AGGREGATE DIVERSION AND MARKET ELASTICITY[1]

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Abstract. *Most quantitative tools for assessing the competitive effects of mergers heavily rely on diversion ratios between merging products as a key parameter. Individual diversion ratios, however, cannot be determined without knowledge of aggregate diversion ratios (ADRs), for which reliable estimates are often not available in real-world antitrust procedures. This paper shows how ADRs can be approximated using their relation to the market elasticity of demand. This permits implementing quantitative tools for assessing the competitive effects of mergers based on simple, observable parameters such as market shares, profit margins and the market elasticity of demand.*

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1. INTRODUCTION

There is a large literature assessing the competitive effects of differentiated products mergers. These papers have developed scoring methods such as "compensating marginal cost reductions" (CMCR) (Werden, 1996) and "upward pricing pressure" (UPP) (Farrell and Shapiro, 2010a) that quantify unilateral effects using simple pre-merger parameters such as profit margins and diversion ratios between competing brands.[2]

The initial objective of these methods was to provide competition authorities with systematic economic tools for screening mergers at the early stages of an investigation, to improve upon more simplistic concentration-based screens (such as market shares and HHIs). However, despite their theoretical elegance and simplicity, these methods have often been difficult to apply in practice. The main reason is that diversion ratios, which are a core ingredient of these approaches, are usually unknown at the early stages of an investigation. Accordingly, these tools are applied only in a limited number of cases, and typically at a much later stage of the investigation, when reliable estimates of diversion ratios may become available (e.g., as the result of demand estimation or due to the submission of switching data).

In the absence of concrete estimates of diversion, competition authorities in practice often assume that diversion is proportional to market shares, which corresponds to the IIA assumption in logit models. For example, consider three competing products A, B and C and suppose that the market share of B is twice as

[2] These scoring methods have been extended in various ways, including the "Price Pressure Index" (O'Brien and Salop, 2000), the "Werden-Froeb Index" (Goppelsroeder, Schinkel and Tuinstra, 2008) and the "Gross Upward Pricing Pressure Index" (GUPPI) (Salop and Moresi, 2009). The latter can also be related to market definition (Moresi, 2010) and to merger simulation (Hausman, Moresi and Rainey, 2011).

large as the market share of C (say, 30% for B and 15% for C).[3] Under the assumption that diversion is proportional to market shares, if the price of product A were to increase unilaterally, then the number of customers that would switch to B is twice as large as the number of customers that would switch to C. Yet, even where such *relative* diversion among "inside goods" can be approximated in this way, the *absolute* level of diversion ratios is usually difficult to determine, because it requires estimating the proportion of lost sales that stays in the market at all (rather than being diverted to "outside goods"). Estimating individual diversion ratios therefore requires knowledge of aggregate diversion ratios (ADRs) (i.e., the total proportion of a product's lost sales that remains in the market).[4]

The problem is easy to see in our example. E.g., suppose diversion to outside goods is zero, and hence the ADR from A to B and C is 100%. Proportional diversion then implies that the diversion ratio from A to B is 66.7% and that from A to C is 33.3%. However, if instead the diversion ratio to outside goods were 30% (i.e., the ADR equals 70%), then proportionality would imply that the diversion ratio from A to B is only 46.7% (instead of 66.7%) and that from A to C would be only 23.3% (instead of 33.3%).

The common practice among antitrust practitioners of using ad hoc assumptions on ADRs (e.g., assuming 100% or 70%) can therefore lead to estimated price effects which significantly misrepresent the true competitive effects of a

[3] We use the term "market shares" loosely here, since A, B and C need not necessarily constitute a relevant antitrust market in a legal sense. As we explain later in more detail, the proportionality assumption can also be based on more sophisticated metrics than market shares. For example, in mobile wireless markets, proportionality is sometimes based on carriers' shares of gross additions of subscribers or on the porting out rates of phone numbers from individual carriers to their respective competitors. Our analysis is applicable in those cases as well.

