The Effectiveness of Leniency Programs when Firms choose the Degree of Collusion∗

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Abstract

An antitrust authority deters collusion using fines and a leniency program. It grants leniency before and after an investigation has started. It chooses the probability of an investigation. Firms pick the degree of collusion: The more they collude, the higher are profits, but so is the probability of detection. Firms thus trade-off higher profits against higher expected fines. If firms are sufficiently patient, leniency is ineffective; it may even increase collusion. Increasing the probability of an investigation at low levels does not increase deterrence. Increasing the probability of an investigation at high levels reduces collusion, yet never completely.

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1 Introduction

A corporate leniency program reduces the sanctions for self-reporting cartel members. In 1993 the US Department of Justice revised its Corporate Leniency Program, committing itself to the lenient prosecution of the first confessor. It allows amnesty to be awarded even when an investigation has already been started. This revision is considered as the most significant policy innovation in antitrust. It substantially increased the number of detected and convicted cartels. The apparent success led the EU to adopt its own leniency program in 1996. Other countries followed suit.\footnote{See, e.g., Harrington and Chang (2009), Spagnolo (2008), or Harrington (2017).}

The literature on leniency typically assumes that firms either fully collude or they do not collude at all: they set, for example, either the monopoly or the competitive price. In this paper we give up this binary choice. Our firms choose the degree of collusion—a continuous variable. They may, e.g., pick the fraction of markets on which they collude; or they may set any price between the competitive and the monopoly one.\footnote{For example, in the Swiss construction bid-rigging cartels, firms colluded on some projects and declared others open for competition. See www.weko.admin.ch/aktuell/00162/index.html?lang=fr....} Firms’ profits are increasing in the degree of collusion, yet so is the probability of detection: the more markets firms collude on, the higher is the probability that the antitrust authority (AA) finds out the illegal behavior once it opened an investigation. Firms thus trade-off higher profits against higher expected fines.

Legislation specifies the fine and full leniency for the first reporting firm. For most of the paper we focus on leniency granted before and after an investigation has started. The fine is proportional to the degree of collusion. The AA picks the probability with which it starts an investigation. For each probability of investigation we determine the corresponding degree of collusion.

We consider two collusive strategies which differ in firms’ behavior in case of an investigation. Either firms do not reveal the illegal behavior once an investigation started; they make the collusive profits, yet both firms pay the...
fine when detected. Or firms exploit leniency: if the AA opens an investiga-
tion, they simultaneously reveal and stop collusion during the investigation;
both firms then have a 50% chance of receiving leniency. Firms continue
collusion after the agreed upon reporting.

First we show that if firms are sufficiently patient, leniency does not
increase deterrence. Either firms collude and do not reveal in case of an
investigation: then the incentive to report the cartel and get leniency is too
small for patient firms. Or firms collude and reveal in case of an investiga-
tion: then firms actually collude on all markets.

Next we look at the degree of collusion as a function of the probability
of an investigation. If an investigation is unlikely, firms collude and reveal
in case of an investigation. Under this strategy firms collude on all markets.
Increasing the probability of an investigation does not lower the degree of
collusion. By contrast, if an investigation is sufficiently likely, firms collude
and do not report in case of an investigation. Here the degree of collusion
decreases with the probability of an investigation. Nevertheless, firms always
choose a positive degree of collusion. The fine is proportional to the degree of
collusion. Slightly colluding has no first-order effect on the fine, yet it raises
profits.

Leniency thus produces mixed results in our set-up. With patient firms it
has no bite and is, therefore, redundant. Moreover, it opens the door for the
strategy collude and reveal in case of an investigation which, in turn, goes
together with full collusion. Firms actually play this strategy for low prob-
abilities of investigation. Thus, in this case leniency induces full collusion.
More statements about the effectiveness of leniency are not possible without
further specifying the model. Yet, our set-up generates another message. As
long as the fine is proportional to the degree of collusion, firms will always
collude: a small increase in the degree of collusion has no first-order effect
on the expected fine but a positive first-order effect on profits.

Two of our results do not hold if firms can only choose between no and full
rather than from a continuum of degrees of collusion: First, that leniency has
no bite with patient firms and second, that there is always some collusion.
The assessment of the efficacy of leniency thus depends on the degree of
collusion firms can choose from.

Our paper builds on the analysis of leniency programs by Motta and Polo (2003), Spagnolo (2004), Aubert et al. (2006), Harrington (2008), and Chen and Rey (2013). This literature analyzes the effects of leniency on the frequency of collusion and derives optimal fine structures.

