect to e and evaluating at c_E^* we get:

$$\frac{\partial \pi_I^R(c_E^*)}{\partial e} = \lambda[1 - F(c_E^*) - f(c_E^*)(c_E^* - c_I)] \quad (32)$$

which shows that π_I^R attains its unconstrained maximum at $c_E^* < \bar{c}_E$. This latter together with the fact that the buyer's participation constraint is less demanding for lower values of c_E^* indicates that the incumbent will strictly set c_E^* below \bar{c}_E .

Proof of Proposition 3

If $T(1 - q_E(0)) \leq v(1 - q_E(0))$ is not binding, then the optimality conditions for the IB coalition problem are $\partial \Pi_{IB} / \partial T(1 - q_E(0)) = 0$, or:

$$\Pi_{IB} \left[\frac{\eta}{(\mathbb{E}\pi_I - \bar{\pi}_I)} - \frac{(1 - \eta)}{(\mathbb{E}\pi_B - \bar{\pi}_B)} \right] = 0 \quad (33)$$

and the Euler-Lagrange condition for the composition of functionals Π_{IB} (see also Castillo et. al. 2008):

$$\begin{aligned} -\Pi_{IB} & \left(\frac{\eta}{(\mathbb{E}\pi_I - \bar{\pi}_I)} + \frac{(1-\eta)}{(\mathbb{E}\pi_B - \bar{\pi}_B)} \right) \left((1-\gamma)(c_E - c_I) - \frac{2[1-F(c_E)]}{f(c_E)} \left(\frac{1}{2} - \gamma \right) + \frac{\gamma \beta F(c_E)}{f(c_E)} \right) \\ & = \theta L(c_E) \geq 0 \end{aligned} \tag{34}$$

where $\theta > 0$ and $\gamma \equiv \frac{(1-\eta)(\mathbb{E}\pi_I - \bar{\pi}_I)}{\eta(\mathbb{E}\pi_B - \bar{\pi}_B) + (1-\eta)(\mathbb{E}\pi_I - \bar{\pi}_I)} \in (0, 1)$. Thus, the sign of the pointwise-derivative is given by

$$L(c_E) \equiv - \left\{ (1-\gamma)(c_E - c_I) - \frac{2[1-F(c_E)]}{f(c_E)} \left(\frac{1}{2} - \gamma \right) + \frac{\gamma \beta F(c_E)}{f(c_E)} \right\}$$

However, from (33) we have $\eta(\mathbb{E}\pi_B - \bar{\pi}_B) = (1-\eta)(\mathbb{E}\pi_I - \bar{\pi}_I)$ so $\gamma = 1/2$. Consequently, the Euler condition simplifies to $-\frac{\theta}{2} \left[c_E - c_I + \frac{\beta F(c_E)}{f(c_E)} \right] \geq 0$ which implies that the optimal allocation $q_E^*(c_E)$ is given by:

$$q_E^*(c_E) = \begin{cases} \lambda & \text{if } c_E \in [0, \tilde{c}_E] \\ 0 & \text{otherwise} \end{cases}$$

where \tilde{c}_E satisfies:

$$f(\tilde{c}_E)(\tilde{c}_E - c_I) + \beta F(\tilde{c}_E) = 0$$

Therefore, $\mathbb{E}\pi_I, \mathbb{E}\pi_B$ depend only on $T^*(1 - q_E(0)) = T^*(1 - \lambda)$ and the threshold \tilde{c}_E :

$$\mathbb{E}\pi_B = v - T^*(1 - \lambda) - \lambda \tilde{c}_E + \lambda(1 - \beta)F(\tilde{c}_E)[\tilde{c}_E - \mathbb{E}(c_E | c_E \leq \tilde{c}_E)] \equiv \mathbb{E}\pi_B(c_E; \tilde{c}_E)$$

$$\mathbb{E}\pi_I = T^*(1 - \lambda) - (1 - \lambda)c_I + \lambda(\tilde{c}_E - c_I)[1 - F(\tilde{c}_E)] \equiv \mathbb{E}\pi_I(c_E; \tilde{c}_E)$$

But condition $\eta(\mathbb{E}\pi_B(c_E; \tilde{c}_E) - \bar{\pi}_B) = (1-\eta)(\mathbb{E}\pi_I(c_E; \tilde{c}_E) - \bar{\pi}_I)$ also implies that $T^*(1 - \lambda)$ is chosen such that $\mathbb{E}\pi_I(c_E; \tilde{c}_E) = \bar{\pi}_I + \eta[\mathbb{E}W_{IB}^*(c_E; \tilde{c}_E) - \bar{W}]$

Proof of Proposition 4

Given (24) and (26) we have

$$T^*(1 - \lambda) = \bar{\pi}_I + \eta[\mathbb{E}W_{IB}^* - \bar{W}] + (1 - \lambda)c_I - \lambda(\tilde{c}_E - c_I)[1 - F(\tilde{c}_E)] \tag{35}$$

But then,

$$\begin{aligned} T^*(1 - \lambda) - v(1 - \lambda) &= \bar{\pi}_I + \eta \Delta W_{IB}(\tilde{c}_E) - (1 - \lambda)(v - c_I) - \lambda(\tilde{c}_E - c_I)[1 - F(\tilde{c}_E)] \\ &= \bar{\pi}_I - \bar{\pi}_I^*(\eta) \end{aligned} \quad (36)$$

which implies that $T^*(1 - \lambda) \leq v(1 - \lambda)$ if and only if $\bar{\pi}_I \leq \bar{\pi}_I^*(\eta)$.

Proof of Proposition 5

The cases $\eta = 0$ and $\eta = 1$ have been already characterized so we analyze $\eta \in (0, 1)$ instead. Fix $\eta \in (0, 1)$ and a pair of outside options $(\bar{\pi}_I, \bar{\pi}_B)$ such that $\bar{\pi}_I \in (\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***})$, and suppose $\check{c}_E(\bar{\pi}_I) < \hat{c}_E(\bar{\pi}_I)$ so the admissible set is non-empty (the case with $\check{c}_E = \hat{c}_E$ is trivial). Now, for $q_E^*(c_E)$ to be a solution is must: (1) belong to the admissible set; and (2) be coherent with (as we will see, a fixed-point of) the Euler-Lagrange condition.