[4] ADRs have also been denoted as "market recapture rates" in the U.S. Merger Guidelines and other policy texts (e.g., see Farrell and Shapiro, 2010b).

merger. For instance, simply assuming zero diversion to outside goods (as is sometimes done for analytical convenience) can easily lead to predictions that overstate the actual price effect of a merger by 100% or more.[5] Thus, it is important for effects-based merger analysis to be able to approximate ADRs in a sensible and rigorous way.[6]

This paper therefore explores the general relationship between ADRs and the price elasticity of market demand.[7] Exploiting this economic relationship permits to sensibly approximate ADRs with simple, observable parameters, which are often readily available even at the early stages of an investigation. Specifically, we derive analytical solutions both for the case of symmetric ADRs and for the case of asymmetric ADRs (in which case we consider the natural benchmark of proportional diversion to outside goods). In these circumstances, ADRs can be directly inferred using no more information than pre-merger markets shares, profit margins and the market elasticity of demand. This substantially facilitates the application of scoring methods such as CMCR, UPP and others to assess the economic effect of mergers when data availability is imperfect.

The paper is structured as follows. Section 2 introduces the model, which builds on the standard model of price competition with differentiated products, and determines the elementary relation between ADRs and market elasticity. Section 3 goes on to analyze symmetric ADRs and derives a closed-form solution for this case,

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[5] It is easy to show that when two out of three symmetric firms merge in an industry with an actual ADR of 2/3, then the assumption of zero diversion to outside goods leads to an overestimation of anticompetitive effects by 100% according to the CMCR (Werden, 1996).

[6] Note that the inability to gauge ADRs early on in an investigation not only afflicts unilateral effects analysis in merger control, but also many other areas of antitrust, e.g., partial ownership (O'Brien and Salop, 2000), market definition (Farrell and Shapiro, 2010b), and vertical integration (Moresi and Salop, 2013), all of which heavily rely on estimates of products' ADRs.

[7] Formally, the market elasticity measures the response of aggregate demand to a proportional increase in the price of *all* products in the market.

which turns out to depend solely on firms' profit margins, market shares and the market elasticity of demand. Section 4 considers the alternative scenario of asymmetric ADRs, derives a solution for the polar case of proportional ADRs, and compares this solution to the case of symmetric ADRs. Section 5 illustrates the implications of these results for applied merger analysis using economic screens. Section 6 goes on to show how the results can also be used in theoretical analyses that explore the interaction of different parameters in determining the competitive effects of mergers. Section 7, finally, concludes.

2. THE MODEL

There are $N \geq 2$ differentiated products that are (imperfect) substitutes from the perspective of customers. The demand for product k is given by:

$$q_k = D_k(p_1, \dots, p_N) \qquad \text{for } k = 1, \dots, N \tag{1}$$

where q_k denotes the quantity demanded, D_k denotes the demand function, and p_j denotes the price of product $j = 1, \dots, N$. The own-price elasticity of product k is given by:

$$\varepsilon_k = \frac{\partial D_k}{\partial p_k} \frac{p_k}{q_k} \tag{2}$$

The total demand for the N products is given by:

$$q = \sum_{k=1}^{N} D_k(p_1, \dots, p_N) \tag{3}$$

and the aggregate elasticity of total demand (with respect to a proportional increase in all prices) is given by:

$$\varepsilon = \frac{t}{q} \frac{d}{dt} \sum_{k=1}^{N} D_k(tp_1, \dots, tp_N) \,|_{t=1} = \sum_{k=1}^{N} \sum_{j=1}^{N} \frac{\partial D_k}{\partial p_j} \frac{p_j}{q} \tag{4}$$

This definition of "market elasticity" is based on total volume and produces an elasticity measure that generally is dependent on the units of measurement chosen for each product. In some cases, one might instead prefer to use total revenue (or total spending):

$$R = \sum_{k=1}^{N} p_k D_k(p_1, \dots, p_N) \tag{3*}$$

and define the elasticity of total demand equal to the elasticity of total revenue minus one:

$$\varepsilon^* = \frac{t}{R}\frac{d}{dt}\sum_{k=1}^{N} tp_k D_k(tp_1, \dots, tp_N)\,|_{t=1} - 1 = \sum_{k=1}^{N}\sum_{j=1}^{N} p_k \frac{\partial D_k}{\partial p_j}\frac{p_j}{R} \tag{4*}$$

This alternative definition can be useful in applications when firms sell multiple products and/or there is no natural way to choose units of measurement.