Our basic set-up is related to Motta and Polo (2003). Besides in the degree of collusion (binary versus continuous), our framework differs from theirs in another respect: In Motta and Polo (2003) the AA chooses the probability that it opens an investigation and the probability that it successfully concludes the investigation. In our set-up the resources the AA puts into an investigation are exogenously given. Firms determine by their choice of the degree of collusion the probability that the investigation leads to a conviction. The AA, in turn, chooses the probability of an investigation. Moreover, following Spagnolo (2004), we take the optimal deviation from collusion as “undercut and report” (using the Bertrand game terminology); the deviator thus gets the entire profits and avoids the fine. By contrast, Motta and Polo (2003) take the deviation from collusion as “compete and report” so that the deviator makes zero profits and avoids the fine.

A few papers look at variable degrees of collusion, their focus is, however, not on leniency. In Block et al. (1981) the probability of detection is an increasing function of the price; in Harrington (2004, 2005) it increases with the price change. In Bos et al. (2018) the probability of detection is a non-decreasing function of price. It is positive even when firms charge the competitive price, and so is the fine. This means firms face a positive expected fine at the competitive price. Therefore, cartels with low overcharges do not form in the first place.

Further theoretical research includes Harrington and Chang (2009), Harrington (2013), Sauvagnat (2015), and Harrington and Chang (2015); empirical and experimental research includes Bigoni et al. (2012, 2015), Brenner (2009), and Miller (2009). For a survey, see Harrington (2017).

"Undercut and report" has been adopted by the literature as the relevant deviation from collusion in the presence of leniency; see, e.g., Aubert et al. (2006) or Chen and Rey (2013).

By contrast, Houba et al. (2010) assume like us that the probability of detection and the fine is zero at the competitive price.
The rest of this paper is organized as follows: The next section describes the model. In Section 3 the AA grants leniency before and after an investigation has started. In Section 4 we briefly look at the case where only preinvestigation leniency is available. Section 5 concludes.

2 Model

Consider two potentially colluding firms. The two firms face a continuum of identical markets with mass 1. The firms can collude on each market. More specifically, they choose the fraction $\nu \in [0, 1]$ of markets on which they collude; $\nu$ thus measures the degree of collusion.$^6$

Next let us describe how firms collude. If the firms do not collude on a market, they compete and make profit 0. If they collude on a market by setting the monopoly price, they make profit $\pi_M$ each. Thus, if they collude on $\nu$ markets, each firm makes profit $\nu\pi_M$. $^7$

Firms support the collusive behavior with grim-trigger strategies. If a firm deviates from collusion, Nash punishment, i.e., competition, starts and continues forever; each firm makes the static Nash profit 0. If a firm deviates while the other firm colludes, the deviating firm reaps the entire monopoly profit $2\nu\pi_M$; the colluding firm’s profit is 0.$^8$

The cartel is stable in the absence of the AA. If $\delta$ denotes the firms’ common discount rate, this means that $\pi_M/(1-\delta) > 2\pi_M$, or $\delta > 1/2$: getting $\pi_M$ forever is better than getting $2\pi_M$ in the first round and from then on nothing. Thus, we assume $\delta \in (1/2, 1)$.

The legislator specifies the antitrust framework that we take as exogenously given. Within this framework the AA chooses its policy: it tries to deter collusion. At the outset the legislator announces the fine $F > 0$ that a

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$^6$Using our notation, most of the existing literature looks at the case where $\nu \in \{0, 1\}$.

$^7$Alternatively, suppose the two firms face one market with demand $1 - q$ where $q$ is the price. Normalizing the firms’ cost to zero, the monopoly price is $q_M = .5$ and the monopoly profit is .25, thus $\pi_M = .125$. Bertrand competition leads to $q = 0$ along with zero profits. If firms collude on prices, the choice of $\nu$ corresponds to setting the price $q = .5[1 - \sqrt{1 - \nu}]$.

$^8$In the Bertrand example firms fix prices: They set $q = .5[1 - \sqrt{1 - \nu}]$ and make profit $\nu\pi_M$. If a firm deviates, it slightly undercuts $q$. If firms compete, they both charge $q = 0$. 