Claim 1. *If the general problem has a solution, then it is given by $q_E^*(c_E) = \lambda$ for all $c_E \leq \omega$ and $q_E(c_E) = 0$ otherwise, where $\omega \in (0, c_E^\circ)$ and satisfies $L(\omega) = 0$ and $L'(\omega) \leq 0$.*

Proof. Suppose a solution exists, and it is given by $q_E^*(c_E)$. Then neither participation constraint can be binding as the objective functional goes to zero, and therefore it is dominated by a threshold allocation with $x \in (\check{c}_E, \hat{c}_E)$. Moreover, the constraint $T(1 - q_E(0)) \leq v(1 - q_E(0))$ is binding since otherwise we would arrive at the A&B “coalitional” solution, which clearly this violates the transfer constraint. Hence, the optimality condition is given by the Euler-Lagrange condition for the composition of functions $\Pi_{IB}: \theta L(c_E)$, where $\theta \equiv \Pi_{IB} \left(\frac{\eta}{(\mathbb{E}\pi_I - \bar{\pi}_I)} + \frac{(1-\eta)}{(\mathbb{E}\pi_B - \bar{\pi}_B)} \right)$, and $L(c_E)$:

$$L(c_E) \equiv - \left\{ (1 - \gamma)(c_E - c_I) - \frac{2[1 - F(c_E)]}{f(c_E)} \left(\frac{1}{2} - \gamma \right) + \frac{\gamma \beta F(c_E)}{f(c_E)} \right\}$$

with,

$$\gamma \equiv \frac{(1 - \eta) (\mathbb{E}\pi_I - \bar{\pi}_I)}{\eta (\mathbb{E}\pi_B - \bar{\pi}_B) + (1 - \eta)(\mathbb{E}\pi_I - \bar{\pi}_I)} \in (0, 1)$$

Since neither participation constraint can be binding, $\theta > 0$ necessarily. Notice furthermore that the allocation rule $q_E^*(c_E)$ enters only as a constant in θ and $L(c_E)$, through $\mathbb{E}\pi_I$, $\mathbb{E}\pi_B$ and

γ . Now,

$$\begin{aligned}
L(0) &= - \left[-(1-\gamma)c_I - \frac{2}{f(0)} \left\{ \frac{1}{2} - \gamma \right\} \right] \geq 0 \\
L(c_E^\circ) &= - \frac{\gamma[1 - (1-\beta)F(c_E^\circ)]}{f(c_E^\circ)} < 0 \\
L'(c_E) &= - \left\{ (1-\gamma) - 2 \left(\frac{1-F(c_E)}{f(c_E)} \right)' \left(\frac{1}{2} - \gamma \right) + \gamma \beta \left(\frac{F(c_E)}{f(c_E)} \right)' \right\} \geq 0
\end{aligned} \tag{37}$$

Given that $L(c_E)$ is not necessarily regular, we construct the optimal allocation schedule $q_E^*(c_E)$ piece by piece beginning by the end. Given that by assumption the problem has a solution and that $L(c_E^\circ) < 0$, then $\exists M \in (0, c_E^\circ)$ such that $L(c_E) < 0$ for all $c_E > M$ (and consequently $L'(M) \leq 0$). Obviously then, $q_E^*(c_E) = 0$ for all $c_E > M$.

Now, given $L'(M) \leq 0$, one possibility is that $L(c_E) \geq 0$ for all $c_E < M$ (the regular case). Then, the problem is trivial and the optimal allocation is $q_E^*(c_E) = \lambda$ for all $c_E \leq M \in (0, c_E^\circ)$ and $q_E(c_E) = 0$ otherwise. On the contrary, suppose that $L'(M) \leq 0$ but that $\exists M' < M$ such that $L(M') = 0$. Notice then, that $q_E^*(c_E) = q_E(M') > 0$ and constant for all $c_E \in [M', M]$.

Let us then move further to the left. We have two options. First, suppose that $L(c_E) < 0$ for all $c_E < M'$. Then, if $\int_0^M L(c_E)f(c_E)dc_E \geq 0$, the optimal allocation would again be (??). On the contrary, if $\int_0^M L(c_E)f(c_E)dc_E < 0$ then $q_E^*(c_E) = 0$ for all c_E , which violates our premise of the existence of a solution. The second possibility is that there exists $M'' < M'$ such that $L(M'') = 0$. Then if $\int_{M''}^M L(c_E)f(c_E)dc_E \geq 0$, $q_E^*(c_E) = q_E(M'') \geq q_E(M) > 0$ constant for all $c_E \in [M'', M]$. On the contrary, if $\int_{M''}^M L(c_E)f(c_E)dc_E < 0$, then $q_E^*(c_E) = 0$ for all $c_E \in [M'', M]$.

Notice then, that the cycle begins again in the interval $[0, M'']$ with either $q_E(M'') = 0$ or $q_E(M'') = k > 0$. By the same argument, it is easy to realize that the optimal allocation has a simple threshold shape $q_E^*(c_E) = \lambda$ for all $c_E \leq \omega \in (0, c_E^\circ)$ and $q_E(c_E) = 0$, where $L(\omega) = 0$ and $L'(\omega) \leq 0$. \square

We proved that if a solution exists, then $T(1 - q_E(0)) = v(1 - q_E(0))$ and it has a simple threshold shape. We now tackle the issue of existence, that is, whether there exists a threshold that is coherent with the optimality conditions. Given $q_E^*(c_E)$ with threshold shape, we can rewrite $\mathbb{E}\pi_I \equiv \mathbb{E}\pi_I(c_E; \omega)$, $\mathbb{E}\pi_B \equiv \mathbb{E}\pi_B(c_E; \omega)$, and therefore $\theta(\omega)$ and $\gamma(\omega)$. Then, for such allocation to be a solution it must: (1) leave both ex-ante participation constraints with slack (i.e $\theta(\omega^*) > 0$), and (2) $L(\omega^*) = 0$ and $L'(\omega^*) \leq 0$.

Claim 2. *There exists some $c_E^* \in (\check{c}_E, \hat{c}_E)$ satisfying all optimality conditions. Hence a solution exists and is given by (29).*

Proof. Condition $\theta(\omega^*) > 0$ restricts $\omega^* \in (\check{c}_E, \hat{c}_E)$. The issue now is whether there exists an ω^* such that

$$L(\omega^*) = - \left[(1 - \gamma(\omega^*))(\omega^* - c_I) - \frac{2[1 - F(\omega^*)]}{f(\omega^*)} \left(\frac{1}{2} - \gamma(\omega^*) \right) + \frac{\gamma(\omega^*)\beta F(\omega^*)}{f(\omega^*)} \right] = 0 \quad (38)$$

and that $L'(\omega^*) \leq 0$. To see it, notice that $\gamma(\check{c}_E) = 0$, so $L(\check{c}_E) = - \left[\check{c}_E - c_I - \frac{1-F(\check{c}_E)}{f(\check{c}_E)} \right] > 0$, while

$$L(\hat{c}_E) = \begin{cases} - \left[\frac{1-(1-\beta)F(\delta)}{f(\delta)} \right] < 0 & \text{if } \hat{c}_E = \delta \\ -\gamma(c_E^o) \left[\frac{1-(1-\beta)F(c_E^o)}{f(c_E^o)} \right] < 0 & \text{if } \hat{c}_E = c_E^o \end{cases}$$

Hence, there exists $c_E^*(\bar{\pi}_I, \eta) \equiv \omega^* \in (\check{c}_E, \hat{c}_E)$ that satisfies all optimality conditions. \square

Proof of Proposition 6

We will only analyze the effect of η , since the proof for $\bar{\pi}_I$ is almost completely analogous. From Proposition 5, we know that $q_E^*(c_E)$ has a simple threshold shape and that the cutoff $c_E^*(\eta, \bar{\pi}_I) \in (\check{c}_E, \hat{c}_E)$ is given by one of the roots of $L(c_E^*) = 0$ where $L'(c_E^*) \leq 0$ as in (38)