The diversion ratio from product j to product k is given by:

$$\delta_{jk} = -\frac{\partial D_k/\partial p_j}{\partial D_j/\partial p_j} \tag{5}$$

This measure also depends on measurement units, and thus one might prefer to use:

$$\delta_{jk}^* = -\frac{p_k}{p_j}\frac{\partial D_k/\partial p_j}{\partial D_j/\partial p_j} \tag{5*}$$

The ADR for product j is the total diversion ratio from j to all the other products:

$$\delta_j = \sum_{k \neq j} \delta_{jk} \tag{6}$$

and the corresponding unit-independent measure is:

$$\delta_j^* = \sum_{k \neq j} \delta_{jk}^* \tag{6*}$$

We define the diversion ratio from product j to "outside goods" to be equal to $1 - \delta_j$ (or $1 - \delta_j^*$). In principle, different products may have different ADRs and thus different diversion ratios to outside goods.

Using these definitions, we are now ready to consider the relationship between elasticities and aggregate diversion ratios. Note first that equation (4) can be rewritten as:

$$\varepsilon = \sum_{j=1}^{N} \left(\frac{p_j}{q} \sum_{k=1}^{N} \frac{\partial D_k}{\partial p_j} \right) \tag{7}$$

by simply reversing the order of summation in (4) and then factorizing p_j/q.

Then, using (5), equation (7) can be rewritten as:

$$\varepsilon = \sum_{j=1}^{N} \left(\frac{p_j}{q} \frac{\partial D_j}{\partial p_j} \left[1 - \sum_{k \neq j} \delta_{jk}\right] \right) \tag{8}$$

Using (2) and (6), equation (8) can be rewritten as:

$$\varepsilon = \sum_{j=1}^{N} s_j \varepsilon_j \left(1 - \delta_j\right) \tag{9}$$

where $s_j = q_j/q$ denotes the quantity share of product j.

Equation (9) shows the general relationship between the aggregate elasticity of demand, own-price elasticities, ADRs and the quantity shares of the individual products. Denoting the revenue share of product j by $s_j^* = p_j\, q_j/R$, this relationship can also be expressed in terms of unit-free, revenue-based variables:

$$\varepsilon^* = \sum_{j=1}^{N} s_j^* \varepsilon_j \left(1 - \delta_j^*\right) \tag{9*}$$

As it turns out, the market elasticity of demand is equal to the weighted average of individual products' own-price elasticities multiplied by the respective diversion to outside goods. Intuitively, the market elasticity goes to zero as ADRs go to zero. After all, when ADRs go to zero, changes in price would merely lead to relative shifts in consumption between inside goods, but no consumer would alter its absolute amount of consumption.

We will now explore in more detail the specific implications of this relationship for the determination of ADRs. The following section begins by considering the case of symmetric ADRs.

3. Symmetric Aggregate Diversion Ratios

If one assumes symmetric ADRs (i.e., $\delta_j = \delta$ for all j), equation (9) implies:

$$\varepsilon = (1 - \delta) \sum_{j=1}^{N} s_j \varepsilon_j \tag{10}$$

If each product is sold by a different firm, the profit margin of each product (in Bertrand-Nash equilibrium) is equal to:[8]

$$m_j = 1/\varepsilon_j \tag{11}$$

Using (11) to substitute for ε_j in (10), one obtains:

$$\delta = 1 - \bar{m}\varepsilon \tag{12}$$

where $\bar{m}$ denotes the volume-weighted harmonic average margin:

$$\bar{m} = \frac{1}{\sum_{j=1}^{N} s_j/m_j} \tag{13}$$

Equation (12) is a remarkably simple expression that only depends on the average margin and the market elasticity. In terms of unit-independent variables, we correspondingly have:

$$\delta^* = 1 - \bar{m}^*\varepsilon^* \tag{12*}$$

where $\bar{m}^*$ denotes the revenue-weighted harmonic average margin:

$$\bar{m}^* = \frac{1}{\sum_{j=1}^{N} s_j^*/m_j} \tag{13*}$$

In the special case with symmetric margins $m_j = m$ for all j and thus equations (12) and (12*) apply with $\bar{m} = \bar{m}^* = m$.