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The legislator grants leniency to the first reporting firm. To get leniency, the reporting firm has to provide evidence of the conspiracy and it has to immediately stop the collusive conduct. If both firms choose to report, nature determines with equal probability who is first. Accordingly, in expectation each firm obtains half the leniency. We look at the case of full leniency so that the reporting firm ends up with no fine while the non-reporting firm pays $F$; if both firms report, each of them pays in expectation $F/2$. The fine is proportional to the degree of collusion. Accordingly, if the firms collude on $\nu$ markets and are convicted, they pay a fine $\nu F$. Concerning leniency we look at two possibilities: Either the AA grants leniency only before an investigation has started, or the AA is generous and grants leniency also after an investigation has been initiated. The AA starts an investigation with probability $\alpha \in [0, 1]$. Then an infinitely repeated game starts. The stage game in each period $t = 0, \ldots$ has the following structure: Knowing $\alpha$, first each firm decides whether it wants to communicate with the other firm or not. Firms make this decision simultaneously. If both firms choose to communicate, they create evidence that —if detected— leads to a conviction by the AA; unless both firms communicate, they do not engage in illegal behavior and there is thus no evidence thereof. The evidence dissolves at the end of the period. If at least one firm chooses not to collude, firms compete and the game ends for that period; otherwise, the stage game continues.

Then firms choose whether they adhere to collusion or whether they deviate. Simultaneously, the firms decide whether they report any communique.
tion or not. If one or both firms report, the evidence of collusion is unveiled and the firms get convicted for sure.\textsuperscript{14} If no firm reports, the stage game continues.

With probability $(1 - \alpha)$ the AA does not launch an investigation and the game ends for that period. With probability $\alpha$ the AA starts an investigation and the game continues. Firms may reconsider their decision whether they collude or whether they deviate. If the AA grants postinvestigation leniency, the firms also decide whether they report any communication or not.

If at least one firm reports, the cartel is detected for sure. If firms do not report, the probability of conviction $p$ depends on the number of markets on which they collude: $p(\nu) \in [0, 1]$ with $p(0) = 0$, $p' > 0$ for $\nu > 0$, and $p'(1)$ finite. If firms do not collude on any market, the probability of conviction is zero.\textsuperscript{15} The more markets the firms collude on, the higher is the probability of conviction. Furthermore, $2p' + \nu p'' > 0$ to ensure the existence of interior solutions for $\nu$ where appropriate.\textsuperscript{16} To summarize, the probability $P$ of detection and conviction if firms communicated,

$$P = \begin{cases} 0, & \text{if there is no investigation and no firm reports;} \\ p(\nu), & \text{if there is an investigation and no firm reports;} \\ 1, & \text{if one or both firms report.} \end{cases}$$

To wrap up the the model:

- The legislator determines the fine $F$ and grants full leniency for the first reporting firm preinvestigation or pre- and postinvestigation; this antitrust framework is exogenously given.

- The AA announces $\alpha$.

- Then the stage game begins:

\textsuperscript{14}Firms thus possess perfect and symmetric evidence of the collusive behavior. See Blatter et al. (2018) for a set-up where firms have imperfect and asymmetric evidence.

\textsuperscript{15}If firms decide to communicate and then pick $\nu = 0$, they engage in illegal behavior that is sanctionable. Yet in our framework firms anticipate that they will choose $\nu = 0$ and they will not communicate in the first place.

\textsuperscript{16}The assumption is satisfied if $p(\nu)$ is convex or concave and $p''$ not too small.
Firms decide whether they communicate or not.

Firms choose whether to adhere to collusion or not. Moreover, they decide whether they report or not before an investigation.

The AA starts an investigation with probability $\alpha$.

Firms may reconsider whether they collude or not.

If firms have the opportunity to report after an investigation has started, they decide whether they report or not.

Firms maximize profits with respect to their communication, price, and reporting decision. The AA strives to deter cartels and sets $\alpha$. We analyze the firms’ behavior for given levels of $\alpha$.

## 3 Pre- and postinvestigation leniency

We will first analyze the scenario where the generous AA grants pre- and postinvestigation leniency. Following the literature we consider two collusive strategies for firms.\(^\text{17}\) Both these strategies differ in the firms’ behavior if there is an investigation. We will, therefore, identify the two strategies with their reporting strategy in case of an investigation:

- Firms agree to collude. Pre- and postinvestigation the firms adhere to collusion and do not report. Call this strategy $N$.

- Firms agree to collude. Preinvestigation they adhere to collusion and do not report. Postinvestigation firms stop collusion and report. After an investigation with agreed upon reporting, firms continue to play the collusive strategy. Call this strategy $R$.\(^\text{18}\)

\(^{17}\)See, e.g., Motta and Polo (2003).

