Multiproduct-Firm Oligopoly:
An Aggregative Games Approach

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Introduction

- Even when defined at the NAICS 5-digit level, multiproduct firms (MPFs) account for 41% of the total number of firms and 91% of total output in the U.S. (Bernard, Redding and Schott, 2010).

- In U.S. manufacturing, the average (resp. median) NAICS 5-digit industry has a C4 of 35% (resp. 33%). (Source: Census of U.S. Manufacturing, 2002).

  Suggests that many markets are characterized by oligopolistic competition.

- Ubiquitousness of MPFs and oligopoly is reflected in modern empirical IO literature.

- Due to technical difficulties (failure of quasi-concavity, (log-)supermodularity, upper semi-continuity), surprisingly little is known about even basic properties of oligopoly models with MPFs.

- This paper: Provide an aggregative games approach to MPF-oligopoly with applications to merger (and trade) policy.
The Baseline Model: Demand

- Set $\mathcal{N}$ of (differentiated) products, and a Hicksian composite commodity.
- Consumers’ indirect utility: $V(p) = \log(H(p)) + y$, where $y$ is income, and $H(p) = \sum_{j \in \mathcal{N}} h_j(p_j) + H^0$.
- Implied demand system:
  \[ D_i(p) = \hat{D}_i(p_i, H(p)) = \frac{-h'_i(p_i)}{H(p)} \]
- Two special cases: CES ($h(p) = ap^{1-\sigma}$) and MNL ($h(p) = e^{\frac{a-p}{\lambda}}$).
- Demand system can equivalently be derived from discrete/continuous choice with i.i.d. Gumbel taste shocks.
The Baseline Model: Firms

- Set of firms, $\mathcal{F}$, is a partition of $\mathcal{N}$.
- Constant marginal cost of product $i \in \mathcal{N}$, $c_i > 0$.
- Each firm $f \in \mathcal{F}$ sets profile of prices $p^f = (p_k)_{k \in f}$.
- Firm $f$’s profit:

$$\Pi^f(p^f, H(p)) = \sum_{j \in f} (p_j - c_j) \hat{D}_j(p_j, H(p)).$$

- Allow for infinite prices: If $p_k = \infty$, $k \in f$, firm $f$ does not make any profit on product $k$.
- Pricing game is aggregative: $\Pi^f(p^f, H(p))$ depends on prices set by rival firms only through uni-dimensional aggregator $H$. 
Existence of a Pricing Equilibrium

Standard approaches to equilibrium existence fail because:

(i) Action spaces are not bounded or payoff functions not upper semi-continuous.

(ii) Payoff functions are not (log-)supermodular.

(iii) Profit functions are not quasi-concave.

Nash/Glicksberg's theorems don't apply due to (i) and (iii).
Topkis/Milgrom-Roberts's theorems don't apply due to (i) and (ii).

Our existence proof relies on an aggregative games approach:

Fix $H$ and look for $(p_k)_{k \in f}$ such that all of firm $f$'s FOCs hold.

Obtain a vector $(p_k(H))_{k \in f}$ for every $f$.

Then, look for an $H$ such that $\sum_{f \in F} \sum_{k \in f} h_k(p_k(H)) = H$. 

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- **First-order condition for product** $k \in f$:

\[
0 = \frac{d\Pi^f}{dp_k} = \hat{D}_k + (p_k - c_k) \frac{\partial \hat{D}_k}{\partial p_k} + \frac{\partial H}{\partial p_k} \left( \sum_{j \in f} (p_j - c_j) \frac{\partial \hat{D}_j}{\partial H} \right),
\]

\[
= \hat{D}_k \left( 1 - \frac{p_k - c_k}{p_k} \left| \frac{\partial \log \hat{D}_k}{\partial \log p_k} \right| \right) + \frac{\partial H}{\partial p_k} \hat{D}_k \left( \sum_{j \in f} (p_j - c_j) \frac{\partial \hat{D}_j}{\partial H} \right) .
\]

- **Re-arranging**:

\[
\frac{p_k - c_k}{p_k} \left| \frac{\partial \log \hat{D}_k}{\partial \log p_k} \right| = 1 + \frac{\partial H}{\partial p_k} \sum_{j \in f} (p_j - c_j) \frac{\partial \hat{D}_j}{\partial H} .
\]

\[
\begin{aligned}
&= p_k \left( -\frac{h''_k(p_k)}{h'_k(p_k)} \right) = \iota_k(p_k) \\
&\text{independent of } k
\end{aligned}
\]
Existence of a Pricing Equilibrium

- The fact that the right-hand side is independent of $k$ follows as the marginal impact on $H$ of an increase in $p_k$ is proportional to the demand of product $k$. (Follows from IIA property, which implies that demand is multiplicatively separable in the aggregator.)