We summarize these results in the following proposition.

Proposition 1. *If aggregate diversion ratios are symmetric, i.e., if* $\delta_j = \delta$ *for all* j*, then* $\delta = 1 - \bar{m}\varepsilon$*, where* $\bar{m}$ *denotes the weighted harmonic mean of firms' profit margins and* ε *denotes the market elasticity of demand.*

[8] In (11) and hereafter, we express all elasticities in absolute value and assume finite own-price elasticities so that $m_j > 0$ for all j.

To illustrate the implications of Proposition 1, consider a group of N products (or "market") with an average margin of 40% and a market elasticity of 0.75. Then, under the assumptions leading to Proposition 1, the aggregate diversion ratio is given by $1 - 0.4 \cdot 0.75 = 70\%$. Thus, following a unilateral price increase of one of the products, 70% of the sales lost by that product are captured by the other $N - 1$ products, whereas the remaining 30% are lost to outside goods.

4. Asymmetric Aggregate Diversion Ratios

In many situations, symmetric ADRs will be a plausible assumption.[9] In other situations, however, ADRs may be asymmetric. In particular, this is true in the polar case when there is proportional diversion to outside goods (i.e., when the IIA assumption holds between inside and outside goods). In this case, ADRs are given by the following expression:[10]

$$\delta_j = \frac{1-s_j}{1-s_j+s_0} \tag{14}$$

where s_0 denotes the "share" of outside goods (i.e., the size of the overall "market" of total potential sales is $1 + s_0$).[11] To see that (14) holds, note that the numerator $1 - s_j$ is the share of inside goods other than the product j from which diversion originates, while the denominator $1 - s_j + s_0$ is the total share of inside and outside goods, again

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[9] In particular, when differentiation between products is mostly horizontal from the perspective of customers rather than vertical, it may be natural to assume that diversion to outside goods does not depend on the particular product considered (no matter its market share).

[10] Note that the derivations that follow solely require the proportionality assumption to hold between inside goods (with diversion δ_j) and outside goods (with diversion $1 - \delta_j$). They do not require that it must necessarily also hold among inside goods.

[11] This attribution of a "share" to outside goods is common in models of competition that satisfy the IIA property (e.g., the logit model). As noted in footnote 3, "shares" may be calculated based on total customers or based on the subset of arriving or departing customers, or some other measure that adds up to 1 for inside products.

excluding product j. Under the proportionality assumption, the aggregate diversion ratio must equal the division of those two shares.[12]

The corresponding unit-independent ADR is accordingly given by:

$$\delta_j^* = \frac{1-s_j^*}{1-s_j^*+s_0^*} \tag{14*}$$

An immediate implication of the proportionality assumption that can be seen from (14) and (14*) is that products with higher market share exhibit lower proportional ADRs than products with a lower market share (i.e., δ_j is decreasing in s_j and, equivalently, δ_j^* is decreasing in s_j^*).

Since s_0 is an unknown variable, we again need to relate equation (14) and (14*) to the market elasticity, in order to be able to substitute for s_0 with observable parameters. From (9), (11) and (14), it follows that:

$$\varepsilon = \sum_j \frac{s_j}{m_j} \frac{s_0}{1+s_0-s_j} \tag{15}$$

In terms of unit-independent variables, we correspondingly obtain:

$$\varepsilon^* = \sum_j \frac{s_j^*}{m_j} \frac{s_0^*}{1+s_0^*-s_j^*} \tag{15*}$$

For a given set of parameter values, equation (15) (respectively, (15*)) cannot generally be solved in closed form for s_0 (respectively, s_0^*). However, with a finite number of brands, its recursive formulation can be solved numerically in applications, and the solution can then be substituted back into equation (14) (respectively, (14*)) to obtain a numerical solution for δ_i. We can therefore state the following result.

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[12] Formally, proportionality implies that diversion to outside goods $(1-\delta_j)$ is proportional to their share (s_0). Similarly, diversion to inside goods (δ_j) is proportional to their share $(1-s_j)$. That is, $1-\delta_j = \mu s_0$ and $\delta_j = \mu(1-s_j)$, where μ is the proportionality factor. Solving these two equations for δ_j and μ yields equation (14).