\(^{18}\)Collude when there is no investigation, report when there is an investigation, and stop collusion after a report is clearly dominated by strategy $R$. Next consider the strategy collude when there is no investigation; when there is an investigation, firm $a$ reports while firm $b$ does not; stop collusion after a firm reported. Postinvestigation both firms play the static Nash equilibrium leading to zero profits. If firm $b$ deviates to reporting, it doesn’t change the probability of conviction, yet lowers its fine by half. Hence, it rather reports. Both firms reporting postinvestigation without returning to collusion thereafter
3.1 Collude and not reveal

Let us first consider strategy $N$ under which firms never report collusion. With probability $(1 - \alpha)$ there is no investigation. Firms collude and make profit $\nu \pi_M$. With probability $\alpha$ there is an investigation. Firms collude, make profit $\nu \pi_M$, and face the expected fine $p(\nu)\nu F$. Next period firms continue with collusion. The expected profit per period is

$$\pi_N = \nu[\pi_M - \alpha p(\nu)F]$$  \hspace{1cm} (1)

and the expected overall profit amounts to $\pi_N/(1 - \delta)$.

3.1.1 Without leniency

Let us first determine the firms’ choice of $\nu$ absent any leniency. Maximizing (1) without any constraints yields

$$\bar{\nu}_N = (\pi_M/\alpha F - p(\bar{\nu}_N))/p' (\bar{\nu}_N).$$ \hspace{1cm} (2)

Call $\bar{\nu}_N$ the unconstrained choice under strategy $N$. $\bar{\nu}_N$ is increasing with $\pi_M$ and decreasing with $\alpha$ and $F$: If the AA increases $\alpha$, the degree of collusion goes down.\(^{19}\) For $\alpha$ and/or $F$ sufficiently small, $\bar{\nu}_N > 1$ so that $\nu \leq 1$ binds. More importantly, note that increasing $\nu$ at $\nu = 0$ has no first-order effect on the expected fine while profits increase. Therefore, firms always choose a positive degree of collusion, i.e., $\bar{\nu}_N > 0$.

Next let us take the firms’ incentive constraint without leniency into account: Pre- and postinvestigation a firm must prefer to adhere to collusion rather than deviate for a onetime increase in profit. Suppose the AA has started an investigation. If firms continue to play $N$, their profit is $\nu[\pi_M - p(\nu)F + \delta(\pi_M - \alpha p(\nu)F)/(1 - \delta)]$. If a firm deviates from collusion, it makes profit $2\nu \pi_M - p(\nu)\nu F$: It gets the deviation profit minus the expected fine; however, from next period on the firms compete and make no profits.

is, however, dominated by strategy $R$. Note that under both strategies $N$ and $R$ firms continue with collusion after an investigation. For empirical evidence on recidivism in price-fixing, see, e.g., Connor (2010).

\(^{19}\)For example, formally we have $\partial \bar{\nu}_N/\partial \alpha = -\pi_M/\alpha^2 F(2p'(\bar{\nu}_N) + \bar{\nu}_N p''(\bar{\nu}_N)) < 0.$
If firms play $N$ preinvestigation, their profit is $\nu(\pi_M - \alpha p(\nu) F)/(1 - \delta)$. If a firm deviates from collusion, it makes profit $2\nu \pi_M - \alpha \nu p(\nu) F$. Firms play $N$ rather than report and deviate, both pre- and postinvestigation, if

$$\nu(\pi_M - \alpha p(\nu) F)/(1 - \delta) \geq 2\nu \pi_M - \alpha \nu p(\nu) F.$$  \hspace{1cm} (3)

To support $N$ without leniency, firms choose the degree of collusion that satisfies the equality in (3),

$$\hat{\nu}_N = p^{-1}((2\delta - 1)\pi_M/\delta F).$$ \hspace{1cm} (4)

3.1.2 With leniency

Consider now the situation with leniency. If firms play $N$ preinvestigation, their profit is $\nu(\pi_M - \alpha p(\nu) F)/(1 - \delta)$. If a firm deviates from collusion and reports, it makes profit $2\nu \pi_M$: It gets the deviation profit and pays no fine; however, from next period on the firms compete and make no profits. Firms play $N$ rather than report and deviate if

$$\nu(\pi_M - \alpha p(\nu) F)/(1 - \delta) \geq 2\nu \pi_M.$$ \hspace{1cm} (5)

To support $N$ with preinvestigation leniency firms choose the degree of collusion that satisfies the equality in (5). We have

$$\hat{\nu}_N = p^{-1}((2\delta - 1)\pi_M/\alpha F).$$ \hspace{1cm} (6)

Now suppose the AA has started an investigation. If firms continue to play $N$, their profit is $\nu[\pi_M - p(\nu) F + \delta(\pi_M - \alpha p(\nu) F)/(1 - \delta)]$. If a firm reports and deviates from collusion, it makes profit $2\nu \pi_M$. Firms play $N$ rather than report and deviate if

$$\nu[\pi_M - p(\nu) F + \delta(\pi_M - \alpha p(\nu) F)/(1 - \delta)] \geq 2\nu \pi_M.$$ \hspace{1cm} (7)

To support $N$ with postinvestigation leniency firms choose the degree of collusion that satisfies the equality in (7). We have

$$\hat{\nu}_N = p^{-1}((2\delta - 1)\pi_M/(1 - \delta + \delta \alpha) F).$$ \hspace{1cm} (8)
Call $\hat{\nu}_N$ the constrained choice under strategy $N$. $\hat{\nu}_N$ is increasing in $\pi_M$ and decreasing in $\alpha$ and $F$.