It is straightforward to prove then that $\gamma(c_E^*)$ is increasing in c_E^* , and that it is decreasing in the direct effect of η . Hence, by the implicit function theorem we have

$$\frac{\partial c_E^*}{\partial \eta} = \frac{(\partial \gamma / \partial \eta) A(c_E^*)}{-L'(c_E^*) - (\partial \gamma / \partial c_E^*) A(c_E^*)}$$

where $A(c_E^*) \equiv c_E^* - c_I - 2 \left(\frac{1-F(c_E^*)}{f(c_E^*)} \right) - \frac{\beta F(c_E^*)}{f(c_E^*)}$. Now, notice that the condition (38) can be rewritten as

$$\left[c_E^* - c_I - \left(\frac{1 - F(c_E^*)}{f(c_E^*)} \right) \right] - \gamma(c_E^*) A(c_E^*) = 0 \quad (39)$$

Moreover $\left[c_E^* - c_I - \left(\frac{1-F(c_E^*)}{f(c_E^*)} \right) \right] < 0$ since $c_E^* \leq c_E^o$, so $-\gamma(c_E^*) A(c_E^*) > 0$, implying that $A(c_E^*) < 0$. Consequently, given that $A(c_E^*) < 0$, $-L'(c_E^*) \geq 0$, $(\partial \gamma / \partial \eta) < 0$ and $(\partial \gamma / \partial c_E^*) > 0$, then $(\partial c_E^* / \partial \eta) > 0$.

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APPENDIX – FOR ONLINE PUBLICATION

Proof of Lemma 1

The proof is organized around 6 claims.

Claim 1. *If $c_E \geq c_I + (1 - \lambda)(v - c_I) \equiv c^*$, then $\{p_I = c_E, p_E = c_E\}$ is a pure-strategy equilibrium.*

Proof. Take the play $p_E = c_E$ and $p_I = c_E - \varepsilon$, where $\varepsilon \rightarrow 0$. Obviously, E does not want to deviate and I does not want to undercut either. However, it may pay I to deviate all the way and charge v for $1 - \lambda$ units. This deviation is profitable only if $(1 - \lambda)(v - c_I) > c_E - c_I$, that is, only if $c_E < c_I + (1 - \lambda)(v - c_I)$. \square

Claim 2. *If $c_E < c^*$, then no pure strategy equilibrium exists.*

Proof. If $p_E < p_I$, firm E has an incentive to rise its price; if $c_E < p_I < p_E$, firm E has an incentive to undercut I 's price; if $p_I \leq p_E = c_E$, the incumbent prefers to act as a residual monopoly and get $(1 - \lambda)(v - c_I)$, as shown in Claim 1; and if $p_I = p_E = v$ the incumbent has an incentive to undercut E 's price. We have discarded all pure-strategy equilibrium candidates. \square

Claim 3. *If $c_E < c^*$, in any mixed strategy equilibrium $\pi_I^e(p_I) = (1 - \lambda)(v - c_I) = \pi_I^{RM}$*

Proof. It is clear that $\pi_I^e(p_I) \geq \pi_I^{RM}$ as the incumbent can always secure at least his residual monopoly profits by charging v for $1 - \lambda$ units; this also implies that $p_I \geq c^*$. We now discard $\pi_I^e(p_I) > \pi_I^{RM}$ by contradiction. Since $\pi_I^e(p_I) = (1 - \lambda)(p_I - c_I) + \lambda(p_I - c_I)\mathbb{P}(p_I \leq p_E)$, $\pi_I^e(p_I) > \pi_I^{RM}$ implies $[(1 - \lambda)(p_I - c_I) - (1 - \lambda)(v - c_I)] + \lambda(p_I - c_I)\mathbb{P}(p_I \leq p_E) > 0$. The term in the square bracket is non-positive which requires $\mathbb{P}(p_I \leq p_E) > 0$ for all possible p_I , which cannot be if firms are randomizing in equilibrium. \square

Claim 4. *If $c_E < c^*$, in any of the mixed strategy equilibria $\pi_E^e(p_E) = \lambda(c^* - c_E)$*

Proof. That $\pi_E^e(p_E) \geq \lambda(c^* - c_E)$ follows from the fact that E can always secure such payoff by playing $p_E = c^* - \varepsilon \equiv c^*$ without risking being undercut by I . As before, we discard $\pi_E^e(p_E) > \lambda(c^* - c_E)$ by contradiction. Suppose then that $\pi_E^e(p_E) > \lambda(c^* - c_E)$, i.e., $[1 - \mathbb{P}(p_I \leq p_E)]\lambda(p_E - c_E) > \lambda(c^* - c_E)$. This latter implies $p_E > c^*$ for all p_E . But if so, I would price right below p_E (and above c^*), making more than π_I^{RM} , a contradiction. \square

Claim 5. *If $c_E < c^*$, there exists a mixed strategy equilibrium.*

Proof. We prove existence by constructing an equilibrium. Suppose E randomizes over the connected interval $[c^*, v]$ according to the pdf

$$g(x) = \frac{c^* - c_I}{\lambda(x - c_I)^2} = \frac{(1 - \lambda)(v - c_I)}{\lambda(x - c_I)^2} \quad (40)$$

It is easy to see that I 's best reply is to randomize over same support $[c^*, v]$. Using $g(x) = G'(x)$ to evaluate $\pi_I^e(p_I) = [1 - G(p_I)](p_I - c_I) + G(p_I)(1 - \lambda)(p_I - c_I)$, it can be shown that $\pi_I^e(p_I) = \pi_I^{RM}$ for all $p_I \in [c^*, v]$. In addition, $\pi_I^e(p_I < c^*) < c^* - c_I = \pi_I^{RM}$ and $\pi_I^e(p_I > v) = 0$.

Suppose now I randomizes over $[c^*, v]$ according to the pdf

$$h(x) = \begin{cases} \frac{c^* - c_E}{v - c_E} & \text{if } x = v \\ \frac{c^* - c_E}{2(x - c_E)^2} & \text{if } x \in [c^*, v) \end{cases}$$

To finish the construction of the equilibrium, it remains to show that E 's best response to I 's play is indeed to randomize over the interval $[c^*, v]$; for example, according to $g(x)$ above. It is easy to see that $\pi_E^e(p_E) = \lambda(c^* - c_E)$ for all $p_E \in [c^*, v]$. In addition, $\pi_E^e(p_E < c^*) = \lambda(p_E - c_E) < \lambda(c^* - c_E)$ and $\pi_E^e(p_E > v) = 0$. \square

Claim 6. *If $c_E < c^*$, in any mixed strategy equilibrium $\pi_B^e < \lambda(v - c_I)$.*

Proof. Consider first $c_E \leq c_I$. The maximum overall surplus available is $W^* = v - \lambda c_E - (1 - \lambda)c_I$, while the overall surplus in equilibrium is equal to $W^e = \pi_I^e + \pi_E^e + \pi_B^e = (1 - \lambda)(v - c_I) + \lambda(c^* - c_E) + \pi_B^e$. But we know $W^e \leq W^*$ (because of the mixing), which implies $\pi_B^e \leq \lambda(v - c^*) = \lambda^2(v - c_I) < \lambda(v - c_I)$.