- IIA also implies: LHS of FOC independent of $H$.

- Hence, if $(p_k)_{k \in f}$ satisfies the FOCs, then for every $i, j \in f$,

$$\frac{p_i - c_i}{p_i} \lambda_i(p_i) = \frac{p_j - c_j}{p_j} \lambda_j(p_j)$$

We say that $(p_k)_{k \in f}$ satisfies the common $\lambda$-markup property.

Within-firm markup structure: Lerner index is inversely proportional to the “perceived” price elasticity of demand.
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Existence of a Pricing Equilibrium

Assume that function \( p_k \mapsto \frac{p_k - c_k}{p_k} \kappa_k(p_k) \) can be nicely inverted for every \( k \in f \).

- Denote the inverse function by \( r_k(\mu^f) \).

- Firm \( f \)'s optimality conditions boil down to a single equation:

\[
\mu^f = 1 + \Pi^f((r_k(\mu^f))_{k \in f}, H).
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  \[
  \mu^f = 1 + \Pi^f ((r_k(\mu^f))_{k \in f}, H).
  \]

Assume that this equation has a unique solution for every \( H \).

- Denote the solution by \( m^f(H) \). \( m^f(.) \) is firm \( f \)'s fitting-in function.
Existence of a Pricing Equilibrium

- Let

\[ \Gamma(H) = \sum_{f \in F} \sum_{k \in f} h_k \left( r_k(m^f(H)) \right). \]

- \( H \) is an equilibrium aggregator level if and only if \( \Gamma(H) = H \).

- \( \Gamma \) is called the aggregate fitting-in function.

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- So the equilibrium existence problem boils down to looking for a fixed point of the aggregate fitting-in function.

Assume that such a fixed point exists.

- Then, the pricing game has an equilibrium.

- The nested fixed point structure gives rise to an efficient way of computing the equilibrium.
Existence of a Pricing Equilibrium

Assumption:

(i) For every $k \in \mathcal{N}$, $\iota_k$ is non-decreasing.

Note:

- Under monopolistic competition, where firms take $H$ as given, Assumption (i) means that the perceived price elasticity of demand is non-decreasing (Marshall’s second law of demand).
- Under MNL demand, $\iota_k(p_k) = \frac{p_k}{\lambda_k}$. Under CES demand, $\iota_k(p_k) = \sigma$. 
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Theorem

Under Assumption (i), the pricing game has an equilibrium for every \((c_i)_{i \in \mathcal{N}}\) and \( F \).
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Theorem

Under Assumption (i), the pricing game has an equilibrium for every \((c_i)_{i \in \mathcal{N}}\) and \( \mathcal{F} \).

We also establish equilibrium uniqueness (under stronger conditions) by showing that \( \Gamma'(H) < 1 \) whenever \( \Gamma(H) = H \).
Other Results

- **Firm scope.** Firm $f$ is “more likely” to offer any given product $k$ in equilibrium, the larger is the equilibrium aggregator $H$ (“fighting brand”). *Intuition:* The more competitive is the market (the larger is $H$), the less the firm cares about self-cannibalizing its more profitable products (and the more it cares about stealing business from rivals).

- **Welfare analysis.** The equilibrium exhibits only two types of distortions:
  1. The equilibrium aggregator, $H^*$, is smaller than the welfare-maximizing aggregator, $H^{FB} = \sum_{k \in \mathcal{N}} h_k(c_k)$.
  2. Conditional on $H^*$, the firm-level aggregators are too small for some firms and too large for others.
Comparing Equilibria

Suppose $H^1$ and $H^2$ are equilibrium aggregator levels with $H^1 < H^2$. Then:

- Consumers prefer $H^2$ to $H^1$.
- Every firm prefers $H^1$ to $H^2$.
- The set of active products at $H^1$ is contained in the set of active products at $H^2$.