First note that $\hat{\nu}_N < \check{\nu}_N$. Playing $N$ postinvestigation is less attractive than preinvestigation. Therefore, deviating to not collude and report is more attractive postinvestigation. Next note that, not surprisingly, $\hat{\nu}_N < \check{\nu}_N$. The introduction of leniency makes the deviation to not collude more attractive: by not collude and report the firm gets the deviation profit and avoids the fine $F$.

Under strategy $N$ firms, therefore, choose either the unconstrained $\check{\nu}_N$ or the constrained degree of collusion $\hat{\nu}_N$. Clearly, they do better with $\check{\nu}_N$ than with $\hat{\nu}_N$. Consequently, they will choose $\hat{\nu}_N$ only when it is smaller than $\check{\nu}_N$. Taking into account that $\nu \in [0, 1]$, we have

$$\nu^*_N = \min\{\check{\nu}_N, \hat{\nu}_N, 1\}.$$ (9)

Let us now state our first result. If firms are sufficiently patient, under strategy $N$ the incentive to deviate plays no role: it is not attractive to give up the future profits from collusion for a onetime profit increase. By contrast, if they are sufficiently impatient, the incentive to deviate alone determines the degree of collusion.

**Lemma 1:**

a) For $\delta$ close to 1, $\check{\nu}_N < \hat{\nu}_N \forall \alpha > 0$;

b) for $\delta$ close to $1/2$, $\hat{\nu}_N < \check{\nu}_N \forall \alpha > 0$.

**Proof:**
a) Rewrite (2) as $p(\check{\nu}_N) = \pi_M/\alpha F - p'(\check{\nu}_N)\check{\nu}_N$ and (8) as $p(\hat{\nu}_N) = (2\delta - 1)\pi_M/(1 - \delta + \delta \alpha) F$. $p(\hat{\nu}_N)$ is increasing in $\delta$ and $\lim_{\delta \uparrow 1} p(\check{\nu}_N) = \pi_M/\alpha F$. For $\delta$ close to one, $p(\check{\nu}_N) > p(\hat{\nu}_N)$, because $\check{\nu}_N > 0$. Since $p' > 0$, $\hat{\nu}_N > \check{\nu}_N$.

b) $\lim_{\delta \downarrow 1/2} p(\hat{\nu}_N) = 0$, hence $\hat{\nu}_N \to 0$ for $\delta$ approaching $1/2$. Since $\check{\nu}_N > 0$, we have $\hat{\nu}_N < \check{\nu}_N$ for $\delta$ close to $1/2$. □

The first part of the Lemma implies that the introduction of leniency does not change patient firms’ behavior. Note that this result does not hold if firms are restricted to choose between no and full collusion. Obviously,
if $\nu \in \{0, 1\}$, firms cannot choose an interior degree of collusion. Suppose without leniency they fully collude. Specifically, let $\alpha$ and $F$ be such that for $\nu = 1$ the equality in (3) is satisfied. Now introduce leniency. The deviation away from collusion becomes more attractive. (5) is not satisfied for $\nu = 1$ and firms opt for no collusion. Thus, leniency increases deterrence in this example.

3.2 Collude and reveal

Let us now turn to the strategy $R$. With probability $(1 - \alpha)$ there is no investigation. Firms collude and make profit $\nu \pi_M$. With probability $\alpha$ there is an investigation. Firms report, stop collusion in this period by playing the static Nash equilibrium, make 0 profit, and pay the fine $\nu F/2$. Next period firms return to collusion. The expected profit per period is

$$\pi_R = \nu[(1 - \alpha)\pi_M - \alpha F/2]$$

and the expected overall profit $\pi_R/(1 - \delta)$.\footnote{Some authors consider collude and reveal as an empirically irrelevant strategy. Simultaneous reporting is rarely seen in reality (in Switzerland we had, however, cases where several firms reported within a short time span at the day of the dawn raid). Experimental work tends not to support their use. With a cartel of $n$ firms, the expected fine when all firms report is $(n - 1)F/n$; the strategy thus becomes less attractive, the larger the cartel. See, e.g., Spagnolo (2008). According to this few our results involving $R$ should be assessed critically.}

Now let us consider the incentive to deviate. Postinvestigation both firms report. A firm does not unilaterally deviate to not report because the other firm reports: the fine it pays increases from $F/2$ to $F$ without any benefits whatsoever.