Consider now $c_E \in (c_I, c^*)$. Now $W^* = v - c_I$, but since $W^e \leq W^*$, we have again $\pi_B^e \leq \lambda(v - c_I) - \lambda(v - c^*) < \lambda(v - c_I)$. \square

Proof of Lemma 4

The first order condition of problem (13) with respect to p_E yields $\beta[\pi_B - v + T(1)] = (1 - \beta)\pi_E$. Rearranging this latter expression and using the definition $\Delta W_{EB} = W_{EB} - v + T(1)$ leads to $\pi_B = v - T(1) + (1 - \beta)\Delta W_{EB}$ and $\pi_E = \beta\Delta W_{EB}$. Since $W_{EB} = v - c_E q_E - T(1 - q_E)$ does not depend on p_E , we can replace the above expressions for π_B and π_E in EB coalition's objective function and rewrite its problem as

$$\max_{q_E \leq \lambda} \Pi_{EB} = \beta^\beta (1 - \beta)^{1 - \beta} [W_{EB}(q_E; c_E) - v + T(1)]$$

which is equivalent as to choose $q_E \leq \lambda$ to maximize $W_{EB}(q_E; c_E)$.

Proof of Lemma 5

We know

$$\mathbb{E}W_{EB}^* = W_{EB}^*(0) - \int_0^{\bar{c}_E} [1 - F(c_E)] q_E(c_E) dc_E \quad (41)$$

Replacing the latter expression in $\mathbb{E}\pi_B$ and using $W_{EB}^*(0) = v - T(1 - q_E(0))$ yields

$$\mathbb{E}\pi_B = v - T(1) + (1 - \beta)[T(1) - T(1 - q_E(0))] - \int_0^{\bar{c}_E} (1 - \beta)[1 - F(c_E)] q_E(c_E) dc_E$$

On the other hand, we know

$$\mathbb{E}W_{EB}^* = \int_0^{\bar{c}_E} \{v - c_E q_E(c_E) - T(1 - q_E(c_E))\} f(c_E) dc_E$$

Moreover, using again (41) we can rewrite $\int_0^{\bar{c}_E} T(1 - q_E(c_E)) f(c_E) dc_E$ as function $q_E(c_E)$. And replacing it in $\mathbb{E}\pi_I$, we get

$$\mathbb{E}\pi_I = T(1 - q_E(0)) - c_I - \int_0^{\bar{c}_E} \left\{ c_E - c_I - \frac{[1 - F(c_E)]}{f(c_E)} \right\} q_E(c_E) f(c_E) dc_E$$

The above leads to the following result:

Claim 7. *The optimal allocation rule satisfies $q_E^*(c_E) = 0$ for all $c_E \geq c_E^\circ \in (c_I, \bar{c}_E)$*

Proof. It is easy to see $\mathbb{E}\pi_B$ is maximized by $q_E(c_E) = 0$ for all $c_E \in (0, \bar{c}_E]$. On the other hand, $\mathbb{E}\pi_I$ is maximized by $q_E(c_E) = \lambda$ for all $c_E \leq c_E^\circ$, and $q_E(c_E) = 0$ for all $c_E > c_E^\circ$, where this latter is defined implicitly by $f(c_E^\circ)(c_E^\circ - c_I) - [1 - F(c_E^\circ)] = 0$, and therefore is strictly between c_I and \bar{c}_E (see also equation (8)). Hence neither the incumbent or the consumer will push for $q_E(c_E) > 0$ for $c_E \in (c_E^\circ, \bar{c}_E]$. \square

The previous result allow us to collapse the constraint $T(q_I) \leq vq_I$, for all $q_I \in [0, 1]$ in a restriction over a single constant. Indeed, notice that the only way $q_E(c_E) = 0$ for all $c_E > c_E^\circ$ is for $\pi_E(c_E) = \beta[W_{EB}^*(c_E) - v + T(1)] \leq 0$ for all c_E 's in the relevant region. By monotonicity of $\pi_E(c_E)$, this is equivalent as asking $\pi_E(c_E^\circ) = 0$. But since $W_{EB}^*(c_E^\circ) =$

$v - T(1 - q_E(0)) - \int_0^{c_E^\circ} q_E(c_E) dc_E$, this implies that

$$T(1) - T(1 - q_E(0)) = \int_0^{c_E^\circ} q_E(c_E) dc_E \quad (42)$$

So replacing this latter condition in $\mathbb{E}\pi_B$ and $\mathbb{E}\pi_I$, and using the fact that $q_E(c_E) = 0$ for all $c_E > c_E^\circ$ is optimal, we get:

$$\begin{aligned} \mathbb{E}\pi_B &= v - T(1 - q_E(0)) - \int_0^{c_E^\circ} \frac{\{1 - (1 - \beta)F(c_E)\}}{f(c_E)} q_E(c_E) f(c_E) dc_E \\ \mathbb{E}\pi_I &= T(1 - q_E(0)) - c_I - \int_0^{c_E^\circ} \left\{ c_E - c_I - \frac{[1 - F(c_E)]}{f(c_E)} \right\} q_E(c_E) f(c_E) dc_E \end{aligned} \quad (43)$$

And the constraint $T(1 - q_E) \leq v(1 - q_E)$, for all $q_E \in [0, \lambda]$ can be simplified to $T(1 - q_E(0)) \leq v(1 - q_E(0))$.

Proof of Lemma 6

We know that

$$\begin{aligned} \pi_E &= \beta \Delta W_{EB}^*(c_E) \\ \pi_B &= (1 - \beta) \Delta W_{EB}^*(c_E) + v - T(1) \end{aligned}$$

so a coalition EB with cost c_E will not be active in equilibrium ($q_E^*(c_E) = 0$) if and only if $\Delta W_{EB}(c_E) < 0$, for all $q_E \in [0, \lambda]$.

Part 1. Now, suppose $T(1 - q_E)$ implements (24), but $(1/\lambda)[T(1) - T(1 - \lambda)] > \tilde{c}_E$, and consider $c_E = \tilde{c}_E + \varepsilon$. Notice that if $q_E(c_E) = \lambda$, then

$$\Delta W_{EB}(c_E) = T(1) - (\tilde{c}_E + \varepsilon)\lambda - T(1 - \lambda) > 0 \iff \frac{T(1) - T(1 - \lambda)}{\lambda} - \tilde{c}_E > \varepsilon$$

which is true for a sufficiently small ε . Hence an entrant with marginal cost $c_E \in [\tilde{c}_E, \tilde{c}_E + \varepsilon]$ will also sell an strictly positive amount in equilibrium, a contradiction. The argument is analogous for $(1/\lambda)[T(1) - T(1 - \lambda)] < \tilde{c}_E$.