Monotone comparative statics: Suppose the aggregate fitting-in function shifts upward (say, because import tariffs are reduced or entry takes place). Then, in the lowest and highest equilibrium:

- Prices go down, consumers are better off, (domestic) firms are worse off.
- The set of active products expands.

Productivity improvements have more ambiguous effects.

An increase in marginal cost can increase $H$ and thus make consumers better off.
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Extensions and Type Aggregation

- **Extensions.**
  - Non-linear pricing.
  - Quantity competition.
  - Generalized IIA demands and nests.

- **(Nested) CES/MNL demands: Type aggregation.**
  - All information about firm $f$’s behavior/performance (markup, market share, profit) can be summarized by its (uni-dimensional) type $T^f$, which is independent of $H$. In CES case: $T^f = \sum_{k \in f} a_k c_k^{1-\sigma}$; in MNL case: $T^f = \sum_{k \in f} \exp(\frac{a_k-c_k}{\lambda})$.
  - Type aggregation useful for:
    - Merger analysis.
    - Defining firm-level productivity.
    - Computational tractability.
Applications to Merger Analysis

For the cases of (nested) CES/MNL demands (for which type aggregation obtains), we apply the model to:

1. Static merger analysis, extending Farrell and Shapiro (1990)
   - Consumer/aggregate surplus effects
   - External effects

2. Dynamic merger analysis, extending Nocke and Whinston (2010)

3. Downward Surplus Pressure (DCP)
Upward Pricing Pressure

- Concept of *upward pricing pressure* (UPP) has been used extensively in applied merger analysis.

- Idea behind UPP: Evaluated at the pre-merger price vector, quantify the effect of the merger on the price of product $k$ due to (i) the internalization of pricing externalities post merger, and (ii) changes in marginal costs.

- For merger $M = \{f, g\}$, the UPP on product $k \in f$ is:

\[
\Delta p_k = \sum_{j \in f} (\bar{c}_j - c_j) \frac{\partial D_j}{\partial p_k} \frac{\partial D_k}{\partial p_k} + \sum_{i \in g} (p_i - \bar{c}_i) \frac{\partial D_i}{\partial p_k} \frac{\partial D_k}{\partial p_k}
\]

- Open questions:
  - How to aggregate UPP over many products in a way that is consistent with a consumer surplus standard?
  - How to deal with merger-specific effects on product qualities?
  - How to deal with effect of merger on product portfolio?
Downward Surplus Pressure

- Under (nested) CES/MNL demands, the market share of firm \( f \) depends on the ratio of the firm’s type to the aggregator, and is equal to the firm’s share of the aggregator:

\[
s^f = S \left( \frac{T^f}{H} \right) = \sum_{i \in f} h_i(p_i) H.
\]

- **Downward surplus pressure** (DCP) summarizes the effect of the merger on consumer surplus, evaluated at the pre-merger aggregator level.

\[
DCP \equiv -\Delta CS = \frac{-\Delta H}{H} = \sum_{j \in f, g} \frac{h_j(r_j(m^f(H)))}{H} - \sum_{j \in f, g} \frac{\overline{h}_j(r_j(\overline{m}(H)))}{H} = S \left( \frac{T^f}{H} \right) + S \left( \frac{T^g}{H} \right) - S \left( \frac{T^M}{H} \right).
\]
Conclusion

- **Main contribution:** Tractable approach to MPF oligopoly.
  - Simple, yet powerful existence, uniqueness, and characterization results.
  - Computationally efficient algorithm.
  - Simple decomposition of welfare distortions.
  - Predictions on how markups and firm scope vary with competitive environment.

- **Secondary contribution:** Complete characterization of class of demand systems derivable from discrete/continuous choice with i.i.d. Gumbel taste shocks.
  - By going beyond CES and MNL demands, allow for richer patterns of markups.

- **Policy contribution:** Merger control with MPFs.
  - Shown how well-known results on static and dynamic merger analysis obtained in homogeneous-goods Cournot settings carry over to price competition with MPFs.
  - Proposed concept of downward surplus pressure (DCP) – which aggregates the upward pricing pressure across products, and allows for merger-induced changes to qualities and firm scope.