Preinvestigation if firms play $R$, they collude and do not report; their profit is $\nu\left[\frac{(1 - \alpha)\pi_M - \alpha F/2}{1 - \delta}\right]$. If a firm reports and deviates from collusion, it makes profit $2\nu \pi_M$: It gets the deviation profit and pays no fine; however, from next period on the firms compete and make no profits. Firms play $R$ rather than report and deviate if

$$\nu\left[\frac{(1 - \alpha)\pi_M - \alpha F/2}{1 - \delta}\right] \geq 2\nu \pi_M.$$
Hence,

\[
\nu_R = \begin{cases} 
0, & \alpha > \pi_M(2\delta - 1)/(F/2 + \pi_M); \\
1, & \text{otherwise.}
\end{cases}
\]

(12)

\(\pi_R\) is linear in \(\nu\). Therefore, the optimal degree of collusion is either zero or one.

3.3 Deterrence

Before we analyze when firms actually play \(N\) or \(R\) under leniency, we may already state the following result: If firms are sufficiently patient, the introduction of leniency does not decrease collusion.

**Proposition 1:** For \(\delta\) close to one, leniency does not increase deterrence.

**Proof:** We know from Lemma 1 that \(\bar{\nu}_N < \hat{\nu}_N\) for \(\delta\) close to 1. If \(\bar{\nu}_N < 1\), firms choose the interior degree of collusion \(\bar{\nu}_N\). If \(\bar{\nu}_N \geq 1\), firms collude on all markets. In both cases the introduction of leniency does not change their behavior as to strategy \(N\).

Leniency introduces the option to play strategy \(R\). Under strategy \(N\) firms always make positive profits. Therefore, if firms prefer \(R\) to \(N\), they must fully collude on all markets. More than full collusion is not possible under strategy \(N\). Thus, collusion does not go down if firms switch from \(N\) to \(R\). □

We know from Lemma 1 that leniency has no bite as to strategy \(N\) if firms are sufficiently patient; the degree of collusion is the same as without leniency. Leniency opens the door for strategy \(R\). If firms actually choose \(R\), then they fully collude. Therefore, if firms continue to play \(N\), the introduction of leniency does not change the degree of collusion. If firms switch to \(R\), they fully collude after the introduction of leniency. Collusion goes up if \(\bar{\nu}_N < 1\); otherwise, it remains unchanged.

Proposition 1 implies that with sufficiently patient firms the introduction of leniency is not a good idea. Given firms play collude and not reveal, the option of getting leniency does not induce them to blow the whistle.
Furthermore, leniency introduces the possibility to play collude and reveal. If firms opt for this strategy, they collude on all markets.

Let us now determine the firms’ strategy choice as a function of $\alpha$.

**Proposition 2:**

a) For $\alpha$ small and $F \geq \pi_M/(p(1) - 1/2)$, firms play $R$ and $\nu^* = \nu_R = 1$;

b) for $\alpha > \pi_M(2\delta - 1)/(F/2 + \pi_M)$, firms play $N$ and $\nu^* = \nu_N^* > 0$.

**Proof:** a) If $\alpha$ small, $\hat{\nu}_N = 1$ and $\pi_N(1) = \pi_M - \alpha p(1) F$. $\hat{\nu}_N \leq \overline{\nu}_N$ and $\pi_N(\hat{\nu}_N) \leq \pi_N(\overline{\nu}_N)$ because $\overline{\nu}_N$ is the unconstrained choice. For $\alpha$ small, $\nu_R = 1$ and $\pi_R(1) = (1 - \alpha) \pi_M - \alpha F/2$. $\pi_R(1) \geq \pi_N(1)$ if $F \geq \pi_M/(p(1) - 1/2)$.

b) For $\alpha > \pi_M(2\delta - 1)/(F/2 + \pi_M)$, $\nu_R = 0$ and $\pi_R(0) = 0$. $\nu^*_N > 0 \forall \alpha$, hence $\pi_R(\nu^*_N) > 0$. □

If $\alpha$ is small, the firms’ unconstrained choice under strategy $N$ is $\bar{\nu}_N = 1$ and $\pi_N(1) = \pi_M - \alpha p(1) F$. Taking the incentive to deviate into account yields $\hat{\nu}_N \leq \bar{\nu}_N$. The profit under the constrained choice cannot exceed the profit under the unconstrained choice, thus $\pi_N(1)$ is the upper bound for profits under strategy $N$. Under strategy $R$ firms pick $\nu_R = 1$ for $\alpha$ small, leading to profit $\pi_R(1) = (1 - \alpha) \pi_M - \alpha F/2$. $R$ yields higher profits than $N$ if $F \geq \pi_M/(p(1) - 1/2)$. Under $R$ a firm gives up $\pi_M$ and pays $F/2$ in the investigation subgame; under $N$ it pays $p(1) F$ in the investigation subgame. Given $p(1) > 1/2$, for $F$ sufficiently large firms prefer $R$.