Part 2. Take any $T(1 - q_E)$ such that $e(q_E)$ is non-decreasing in q_E for all $q_E \in [0, \lambda]$ and $e(\lambda) = \tilde{c}_E$, and rewrite the surplus as $\Delta W_{EB} = q_E[e(q_E) - c_E]$. Therefore, the EB coalition

problem is

$$\max_{q_E \leq \lambda} \Delta W_{EB} = q_E [e(q_E) - c_E] \quad (44)$$

Noticing that $e(q_E)$ is maximized at $q_E = \lambda$ we get that if $c_E \leq \tilde{c}_E = e(\lambda)$, ΔW_{EB} is maximized at λ . That surplus moreover is non-negative $\Delta W_{EB}^* = \lambda[\tilde{c}_E - c_E] \geq 0$, so all types $c_E \leq \tilde{c}_E$ will choose $q_E(c_E) = \lambda$. On the other hand, if $c_E > \tilde{c}_E$, then $e(q_E) \leq \tilde{c}_E < c_E$ for all $q_E \in [0, \lambda]$, so ΔW_{EB} is strictly decreasing in q_E , implying that all types $c_E > \tilde{c}_E$ will chose $q_E^*(c_E) = 0$. Hence, this price schedule implements (24).

Proof of Lemma 7

To simplify notation, let $J(c_E) = c_E - c_I - \frac{1-F(c_E)}{f(c_E)}$ and $H(c_E) \equiv \frac{1-(1-\beta)F(c_E)}{f(c_E)}$, so

$$\begin{aligned} \mathbb{E}\pi_B &= v - T(1 - q_E(0)) - \int_0^{c_E^\circ} H(c_E) q_E(c_E) f(c_E) dc_E \\ \mathbb{E}\pi_I &= T(1 - q_E(0)) - c_I - \int_0^{c_E^\circ} J(c_E) q_E(c_E) f(c_E) dc_E \end{aligned} \quad (45)$$

Then, $J(c_E) \leq 0$ and increasing in $[0, c_E^\circ]$, and $H(c_E) \geq 0$. Moreover $I(c_E) \equiv J(c_E) + H(c_E) = c_E - c_I + \frac{\beta F(c_E)}{f(c_E)} > 0$ for $c_E > \tilde{c}_E$, and increasing in c_E .

Part 1: $\eta = 1$

Claim 8. *Suppose $\eta = 1$. Fix \bar{W} and some $\bar{\pi}_I \in (\bar{\pi}_I^*(\eta = 1), \bar{\pi}_I^{***}]$. Then, if an optimal schedule exists, it involves $T^*(1 - q_E(0)) = v(1 - q_E(0))$.*

Proof. Suppose the problem has a solution, and that the optimal schedule implements $q_E^*(c_E)$ and has $T^* \equiv T(1 - q_E(0)) < v(1 - q_E(0))$. Denote $\mathbb{E}\pi_B^*$ and $\mathbb{E}\pi_I^*$ the payoffs prescribe by such optimal schedule. Assume first that $q_E^*(c_E) = 0$ for all $c_E \geq \tilde{c}_E$, this would imply

$$\mathbb{E}\pi_B^* \geq (v - T^* - \lambda \tilde{c}_E) + \lambda(1 - \beta)F(\tilde{c}_E)[\tilde{c}_E - \mathbb{E}(c_E | c_E \leq \tilde{c}_E)]$$

But it is easy to prove that

$$(v - T^* - \lambda \tilde{c}_E) + \lambda(1 - \beta)F(\tilde{c}_E)[\tilde{c}_E - \mathbb{E}(c_E | c_E \leq \tilde{c}_E)] \geq \bar{W} - \bar{\pi}_I^*(\eta = 1) > \bar{W} - \bar{\pi}_I = \bar{\pi}_B$$

which implies that $\mathbb{E}\pi_B^* > \bar{\pi}_B$. Therefore, since $\mathbb{E}\pi_I$ is increasing in T^* there is room to improve I 's payoff while still satisfying all constraints, contradicting the claim that $\{q_E^*(c_E), T^*\}$ was

indeed optimal.

On the other hand, assume instead that $q_E^*(c_E) > 0$ for some measurable interval $[\tilde{c}_E, \tilde{c}_E + \xi]$. Then, $\exists M \in (\tilde{c}_E, c_E^\circ)$ such that $0 < \Delta < v(1 - q_E(0)) - T^*$, where

$$\Delta \equiv \int_M^{c_E^\circ} H(c_E) q_E^*(c_E) f(c_E) dc_E \quad (46)$$

Consider then, the alternative policy with allocation rule

$$q_E^{**}(c_E) = \begin{cases} q_E^*(c_E) & \text{if } c_E \leq M \\ 0 & \text{otherwise} \end{cases}$$

and payment $T^{**} = T^* + \Delta$. Such policy clearly satisfies the sorting and ex-post participation restrictions, while leaving the buyer indifferent in terms of ex-ante surplus (so B 's ex-ante participation is also met). However, the change in I 's profits produced by the change in scheme is

$$\mathbb{E}\pi_I^{**} - \mathbb{E}\pi_I^* = T^{**} - T^* + \int_M^{c_E^\circ} J(c_E) q_E^*(c_E) f(c_E) dc_E$$

But using $T^{**} = T^* + \Delta$ this can be rewritten as

$$\mathbb{E}\pi_I^{**} - \mathbb{E}\pi_I^* = \int_M^{c_E^\circ} I(c_E) q_E^*(c_E) f(c_E) dc_E \quad (47)$$

which is strictly positive since $I(c_E) > 0$ for all $c_E \in [M, c_E^\circ]$, as $M > \tilde{c}_E$, contradicting the claim that the original policy was indeed optimal. \square

Having proved that $T^*(1 - q_E(0)) = v(1 - q_E(0))$, we now characterize $q_E^*(c_E)$. The problem faced by the incumbent in this setting is

$$\max_{q_E(c_E)} \mathbb{E}\pi_I = v(1 - q_E(0)) - c_I - \int_0^{c_E^\circ} J(c_E) q_E(c_E) f(c_E) dc_E \quad (48)$$

s.t.

$$v q_E(0) - \int_0^{c_E^\circ} H(c_E) q_E(c_E) f(c_E) dc_E \geq \bar{\pi}_B \quad (49)$$

and $q'(c_E) \leq 0$. Now, in this particular case, the buyer's participation constraint may or may not be binding. If it is not, then the solution is straightforward as point-wise optimization

indicates that the optimal schedule is given by

$$q_E^*(c_E) = \begin{cases} \lambda & \text{if } c_E \leq c_E^\circ \\ 0 & \text{otherwise} \end{cases}$$

The more difficult case arises when the constraint is binding. The proof is by contradiction. Suppose that the optimal schedule $q_E^*(c_E)$ is an arbitrary weakly decreasing function of c_E that does not take the following two-step shape

$$q_E(c_E) = \begin{cases} \lambda & \text{if } c_E \leq \omega \\ 0 & \text{otherwise} \end{cases}$$

for some $\omega \in [0, c_E^\circ]$. Define then the sets

$$\begin{aligned} \mathfrak{s}_1 &= \{c_E \in [0, c_E^\circ] : H'(c_E) \geq 0 \text{ and } q(c_E) \in (0, \lambda)\} \\ \mathfrak{s}_2 &= \{c_E \in [0, c_E^\circ] : H'(c_E) < 0 \text{ and } q(c_E) \in (0, \lambda)\} \end{aligned} \tag{50}$$