Proposition 2. *If aggregate diversion ratios are proportional to the share of inside and outside goods, then* $\delta_j = (1 - s_j)/(1 - s_j + s_0)$ *for all* j, *where* $s_0 \geq 0$ *is the unique solution of the equation* $\varepsilon = \sum_j s_j s_0 / [m_j(1 + s_0 - s_j)]$.

Proof. In order to prove this proposition, we still need to show that (15) has a unique solution $s_0 \geq 0$.

(i) If $\varepsilon = 0$ then it is easy to see that $s_0 = 0$ is the unique solution of (15) such that $s_0 \geq 0$.

(ii) If $\varepsilon > 0$ then the right-hand side of (15) is smaller than ε at $s_0 = 0$, and greater than ε if $s_0 \to \infty$ since it converges to $\sum_j s_j \varepsilon_j > \varepsilon$. The latter inequality follows because the average own-price elasticity must be greater (in absolute value) than the market elasticity since the products are substitutes (see also equation (9)). As $m_j > 0$ and $0 < s_j < 1$, the right-hand side of (15) is also continuous and strictly increasing in s_0 for all $s_0 \geq 0$. Hence, it has a unique solution $s_0 > 0$. ■

It is instructive to compare how estimated ADRs differ depending on whether the assumption of symmetric ADRs (Proposition 1) or proportional ADRs (Proposition 2) is made. Denoting the former by δ^S (for "symmetric" ADR) and the latter by δ_i^P (for "proportional" ADR), we obtain the following result.

Proposition 3. *(i) When* $\varepsilon = 0$ *or when market shares are symmetric (i.e.,* $s_j = 1/N$ *for all* j*), then proportional ADRs are equal to the symmetric ADR, i.e.,* $\delta_j^P = \delta^S = 1 - \overline{m}\varepsilon$ *for all* $j = 1, \ldots, N$. *(ii) Conversely, when* $\varepsilon > 0$ *and market shares are asymmetric,* $\delta_j^P < \delta^S$ *for products with high market share and* $\delta_j^P > \delta^S$ *for products with low market share.*

Proof. (i) If $\varepsilon = 0$ then (10) implies $\delta^S = 1$, and (15) implies $s_0 = 0$ so that (14) implies $\delta_j^P = 1$. If $s_j = 1/N$ for all j, then solving (15) for s_0 yields:

$$s_0 = \frac{N-1}{N}\frac{\varepsilon}{\sum_j \frac{1}{m_j} - \varepsilon} \tag{16}$$

Substituting this expression and $s_j = 1/N$ into (14), using (13) and rearranging then yields $\delta_j^P = 1 - \bar{m}\varepsilon$. Thus, in both cases, $\delta_j^P = \delta^S = 1 - \bar{m}\varepsilon$, as claimed.

(ii) From (12) and (14), $\delta_i^P < \delta^S$ if and only if:

$$\frac{1-s_i}{1-s_i+s_0} < 1 - \bar{m}\varepsilon \tag{17}$$

Solving this for s_i yields:

$$s_i > 1 - \frac{1-\bar{m}\varepsilon}{\bar{m}\varepsilon} s_0 \tag{18}$$

Hence, $\delta_j^P < \delta^S$ if and only if s_i is sufficiently large. What remains to show is that (18) must hold for the product with the highest market share $\bar{s}$ and must fail to hold for the product with the lowest market share $\underline{s}$. To do so, substitute for $\bar{m}$ and ε from (13) and (15) into (18) and rearrange, which yields:

$$\sum_j \frac{s_j}{m_j} \frac{s_0(s_j - s_i)}{(1+s_0-s_j)(1+s_0-s_j)} < 0 \tag{19}$$

Inequality (19) must hold for $s_i = \bar{s}$ since by the definition of $\bar{s}$, we must have $s_j - \bar{s} \leq 0$ for all j, with $s_j - \bar{s} < 0$ for some j. Similarly, the inequality sign in (19) must be reversed for $s_i = \underline{s}$. ■

Proposition 3 shows that when firms' shares are symmetric or when the market elasticity is zero, the assumption of symmetric ADRs and the assumption of proportional ADRs generate identical results. However, in most real-world merger cases, market shares are asymmetric and the market elasticity is not equal to zero.