For $\alpha > \pi_M(2\delta - 1)/(F/2 + \pi_M)$, firms do not collude under $R$ which, in turn, implies zero profits. Under strategy $N$ there is always collusion leading to positive profits. From Lemma 1 we know that for $\delta$ close to one, firms choose the unconstrained $\bar{\nu}_N$; leniency has thus no bite. By contrast, for $\delta$ close to $1/2$, firms pick $\hat{\nu}_N$: the degree of collusion is constrained by leniency.

For all other cases the firms’ choices cannot be determined without specifying the detection probability $p(\nu)$. Firms may play strategy $N$ with the unconstrained $\bar{\nu}_N$ or the constrained choice $\hat{\nu}_N$, both of which are positive for all $\alpha$; or firms play strategy $R$ with $\nu_R = 1$. Yet, despite this vagueness, we can make the following statement: Whatever detection probability $\alpha$ the
AA chooses, firms will answer with a positive degree of collusion; complete deterrence is not possible in our set-up.

Proposition 2 has the following policy messages. In case a) slightly increasing $\alpha$ does not lower the degree of collusion: firms continue to collude on all markets playing $R$. However, it increases cartel desistence, i.e., firms interrupt their collusion more often and the AA collects the fine more often. In case b) increasing $\alpha$ lowers the degree of collusion (though never to zero). Whether the AA collects more fines is, however, unclear: $\alpha$ goes up, $\nu_N^*$ goes down, and so does the probability of detection $p(\nu_N^*)$ and the fine $\nu_N^*F$.

As to the introduction of leniency, Proposition 2 implies the following. For small values of $\alpha$ introducing leniency is not a good idea: firms may switch from collude and not reveal with $\nu_N^* < 1$ to collude and reveal with $\nu_R = 1$, i.e., collusion goes up. For large values of $\alpha$ firms choose collude and not reveal. Suppose without leniency firms do not choose the unconstrained degree of collusion, i.e., $\tilde{\nu}_N < \tilde{\nu}_N$ with $\tilde{\nu}_N$ given by (4). Then leniency reduces collusion because it reinforces the incentive to deviate ($\nu_N < \tilde{\nu}_N$).

Nevertheless, with and without leniency firms may pick the unconstrained $\tilde{\nu}_N$, so that leniency has no effect on collusion.

4 Preinvestigation leniency

Let us now briefly look at the case where the AA does not grant postinvestigation leniency. If only preinvestigation leniency is available, firms will not follow the strategy collude and reveal $R$: If a firm stops colluding and reports in case of an investigation, it lowers its profits by $\pi_M$ and increases the probability of conviction by $1 - p(\nu)$.

We are thus left with strategy collude and not reveal $N$. Since postinvestigation leniency is not an issue, we need not consider $\hat{\nu}_N$ as defined by (8). First note that $\tilde{\nu}_N$ as defined by (6) is smaller than $\tilde{\nu}_N$ as given by (4). Also with preinvestigation leniency the deviation to not collude is more attractive than without leniency. The optimal degree of collusion with preinvestigation leniency is thus

$$\nu_N^{**} = \min\{\tilde{\nu}_N, \tilde{\nu}_N, 1\}.$$ 

(13)
Next note that the analogue to Lemma 1 holds with $\tilde{\nu}_N$ substituted by $\hat{\nu}_N$. Since strategy $R$ is no valid option without postinvestigation leniency, this result immediately implies: Switching from no to preinvestigation leniency has no effect on deterrence if firms are sufficiently patient because they choose the unconstrained degree of collusion $\bar{\nu}_N$. By contrast, for $\delta$ close to 1/2, firms pick $\check{\nu}_N$; the degree of collusion is constrained by the introduction of preinvestigation leniency. We may, therefore, conclude that preinvestigation leniency can increase deterrence. There are no counterproductive effects as under pre- and postinvestigation leniency where firms may choose strategy $R$ along with full collusion. Yet note that $\hat{\nu}_N < \check{\nu}_N$. If parameters are such that with the generous AA firms choose $\check{\nu}_N$ whilst with the non-generous one they pick $\hat{\nu}_N$, allowing for postinvestigation leniency increases deterrence.