Obviously, at least one of the set must be non-empty. Suppose \mathfrak{s}_1 is not empty and take then any connected subinterval $[n, N] \subseteq \mathfrak{s}_1$, fix $\delta > 0$ such that $H(x) \leq H(y)$ for all $x \in [n, n + \delta]$ and $y \in [N - \delta, N]$, and a sufficiently small $\varepsilon > 0$. And consider the alternative schedule

$$q_E^{**}(c_E) = \begin{cases} q_E^*(c_E) + \xi(\varepsilon) & \text{if } c_E \in [n, n + \delta] \\ q_E^*(c_E) - \varepsilon & \text{if } c_E \in [N - \delta, N] \\ q_E^*(c_E) & \text{otherwise} \end{cases} \tag{51}$$

where $\xi(\varepsilon)$ is chosen such that $\mathbb{E}\pi_B^{**} = \mathbb{E}\pi_B^*$, in other words

$$\int_n^{n+\delta} \xi(\varepsilon) H(c_E) f(c_E) dc_E = \int_{N-\delta}^N \varepsilon H(c_E) f(c_E) dc_E$$

This alternative is feasible: it satisfies B 's participation constraint, $q_E'(c_E) \leq 0$, and both $q_E(c_E) - \varepsilon$ and $q_E(c_E) + \xi(\varepsilon)$ are between 0 and λ for a sufficiently small ε .

Then, the change in the objective function would be

$$\mathbb{E}\pi_I^{**} - \mathbb{E}\pi_I^* = - \int_n^{n+\delta} \xi(\varepsilon) J(c_E) f(c_E) dc_E + \int_{N-\delta}^N \varepsilon J(c_E) f(c_E) dc_E$$

But using the definition of $\xi(\varepsilon)$, this can be rewritten as:

$$= \left[\int_{N-\delta}^N \varepsilon H(c_E) f(c_E) dc_E \right] \left[-\frac{\int_n^{n+\delta} J(c_E) f(c_E) dc_E}{\int_n^{n+\delta} H(c_E) f(c_E) dc_E} + \frac{\int_{N-\delta}^N J(c_E) f(c_E) dc_E}{\int_{N-\delta}^N H(c_E) f(c_E) dc_E} \right]$$

Given that the first term in bracket is strictly positive, the sign of the expression comes from the second term. Taking then the limit when $\delta \rightarrow 0$, this becomes $-\frac{J(n)}{H(n)} + \frac{J(N)}{H(N)}$. But,

$$\left(\frac{J(x)}{H(x)} \right)' = \frac{J'(x)}{H(x)} - \frac{J(x)H'(x)}{H(x)^2} > 0$$

Since $H'(x) \geq 0$, $J'(x) > 0$ and $J(x) \leq 0$, and therefore

$$-\frac{J(n)}{H(n)} + \frac{J(N)}{H(N)} > 0 \quad (52)$$

which implies that $\mathbb{E}\pi_I^{**} - \mathbb{E}\pi_I^* > 0$, so $q_E^*(c_E)$ cannot be the optimal schedule.

On the contrary, suppose instead that \mathfrak{s}_1 is empty and therefore $\mathfrak{s}_2 \neq \emptyset$. Again, take any connected subinterval $[n, N] \subseteq \mathfrak{s}_2$, fix $\delta > 0$ such that $H(x) > H(y)$ for all $x \in [n, n + \delta]$ and $y \in [N - \delta, N]$, and a sufficiently small $\varepsilon > 0$. And consider the alternative schedule

$$q_E^{**} = \begin{cases} q_E^*(c_E) + \xi(\varepsilon) & \text{if } c_E \in [n, n + \delta] \\ q_E^*(c_E) - \varepsilon & \text{if } c_E \in [N - \delta, N] \\ q_E^*(c_E) & \text{otherwise} \end{cases} \quad (53)$$

where $\xi(\varepsilon)$ satisfies

$$\int_n^{n+\delta} \xi(\varepsilon) H(c_E) f(c_E) dc_E = \int_{N-\delta}^N \varepsilon H(c_E) f(c_E) dc_E$$

Proceeding as before, the change in the objective function $\mathbb{E}\pi_I^{**} - \mathbb{E}\pi_I^*$ is again

$$= \left[\int_{N-\delta}^N \varepsilon H(c_E) f(c_E) dc_E \right] \left[-\frac{\int_n^{n+\delta} J(c_E) f(c_E) dc_E}{\int_n^{n+\delta} H(c_E) f(c_E) dc_E} + \frac{\int_{N-\delta}^N J(c_E) f(c_E) dc_E}{\int_{N-\delta}^N H(c_E) f(c_E) dc_E} \right]$$

Taking then the limit of the second term when $\delta \rightarrow 0$, this becomes $-\frac{J(n)}{H(n)} + \frac{J(N)}{H(N)}$. But we know that

$$I(x) = H(x) + J(x) = H(x) \left[1 + \frac{J(x)}{H(x)} \right]$$

is increasing in x . Consequently, given that $H(x)$ is decreasing by assumption, $J(x)/H(x)$ must be increasing, and therefore

$$-\frac{J(n)}{H(n)} + \frac{J(N)}{H(N)} > 0 \quad (54)$$

which in turn implies that $\mathbb{E}\pi_I^{**} - \mathbb{E}\pi_I^* > 0$, so $q_E^*(c_E)$ cannot be the optimal schedule.

Hence, when $\eta = 1$ the optimal schedule must induce an allocation

$$q_E^*(c_E) = \begin{cases} \lambda & \text{if } c_E \leq \hat{c}_E(\bar{\pi}_I) \\ 0 & \text{otherwise} \end{cases} \quad (55)$$

implying that $\mathbb{E}\pi_B$ has again a threshold shape:

$$\mathbb{E}\pi_B(c_E; \hat{c}_E) = \lambda(v - \hat{c}_E) + \lambda(1 - \beta)F(\hat{c}_E)[\hat{c}_E - \mathbb{E}(c_E \mid c_E \leq \hat{c}_E)]$$

When B 's participation is not binding, then $\hat{c}_E = c_E^\circ$. On the other hand, when the constraint is binding, then $\hat{c}_E = \delta < c_E^\circ$, where δ is the cut-off that makes consumer participation hold:

$$\lambda(v - \delta) + \lambda(1 - \beta)F(\delta)[\delta - \mathbb{E}(c_E \mid c_E \leq \delta)] = \bar{W} - \bar{\pi}_I$$

Hence, the optimal schedule with $\eta = 1$ is (55), where $\hat{c}_E(\bar{\pi}_I) = \min\{c_E^\circ, \delta\}$.