Proposition 3 then shows that proportional ADRs will be smaller (larger) than symmetric ADRs for products with higher (lower) market share.

The intuition for this result is easy to see if one considers the extreme example of a quasi-monopoly, where a dominant firm has a market share $s_1 \to 1$ and a small fringe competitor has a market share $s_2 \to 0$. With only two firms, the ADR of a firm is simply the diversion ratio from that firm to the other firm. Thus, when ADRs are symmetric, the diversion ratio from the fringe competitor to the dominant firm is identical to the diversion ratio from the dominant firm to the fringe competitor. Conversely, when ADRs are proportional to shares, lost sales by the dominant firm will be mostly diverted to the outside good (since the fringe competitor is so small), whereas lost sales by the fringe competitor will be substantially diverted to the dominant firm (since the dominant firm is so large). Hence, the diversion ratio from the dominant firm to the fringe competitor is close to zero, whereas the diversion ratio from the fringe competitor to the dominant firm is substantial.

Finally, note that Proposition 3 implies that, all else equal, proportional ADRs lead to less (more) pronounced merger effects than symmetric ADRs when the merging firms' market shares are large (small). Indeed, with proportional ADRs, large (small) shares imply lower (higher) ADRs, which in turn imply lower (higher) individual diversion ratios. Consequently, the predicted closeness of competition is smaller (larger) and hence the predicted merger effect is less (more) pronounced.

5. Application to Competitive Effects in Merger Control

The results derived in the previous sections can be readily applied to competitive effects analyses in merger control and other areas of antitrust policy where diversion ratios play a central role. Note, in particular, that the diversion ratio from product i to product j is given by the following expression when the proportionality assumption holds between inside goods (i.e., when goods are equally close substitutes to each other):[13]

$$\delta_{ij} = \frac{s_j}{1-s_i}\delta_i \tag{20}$$

From (20) it is obvious that individual diversion ratios δ_{ij} cannot be meaningfully quantified without sensible estimates of ADRs δ_i. Propositions 1 and 2 above can then be readily applied to obtain such estimates.

When it is sensible to assume symmetric ADRs (i.e., when there is no reason to presume that some inside goods have appreciably different diversion to outside goods than others), Proposition 1 implies that (20) can be expressed as follows:

$$\delta_{ij} = \frac{s_j}{1-s_i}(1-\bar{m}\varepsilon) \tag{21}$$

This formulation of diversion ratios solely depends on market shares, margins and the market elasticity, all of which can typically be observed with a reasonable degree of accuracy even at the early stages of antitrust investigations.[14] Note, in particular, that (21) is independent of cross-price elasticities and more complex curvature

[13] When goods are asymmetrically positioned in terms of closeness of competition, equation (20) continues to hold if one replaces market shares with shares of relative diversion between inside goods. As noted earlier, estimates of such shares often become available at the later stages of an investigation, since many firms track the identity of customer gains and losses from and to rivals.

[14] Market shares and margins are routinely requested in merger control proceedings. Moreover, there is a large literature surveying empirical estimates of market elasticities in a plethora of different industries (e.g., see Clements, 1998, for a meta-study). Clements proposes to use an elasticity of minus one-half as a prior in the absence of more specific information, since real-world market elasticities tend to be scattered around minus one-half in the majority of cases.

properties of demand functions, whose estimation would require the availability of much richer data sets and the use of advanced econometric techniques.

Similarly straightforward diversion metrics can be derived in the case of proportional diversion between inside and outside goods (in which case ADRs are asymmetric). In this case, Proposition 2 implies that (20) can be expressed as follows:

$$\delta_{ij} = \frac{s_j}{1 - s_i + s_0(\cdot)} \tag{22}$$

where $s_0(\cdot)$ is defined implicitly by equation (15). As it turns out, this expression depends on the same simple parameters as (21).