Two final remarks are in order. The result that $\bar{\nu}_N > 0$ is of course driven by our assumption that the fine $\nu F$ is proportional to the degree of collusion. We are not aware of any legal system where this is not the case in one form or the other. For example, in Switzerland the fine is a percentage (up to 10%) of the revenue made in Switzerland in the last 3 years. The actual percentage is adjusted for the severity of collusion (horizontal price-fixing is most severe offense), whether the firms cooperated during the investigation etc. The revenue should be positively correlated with the degree of collusion, thus so is the fine. Accordingly, our assumption that the fine goes up with the degree of collusion seems a reasonable approximation for Swiss institutions. Note that even with a non-proportional fine the result continues to hold if $p'(0) = 0$ which is, e.g., the case for $p(\nu) = \nu^\beta$, $\beta > 1$. For example, many jurisdictions have a positive baseline fine for communicating, which is then augmented with the degree or the length of collusion. Obviously, the result $\bar{\nu}_N > 0$ is not true if we restrict $\nu \in \{0, 1\}$.

In our framework the AA chooses the probability of an investigation $\alpha$. Suppose, for example, to investigate an industry the AA needs a certain amount of manpower. By choosing the overall size of its staff, the AA determines the fraction $\alpha$ of industries it can investigate. Each industry is equally likely to be investigated, independently of its degree of collusion $\nu$; $\nu$ only plays a role for the probability of conviction $p$. 

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It seems conceivable that $\alpha$ is also an increasing function of $\nu$: the more firms collude, the more suspicious the industry, the more likely it is to be investigated. Let us first look at the dual of our framework: the AA picks $p$, which is independent of $\nu$, and firms determine by their choice of $\nu$ the probability of an investigation $\alpha(\nu)$. The analysis of strategy $N$ changes qualitatively only slightly, the probability of being investigated and convicted, $\alpha p$, as a deterrent device works similarly as in our set-up. Yet, the results as to strategy $R$ change drastically. First, the profit $\pi_R = \nu[(1 - \alpha(\nu))\pi_M - \alpha(\nu)F/2]$ is not linear in $\nu$. Accordingly, we no longer have the result that the optimal $\nu_R^* \alpha$ is either 0 or 1. Second, and more importantly, firms do not care about $p$: in case of an investigation they report anyway. Only $\alpha$ deter firms. However, in the dual $\alpha(\nu)$ set-up firms themselves choose the probability of an investigation. Therefore, for all levels of $p$, firms pick $\nu_R^*$.

Next consider the case where both, $p$ and $\alpha$ are increasing in $\nu$. While such a set-up is certainly of interest, it raises a couple of problems: Is $\nu$ more important for $\alpha$ or for $p$? How does $\alpha$ depend on the efforts by the AA and the degree of collusion $\nu$? We have avoided these modeling issues by assuming separability: the AA picks $\alpha$ while the firms strategically determine $p$.

Following the literature we focus on the incentive of firms to deviate to “undercut and report”. This allows us to relate our results to earlier findings of the literature and highlight the effects of variable degrees of collusion. Experimental evidence suggests that strategic risk, i.e., the risk of being the sucker, is another important channel to deter collusion; see Bigoni et al. (2015). Analyzing this alternative deterrence avenue is beyond the scope of the paper and is left for future research.

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21 Abusing notation we could, e.g., use a contest function to define the probability of an investigation as $\alpha/(\alpha + \nu)$. With the same approach we could define $p$.
22 The general results will change when we allow for this additional deterrence channel. We do not, however, expect major changes as to the effects of variable compared to binary degrees of collusion. Spagnolo (2004) provides conditions for an optimal policy if agents do not play risk dominated strategies, two of which are satisfied in our framework: Only the first reporting firm is eligible for leniency and the fine is maximal. We do not allow for rewards which should also be maximal.
5 Conclusions

In most of the existing literature on leniency firms have to decide between colluding and not colluding, a binary choice. Yet, often firms have more than two options as to collusion: they can, e.g., collude on some markets and compete on the others or they can set any price between the competitive and the monopoly one. The purpose of this paper is to study leniency programs when firms choose the degree of collusion, a continuous variable. It turns out that this affects the effectiveness of leniency programs. To assess leniency it seems thus a good idea to keep the firms’ collusion possibilities in mind.

As to the introduction of leniency, our results are mixed. If only preinvestigation leniency is granted, leniency can increase deterrence; there are no counterproductive effects. With pre- and postinvestigation leniency, if the probability of an investigation is small or if firms are sufficiently patient, leniency is ineffective and may even increase collusion. In all other cases leniency may or may not reduce collusion. Firms, choose, however, always a positive degree of collusion. Complete deterrence is not possible in our framework.
References


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