Part 2: $\eta = 0$

The case when $\eta = 0$ is almost completely analogous. The only difference is that in this case the incumbent participation constraint will always be binding, and therefore the optimal schedule must induce an allocation

$$q_E^*(c_E) = \begin{cases} \lambda & \text{if } c_E \leq \check{c}_E \\ 0 & \text{otherwise} \end{cases} \quad (56)$$

where the threshold \check{c}_E ensures I 's participation:

$$\mathbb{E}\pi_I(c_E; \check{c}_E) = (1 - \lambda)(v - c_I) + \lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)] = \bar{\pi}_I$$

Proof of Lemma 8

Fix \bar{W} , and a $\bar{\pi}_I \in (\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***})$. Suppose $\check{c}_E(\bar{\pi}_I) \leq \hat{c}_E(\bar{\pi}_I)$. Then, any schedule $T(\cdot)$ with $T(1 - \lambda) = v(1 - \lambda)$ that implements an allocation $q_E(c_E) = \lambda$ for $c_E \leq x \in [\check{c}_E, \hat{c}_E]$, and 0

otherwise, satisfies both participation constraints and the sorting condition $q'(c_E) \leq 0$ for all c_E . Hence a solution exists.

For the converse, let

$$\mathbb{E}\pi_I^M \equiv (1 - \lambda)(v - c_I) + \lambda(\hat{c}_E - c_I)[1 - F(\hat{c}_E)]$$

Notice that $\mathbb{E}\pi_I^M$ is the maximum payoff I could obtain while satisfying B 's participation constraint. Now, suppose the admissible set $\tilde{\Theta}$ is non-empty for a given \bar{W} and some $\bar{\pi}_I \in (\bar{\pi}_I^*(\eta), \bar{\pi}_I^{***})$, and let $q_E(c_E) : [0, \bar{c}_E] \rightarrow [0, \lambda] \in \tilde{\Theta}$. Denote by $\mathbb{E}\pi_I^{\tilde{\Theta}}$ the payoff the incumbent gets under this latter allocation rule. It must be true that $\mathbb{E}\pi_I^M \geq \mathbb{E}\pi_I^{\tilde{\Theta}} \geq \bar{\pi}_I$, so $\mathbb{E}\pi_I^M \geq \bar{\pi}_I$. Hence $\mathbb{E}\pi_I^M + \bar{\pi}_B \geq \bar{\pi}_I + \bar{\pi}_B = \bar{W}$.

Now, if $\hat{c}_E = c_E^\circ$ the results follows immediately since $\check{c}_E \leq c_E^\circ$ for all $\bar{\pi}_I \leq \bar{\pi}_I^{***}$. We focus then on the case $\hat{c}_E = \delta$. We then have:

$$\begin{aligned} \check{c}_E : (1 - \lambda)(v - c_I) + \lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)] &= \bar{\pi}_I \\ \hat{c}_E : \lambda(v - \hat{c}_E) + \lambda(1 - \beta)F(\hat{c}_E)[\hat{c}_E - \mathbb{E}(c_E \mid c_E \leq \hat{c}_E)] &= \bar{W} - \bar{\pi}_I \end{aligned} \tag{57}$$

Consequently

$$\bar{W} - \lambda(v - \hat{c}_E) - \lambda(1 - \beta)F(\hat{c}_E)[\hat{c}_E - \mathbb{E}(c_E \mid c_E \leq \hat{c}_E)] = (1 - \lambda)(v - c_I) + \lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)]$$

Using $\mathbb{E}\pi_I^M = (1 - \lambda)(v - c_I) + \lambda(\hat{c}_E - c_I)[1 - F(\hat{c}_E)]$ and $\lambda(v - \hat{c}_E) + \lambda(1 - \beta)F(\hat{c}_E)[\hat{c}_E - \mathbb{E}(c_E \mid c_E \leq \hat{c}_E)] = \bar{\pi}_B$ yields

$$\bar{W} - [\mathbb{E}\pi_I^M + \bar{\pi}_B] = \lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)] - \lambda(\hat{c}_E - c_I)[1 - F(\hat{c}_E)] \tag{58}$$

But $\bar{W} \leq \mathbb{E}\pi_I^M + \bar{\pi}_B$, so $\lambda(\check{c}_E - c_I)[1 - F(\check{c}_E)] \leq \lambda(\hat{c}_E - c_I)[1 - F(\hat{c}_E)]$. Finally, since $\lambda(r - c_I)[1 - F(r)]$ is increasing in r for all $r \leq c_E^\circ$, we can conclude $\hat{c}_E \geq \check{c}_E$.

Proof of Lemma 9

We already showed that without upfront payments $T(0, q_E) = 0$ for all possible q_E . Moreover since rent extraction comes from marginal penalties $T_{q_E}(q_I, q_E)$, and not from marginal prices $T_{q_I}(q_I, q_E)$, the IB coalition does not want to induce a bilateral inefficiency $P(q_I + q_E) - T_{q_I}(q_I, q_E) \geq 0$, for all $q_I + q_E \leq D(c_I)$ and a fixed q_E (this is analogous to $T'(q_I) \leq v$ in the

unit-demand model). Using both, we have that a necessary and sufficient condition for B 's ex-post participation to be met is

$$T(q_I, q_E) = T(0, q_E) + \int_0^{q_I} T_{q_I}(s, q_E) ds \leq \int_0^{q_I} P(s + q_E) ds = S(q_I + q_E) - S(q_E) \quad (59)$$

This is intuitive: the total amount charged by I to B including penalties, must be less or equal to I 's contribution to B 's total surplus.

Now, using the same backward induction and mechanism design approach outlined in section 3, and analogous steps as the ones used in the proof of Lemma 6, we have the following result:

Claim 9. *The problem faced by the IB coalition with a downward sloping demand can be rewritten as choosing an allocation rule $[q_I(c_E), q_E(c_E)] : [0, \bar{c}_E] \rightarrow \mathbb{R}_+ \times [0, \lambda]$ and a transfer $T(q_I(0), q_E(0))$ to solve:*

$$\max_{\substack{q_I(c_E) \\ q_E(c_E) \leq \lambda \\ T(q_I(0), q_E(0))}} \Pi_{IB} = (\mathbb{E}\pi_I - \bar{\pi})^\eta (\mathbb{E}\pi_B - \bar{\pi}_B)^{1-\eta} \quad (60)$$

s.t.

$$\begin{aligned} \mathbb{E}\pi_B &= S(q_I(0) + q_E(0)) - T(q_I(0), q_E(0)) - \int_0^{\bar{c}_E} \left\{ \frac{1 - (1 - \beta)F(c_E)}{f(c_E)} \right\} q_E(c_E) f(c_E) dc_E \geq \bar{\pi}_B \\ \mathbb{E}\pi_I &= -S(q_I(0) + q_E(0)) + T(q_I(0), q_E(0)) \\ &\quad + \int_0^{\bar{c}_E} \left\{ S(q_I(c_E) + q_E(c_E)) - c_E q_E(c_E) - c_I q_I(c_E) + \left[\frac{1 - F(c_E)}{f(c_E)} \right] q_E(c_E) \right\} f(c_E) dc_E \geq \bar{\pi}_I \\ q'_E(c_E) &\leq 0 \text{ for all } c_E \in [0, \bar{c}_E] \\ T(q_I(0), q_E(0)) &\leq S(q_I(0) + q_E(0)) - S(q_E(0)) \end{aligned}$$