In applied analyses, equations (21) and (22) can be used in two ways. First, at the early stages of an investigation, they permit computing concrete estimates of competitive effects via established scoring methods such as UPP, CMCR or GUPPI, using only minimal information (market shares, margins and the market elasticity of demand). By construction, assuming proportionality for inside goods on the basis of market shares implies that such analysis does not take account of the relative closeness of competition of different products. However, this technique can nonetheless constitute a useful and simple way of calculating a *lower bound* for merger effects in instances where there is no reason to assume that the merging parties offer particularly remote substitutes.[15]

Second, at more advanced stages of a merger proceeding, patterns of substitution from one product to another often become available (e.g., in the form of switching data such as gross additions of subscribers or port-out rates). It is then possible to

[15] Moreover, it will in any event be a more precise measure of competitive effects than alternative, purely concentration-based metrics such as market shares or HHIs. After all, concentration-based measures ignore the additional information contained in margins and market elasticities, although they tend to be highly informative about the degree of competition in a market.

replace market shares in Equations (21) and (22) with those more precise estimates of diversion between inside goods. This addresses the potential shortcoming of the proportionality assumption and produces more reliable estimates of merger effects by taking proper account of differentiated patterns of closeness of substitution.

6. Application to Market Share Thresholds

Beside their use in antitrust practice, the results of this paper can also be used in more general analyses of the economic effects of mergers. For instance, a series of recent empirical papers has documented a persistent increase in corporate profits in western economies over the past 30 years and a corresponding decrease in the share of returns obtained by labor.[16] This secular shift in rents from labor to capital has led to public calls by prominent economists for more vigorous merger control enforcement.[17] In particular, it has been argued that antitrust authorities should more aggressively oppose industrial concentration in a world with higher profit margins, to prevent anticompetitive rent extraction.

Unfortunately, the economic literature contains few workable results on the relation of concentration and margins with general applicability.[18] Yet, the results of the previous sections do provide some insight into how profit margins and market concentration interact when one assesses the economic effects of a merger. We will illustrate this in a simple variant of our model with symmetric firms.

[16] See Karabarbounis and Neiman (2014) and Autor et al. (2017) and the references cited therein.

[17] E.g., see Baker and Shapiro (2008), Stiglitz (2012) and Piketty (2014).

[18] It has even been argued that concentration-based measures should be avoided entirely in assessing the economic effects of mergers (see Kaplow 2010, 2015).

Consider an industry with N symmetric firms where two of the firms intend to merge.[19] Symmetry implies $\delta_{ij} = d$ and $m_i = m$ for all i, j. Under these circumstances, Werden (1996) has shown that the compensating marginal cost reduction of a merger is given by:

$$\frac{\Delta c}{p} = m \frac{d}{1-d} \tag{23}$$

The CMCR measures the price-increasing effect of a merger in terms of the countervailing efficiency that would be required to offset it. For instance, if $\Delta c/p =$ 2% and the pre-merger price of the product is 100 USD, then the transaction would have to permit a reduction in marginal cost of 2 USD to ensure that post-merger prices do not increase on account of enhanced market power.

Equation (23) shows the *direct effect* of higher margins on the competitive consequences of mergers. As the right-hand side of (23) is increasing in m, the direct effect is positive, as one might expect: the higher the merging firms' margins, the more damaging a merger will be for competition (all else equal). This is because higher margins induce merging parties to be more reluctant to compete for incremental sales post-transaction, since aggressive pricing would then jeopardize the more profitable sales of their partner. All else equal, higher margins therefore imply that a given merger will lead to stronger price increases.

It is important to realize, however, that there is also an *indirect effect* caused by higher margins, since in fact "all else is not equal". Specifically, note that diversion ratios in (23) are endogenous and in general depend on the size of margins, so $d = d(m)$. To determine this indirect effect, note that by Proposition 1, the ADRs of symmetric firms are given by (12). As a result, diversion ratios are given by (21).

[19] By focusing on the number of firms, we can consider simple questions such as "what does a 4-to-3 merger correspond to in terms of economic effects in a world where margins have increased?"

According to (12), higher margins imply a lower ADR, which by (21) leads to lower diversion ratios. From (23), however, it then follows that the anticompetitive effect of a merger is *reduced* when margins are higher. The intuition for this countervailing effect is that firms with higher margins must face less competition pre-merger (otherwise they could not have charged higher margins). This lack of closeness of competition softens the price-increasing effect of mergers, since the merging parties produce more remote substitutes.