Assuming $T(q_I(0), q_E(0))$ is interior, the optimality conditions are given by

$$\frac{\partial}{\partial T} = \Pi_{IB} \left[\frac{\eta}{\mathbb{E}\pi_I - \bar{\pi}_I} - \frac{(1 - \eta)}{\mathbb{E}\pi_B - \bar{\pi}_B} \right] = 0 \quad (61)$$

and the system of Euler-Lagrange equations

$$\begin{aligned} &\frac{\eta \Pi_{IB}}{\mathbb{E}\pi_I - \bar{\pi}_I} \{P(q_I(c_E) + q_E(c_E)) - c_I\} \\ \Pi_{IB} &\left(\frac{\eta}{(\mathbb{E}\pi_I - \bar{\pi}_I)} + \frac{(1 - \eta)}{(\mathbb{E}\pi_B - \bar{\pi}_B)} \right) \left((1 - \gamma)(P(q_I(c_E) + q_E(c_E)) - c_E) + 2 \frac{[1 - F(c_E)]}{f(c_E)} \left(\frac{1}{2} - \gamma \right) - \frac{\gamma \beta F(c_E)}{f(c_E)} \right) \end{aligned}$$

Notice however that the first Euler condition implies $P(q_I(c_E) + q_E(c_E)) = c_I$ for all $c_E \in [0, \bar{c}_E]$, so we can collapse the two of them and conclude that $q_I(c_E) = D(c_I) - q_E(c_E)$. Moreover (61) implies $\gamma = 1/2$, so the optimal allocation would then be given by

$$[q_I^*(c_E), q_E^*(c_E)] = \begin{cases} [D(c_I), \lambda] & \text{if } c_E \in [0, \tilde{c}_E] \\ [D(c_I), 0] & \text{otherwise} \end{cases} \quad (62)$$

where \tilde{c}_E satisfies:

$$f(\tilde{c}_E)(\tilde{c}_E - c_I) + \beta F(\tilde{c}_E) = 0$$

This in turn implies that:

$$\mathbb{E}\pi_I = T(D(c_I) - \lambda, \lambda) - c_I[D(c_I) - \lambda] + \lambda[1 - F(\tilde{c}_E)](\tilde{c}_E - c_I) \equiv \mathbb{E}\pi_I(c_E; \tilde{c}_E)$$

$$\mathbb{E}\pi_B = S(D(c_I)) - T(D(c_I) - \lambda, \lambda) - \lambda\tilde{c}_E + \lambda(1 - \beta)F(\tilde{c}_E)[\tilde{c}_E - \mathbb{E}(c_E \mid c_E \leq \tilde{c}_E)] \equiv \mathbb{E}\pi_B(c_E; \tilde{c}_E)$$

Finally, $T(q_I(0), q_E(0)) = T^*(D(c_I) - \lambda, \lambda)$ is set such that:

$$\mathbb{E}\pi_I(c_E; \tilde{c}_E) = \bar{\pi}_I + \eta \Delta W_{IB}^*(c_E; \tilde{c}_E) \quad (63)$$

$$\mathbb{E}\pi_B(c_E; \tilde{c}_E) = \bar{\pi}_B + (1 - \eta) \Delta W_{IB}^*(c_E; \tilde{c}_E) \quad (64)$$

with $\Delta W_{IB}^*(c_E; \tilde{c}_E) \equiv \mathbb{E}\pi_I(c_E; \tilde{c}_E) + \mathbb{E}\pi_B(c_E; \tilde{c}_E) - \bar{W}$. However, for this to be a solution we still have to check whether $T^*(D(c_I) - \lambda, \lambda)$ is indeed interior. From (63):

$$T^*(D(c_I) - \lambda, \lambda) = \bar{\pi}_I + c_I[D(c_I) - \lambda] - \lambda[1 - F(\tilde{c}_E)](\tilde{c}_E - c_I) + \eta \Delta W_{IB}^*(c_E; \tilde{c}_E)$$

So $T^*(D(c_I) - \lambda, \lambda) \leq S(D(c_I)) - S(\lambda)$ if and only if $\bar{\pi}_I \leq \bar{\pi}_I^{D^*}(\eta)$, where

$$\bar{\pi}_I^{D^*}(\eta) \equiv S(D(c_I)) - S(\lambda) - c_I[D(c_I) - \lambda] + \lambda(\tilde{c}_E - c_I)[1 - F(\tilde{c}_E)] - \eta \Delta W_{IB}^*(c_E; \tilde{c}_E) \quad (65)$$

Notice that $\bar{\pi}_I^{D^*}(\eta)$ is certainly decreasing in η . We still have to characterize the solution then when $\bar{\pi}_I > \bar{\pi}_I^{D^*}(\eta)$, obviously then $T(q_I(0), q_E(0)) = S(q_I(0) + q_E(0)) - S(q_E(0))$. For simplicity, we will derive the solution only for $\eta \in (0, 1)$. The optimality conditions would then be given

by the system of Euler-Lagrange equations

$$\frac{\eta \Pi_{IB}}{\mathbb{E}\pi_I - \bar{\pi}_I} \{P(q_I(c_E) + q_E(c_E)) - c_I\}$$

$$\Pi_{IB} \left(\frac{\eta}{(\mathbb{E}\pi_I - \bar{\pi}_I)} + \frac{(1-\eta)}{(\mathbb{E}\pi_B - \bar{\pi}_B)} \right) \left((1-\gamma)(P(q_I(c_E) + q_E(c_E)) - c_E) + \frac{2[1-F(c_E)]}{f(c_E)} \left(\frac{1}{2} - \gamma \right) - \frac{\gamma \beta F(c_E)}{f(c_E)} \right)$$

This means that in this case it is again true that $P(q_I(c_E) + q_E(c_E)) = c_I$ for all $c_E \in [0, \bar{c}_E]$, so $q_I(c_E) = D(c_I) - q_E(c_E)$ and the conditions collapse into

$$\Pi_{IB} \left(\frac{\eta}{(\mathbb{E}\pi_I - \bar{\pi}_I)} + \frac{(1-\eta)}{(\mathbb{E}\pi_B - \bar{\pi}_B)} \right) \left((1-\gamma)(c_I - c_E) + 2 \frac{[1-F(c_E)]}{f(c_E)} \left(\frac{1}{2} - \gamma \right) - \frac{\gamma \beta F(c_E)}{f(c_E)} \right)$$

$$= \theta L(c_E) \geq 0$$
(66)

Which is the exact same expression as when we were dealing with the inelastic unitary demand. Hence, by a similar argument as lemma 8 and proposition 4, and using the fact that $q_I(c_E) = D(c_I) - q_E(c_E)$ we have that the solution in this case is given by:

$$(q_I^*(c_E), q_E^*(c_E)) = \begin{cases} (D(c_I), \lambda) & \text{if } c_E \in [0, z(\eta, \bar{\pi}_I)] \\ (D(c_I), 0) & \text{otherwise} \end{cases} \quad (67)$$

where we know that $z(\eta, \bar{\pi}_I) > \tilde{c}_E$ and is increasing in both η and $\bar{\pi}_I$ (see Proposition 6). Hence, combining all of the above, we finish the proof.