Noting that $\bar{m} = m$, $s_i = 1/N = s$ and $\delta_{ij} = d$, we can substitute (21) into (23) and rearrange, which yields an expression that takes account of the endogeneity of diversion ratios:

$$\frac{\Delta c}{p} = m \frac{sm(1-m\varepsilon)}{1-s(2-m\varepsilon)} \tag{24}$$

Equation (24) reflects the fact that—for a given level of market shares and demand elasticity—the size of margins affects competitive effects in the two opposing ways just described. Indeed, it is simple to show that the derivative of (24) with respect to m is generally indeterminate. Perhaps surprisingly, therefore, higher margins must not generally imply that concentration will be more harmful to competition, as is often assumed in policy debates.

This notwithstanding, using realistic values for the parameters in (24) indicates that the actual increase in profit margins that has been observed in recent years appears likely to make stricter merger control enforcement desirable (taking as given the standard on preventing anticompetitive effects).

To see this, consider the following numerical example. Suppose the two merging companies have a combined market share of 50% (which in legal procedures is often considered to be indicative of a competition concern).[20] Noting that net profit margins in the US have roughly increased from 6% to 9% over the course of the last 25 years, let us consider an increase in margins by 50% in the model.[21] We will therefore consider the competitive effects of a merger of two firms with combined market share of 50% before and after an increase of pre-merger margins by 50% (taking as given the level of market elasticity).

On the basis of these parameter values, we can now use equation (24) to consider which level of concentration corresponds to a 50% combined share if one wants to prevent an anticompetitive effect of the same order of magnitude as before. Table 1 below displays the market share threshold that corresponds to a 50% threshold in terms of anticompetitive effect when margins have increased by 50% according to (24).

	Elasticity		
Profit margin	$\varepsilon = 0.25$	$\varepsilon = 0.5$	$\varepsilon = 0.75$
m = 10%	40%	40%	41%
m = 20%	40%	41%	41%
m = 30%	41%	41%	42%
m = 40%	41%	42%	44%
m = 50%	41%	43%	45%

TABLE 1: IMPLIED CONCENTRATION THRESHOLDS EQUIVALENT TO A 50% THRESHOLD AFTER AN INCREASE OF MARGINS BY 50%

As the entries in Table 1 indicate, an increase in margins by 50% significantly lowers the market share threshold that corresponds to a 50% combined

[20] A 50% concentration threshold is regarded as indicative of market dominance both under EU and US antitrust law. See Case C-62/86 AKZO Chemie BV v. Commission [1991] E.C.R. I-3359, para. 60 (holding that a 50% market share is in itself evidence of a dominant position "save in exceptional circumstances") and Werden (2002) (summarizing US case law requiring a 50% market share in monopolization cases). Here, we do not take any position on the sensibility of this standard (or lack thereof), but merely take the prominent benchmark as a numerical example.

[21] See https://www.yardeni.com/pub/sp500margin.pdf for evidence on the change in profit margins.

share before the increase in margins (taking as given the anticompetitive effect that is supposed to be prevented). Indeed, the anticompetitive effect caused by a merger with 50% share is equivalent to the anticompetitive effect caused by a merger with only 40-45% when margins have increased. This suggests that substantial increases in economy-wide profit margins may well call for a less permissive approach towards industrial concentrations in order to meet the same economic standard on anticompetitive effects.

7. CONCLUSION

In this paper, we have determined the general relation between aggregate diversion ratios and the market elasticity of demand. Moreover, we have provided simple solutions for estimating ADRs in merger control proceedings, both for the case of symmetric ADRs and for the case of asymmetric ADRs. These results allow approximating firms' diversion ratios on the basis of simple, observable variables such as market shares, profit margins and the market elasticity of demand, without having to resort to ad-hoc assumptions on ADRs. In applied work, this is useful because most quantitative tools for assessing the competitive effects of mergers heavily rely on diversion ratios between merging products as a key parameter. Moreover, the relation between aggregate diversion ratios and market elasticity can also be fruitfully used in theoretical work focusing on the impact of different parameters on the competitive effects of mergers. We provide an example that analyzes the implications of higher profit margins for market share thresholds in merger control.

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